Active Control of Thermocapillary Convection

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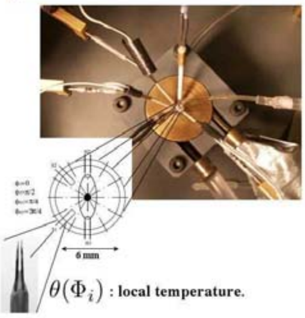
General aspects of the study

We have studied the active control of instabilities of a thermocapillary-driven flow, in a spatially-constrained geometry. The liquid (silicon oil, 1 cSt) lies in a cylindrical container, where a temperature gradient is applied: the periphery is warmed and the center is cooled. As the surface tension (γ) is temperature-dependent, a force is resulting from the temperature gradient on the surface, driving the fluid from zones of low γ to zones of high γ. The control parameter is the temperature difference ΔT. At low temperature difference, the flow is steady and axisymmetric, from periphery to center. When ΔT overcomes a certain threshold, the flow becomes unstable and undergoes successive bifurcations, from standing and traveling waves, to temporal chaos, through a period-doubling scenario. The control of such instabilities is achieved by a feedback scheme, using hot wires: a pair of sensors acquire the local temperature, and a pair of heaters inject a local heating. The optimal conditions are obtained by playing on sensors/heaters relative positions as well as on weights of *linear, non-linear and derivative terms* in the feedback law. Besides fundamental interests of control achievement in a benchmark flow, the study is motivated by practical purposes of eliminating striations during crystal productions and on undesirable flows in welding pools.

CONTROL PROCEDURE



$\theta(\Phi_i)$: local temperature.

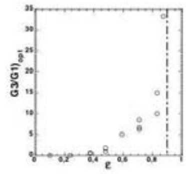
Control law with 2 sensors/ 2 heaters :

$$S = -G_1\theta(\Phi_i) - G_3\theta(\Phi_i)(\theta(\Phi_1)^2 + \theta(\Phi_2)^2)$$

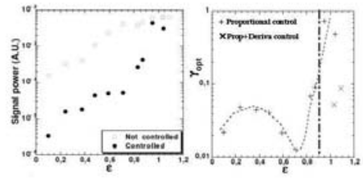
(S: locally injected power)

⇒Seek for an optimum, playing with G1 and G3

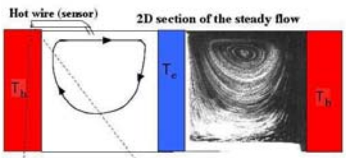
CONTROL PERFORMANCES

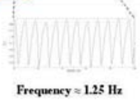


Optimal conditions need a more and more non-linear contributions at increasing ε.



STRUCTURE OF THE FLOW: STEADY AND OSCILLATORY STATES



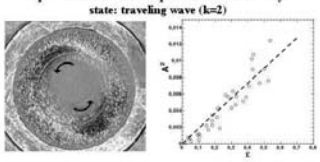


T_h-T_c = $\Delta T > 17$°C, Appearance of a oscillatory flow

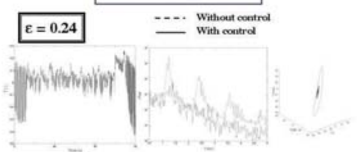
Frequency ≈ 1.25 Hz

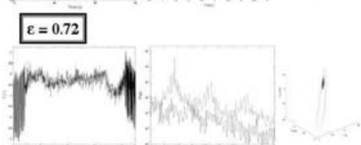
$$\left(Ma = \frac{\gamma \Delta T R}{\mu\alpha} \simeq 44000\right)$$

Spatial structure and amplitude of the oscillatory state: traveling wave (k=2)

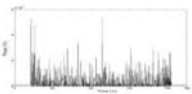


RESULTS





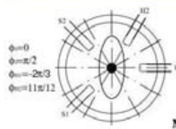
Electrical power, injected during the control



Measure of control performances, from frequency power spectra:

$$\gamma_{opt} = \frac{\int_{0.5}^{5} Psd_c(f)df}{\int_{0.5}^{5} Psd_{nc}(f)df}$$

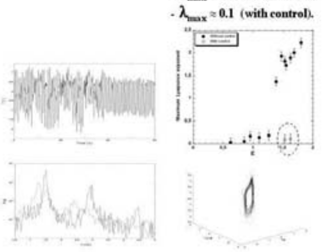
SYNCHRONISATION OF CHAOTIC STATES



Suitable sensors/heaters relative positions

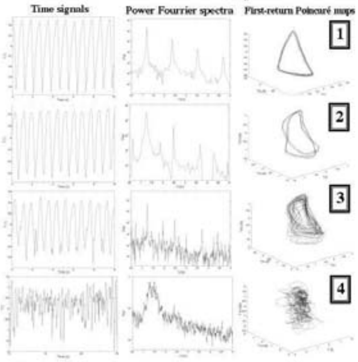
Maximal Lyapunov exponents

- $\lambda_{max} \approx 2$ (without control).
- $\lambda_{max} \approx 0.1$ (with control).



STRUCTURE OF THE FLOW: SUCCESSIVE INSTABILITIES

Dimensionless control parameter: $\epsilon = \dfrac{\Delta T - \Delta T_c}{\Delta T_c}$

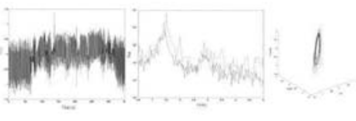


1 : Oscillations ($\Delta T = 26$°C, $\varepsilon = 0.53$).
2 : Period Doubling ($\Delta T = 33.9$°C, $\varepsilon = 0.99$).
3 : Complex combination of frequencies ($\Delta T = 35.75$°C, $\varepsilon = 1.10$).
4 : Chaos ($\Delta T = 47.6$°C, $\varepsilon = 1.80$).

RESULTS - PROPORTIONAL/DERIVATIVE CONTROL

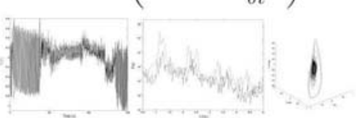
ε = 1.03

Limit of efficiency for a proportional control law



Alternative feedback law, suitable far from threshold: proportional+derivative combination

$$S = -G_1\left(\theta(\phi_i) + a\frac{\delta\theta(\phi_i)}{\delta t}\right)$$



CONCLUSIONS

-Study of successive bifurcations in a thermocapillary flow in a confined geometry: steady flow, oscillatory flow (standing or traveling wave), period-doubling and chaos.

-Significative atenuation of oscillations, even far from threshold ($\varepsilon = 0.9$), until occurences of secondary bifurcations (period-doubling).

-Feedback law needs more and more complex terms (nonlinearities, time-derivative terms) as ε increases, for an optimal control.

-Possible synchronisation of chaotic states, and reduction of impredictable behaviors.