Controlling wave packets on swept wings

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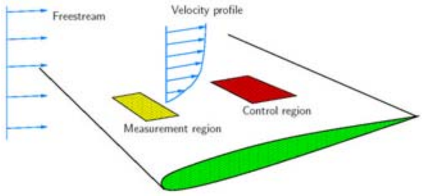
INTRODUCTION

We focus on the problem of controlling travelling wave packets on an infinite swept wing using linear control theory. Based on wall measurements , an extended Kalman filter is used to estimate the 3D wave packet. The estimated field is in turn used to calculate a feedback control which changes the growth of the disturbance into decay. It is the first time that optimal control and estimation concepts is successfully applied to construct a dynamic output feedback compensator which is used to control disturbances in spatially-developing boundary layers.



Applications:

- Maintain laminar flow on aircraft wings.
- Relaminarize/ decrease drag in turbulent flows.
- Enhance mixing in turbulent flows / separation control

CONTROL THEORY

We use the linearized Navier–Stokes equations in the form of the Orr–Sommerfeld/Squire equations. The blowing and suction boundary condition (φ) is lifted through linear super-position into the domain and the equations can be expressed in the standard form for control theory for each wavenumber pair as,

$$\dot{\hat{\mathbf{x}}} = \hat{\mathbf{A}}\hat{\mathbf{x}} + \hat{\mathbf{B}}\hat{\phi}, \quad \hat{\mathbf{x}} = [\hat{\mathbf{v}}, \hat{\eta}, \hat{\varphi}]^{\mathbf{T}}, \quad \hat{\phi} = \frac{\partial \hat{\varphi}}{\partial \mathbf{t}}.$$

Our goal is to minimize the objective function:

$$\begin{aligned} J(\varphi) &= \frac{1}{2}\int_0^T \int_\Omega |\mathbf{u}|^2 \, d\Omega \, dt + \frac{\ell^2}{2}\int_0^T \int_\Gamma \left|\frac{\partial \varphi}{\partial t}\right|^2 d\Gamma \, dt \\ &= \frac{1}{2}\int_0^T (\hat{\mathbf{x}}^* Q \hat{\mathbf{x}} + \ell^2 \hat{\phi}^* \hat{\phi}) \, dt, \end{aligned} \quad (1)$$

If $T \to \infty$ the optimal controller is given through

$$\hat{\phi} = \hat{K}\hat{\mathbf{x}}, \quad \text{where} \quad \hat{K} = -\frac{1}{\ell^2}\hat{B}\hat{X}$$

and $\hat{X}$ is the positive self adjoint solution to the Riccati equation,

$$\left(\hat{X}\hat{A} + \hat{A}^*\hat{X} - \hat{X}\frac{1}{\ell^2}\hat{B}\hat{B}^*\hat{X} + \hat{Q}\right)\hat{\mathbf{x}} = 0, \quad \forall \ \text{admissible} \ \hat{\mathbf{x}}.$$

To find the optimal estimator we solve a similar problem for the estimated flow $\hat{\mathbf{x}}_e$:

Model

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_h &= \hat{A}\hat{\mathbf{x}}_h + \hat{B}_1\hat{\mathbf{w}} + \hat{B}\hat{\phi} \\ \hat{\mathbf{y}}_h &= \hat{C}\hat{\mathbf{x}}_h + \hat{D}\hat{\mathbf{w}}, \end{aligned}$$

Estimator

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_e &= \hat{A}\hat{\mathbf{x}}_e + \hat{B}\hat{\phi} - \hat{\psi}, \\ \hat{\mathbf{y}}_e &= \hat{C}\hat{\mathbf{x}}_e, \\ \hat{\psi} &= \hat{L}\Delta\hat{\mathbf{y}} = \hat{L}(\hat{\mathbf{y}}_h - \hat{\mathbf{y}}_e). \end{aligned}$$

With the feedback law:

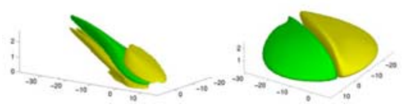
$$\hat{L} = -\frac{1}{\alpha^2}\hat{Y}\hat{C}^*,$$

where $\hat{Y}$ is the positive self-adjoint solution to the Riccati equation,

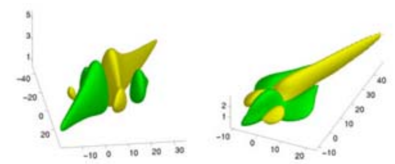
$$\left(\hat{A}^*\hat{Y} + \hat{Y}\hat{A} - \hat{Y}\frac{1}{\alpha^2}\hat{C}^*\hat{C}\hat{Y} + \hat{B}_1\hat{B}_1^*\right)\hat{\mathbf{x}}_e = 0, \quad \forall \ \text{admissible} \ \hat{\mathbf{x}}_e$$

CONVOLUTION KERNELS

Get a physical space representation of the control law, by inverse Fourier transform



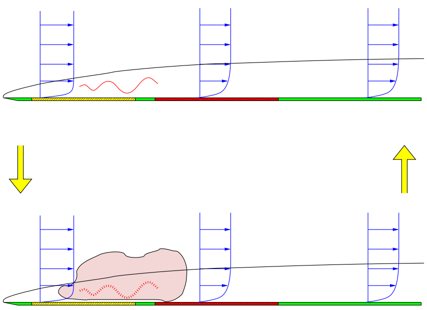
Control convolution kernels for normal velocity (left) and normal vorticity (right). The control signal for each point on the wall is computed through a three dimensional convolution integral of these kernels and the corresponding velocity/vorticity components at each time step.



Estimator kernels based on measurement of wall normal derivative of normal vorticity for forcing the normal velocity component (left) and the normal vorticity (right). The forcing is computed through a two dimensional convolution of these forcing kernels and the measurement error.

DYNAMIC COMPENSATOR

We combine the controller and estimator into a compensator. We apply the estimator forcing given from the linear problem in nonlinear DNS, *i.e.* extended Kalman filter. The schematic figure below illustrates the process of the compensator where the upper boundary layer represents the flow we wish to control and the lower one represents the estimator.



1. Get difference in measurements from both flows
2. Apply estimator forcing and compute control signal
3. Apply control signal in both simulations

TEST CASE

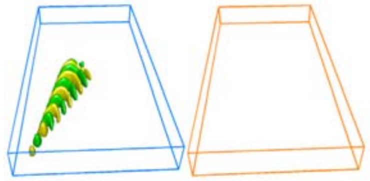
In the simulation we march the flow and its estimator simultaneously. A well established spectral DNS code has been used in all simulations.

- Spatial Falkner–Skan–Cooke flow, $Re_{\delta^*} = 499$, $m = -0.0312$, $W_\infty = 0.5\,U_\infty$
- Box: $x \in [0, 400]$, $z \in [-100, 100]$, $y \in [0, 10]$
- Flow solver and estimator simulated in parallel
- Localized volume force generates wave-packet in flow solver
- Wall measurement in $x \in [0, 100]$ in both simulations
- Volume force in estimator given by measurement error.
- Control from estimator state, applied in $x \in [100, 200]$ in both simulations

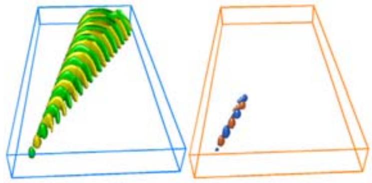
RESULTS

Snapshots showing iso-surfaces of the normal velocity at instants of time.

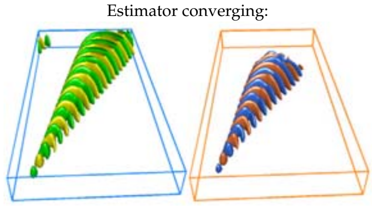
Perturbation propagating:



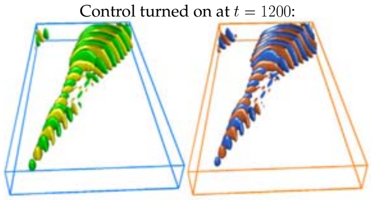
Estimator turned on at $t = 600$:



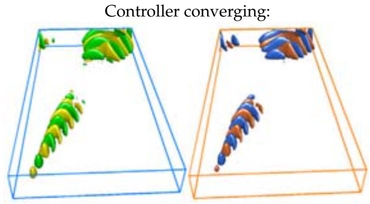
Estimator converging:



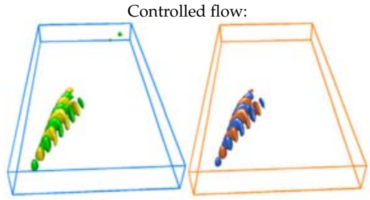
Control turned on at $t = 1200$:



Controller converging:



Controlled flow:



PUBLICATIONS

- Linear optimal control applied to instabilities in spatial boundary layers. *J.* Fluid Mech. 470 151-179 (2002)
- Linear compensator control of a pointsource induced perturbation in a Falkner-Skan-Cooke boundary layer *P*hys. Fluids 15, 2449 (2003)
- Relaminarization of $Re_\tau = 100$ turbulence using gain scheduling and linear state-feedback control *P*hys. Fluids 15, 3572 (2003)
- 2002 Linear feedback control and estimation of transition in plane channel flow *J.* Fluid Mech. 481, 149 - 175 (2003)
- State estimation of wall bounded flow systems. Part 1. Laminar flows *J.* Fluid Mech. 534 263–294, (2005)