A Simple Model of the Mechanics of Trombone Playing

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Abstract

To study human motion in general, the motion when playing the trombone has been examined. This makes it possible to work with a simple mechanical model and thus get results that are easier to interpret. Calculations from the model have been compared to measurements from experiments.

The arm-trombone system consists of a rod, tilted at a fixed angle, and two bars, connected by a hinge, that represents the arm. The shoulder consists of another hinge and is placed level with the trombone. The hand is allowed to slide without friction along the trombone. The system has only one degree of freedom and the behavior is similar to that of a pendulum. Energy can be added to the system, by applying an impulse in the beginning of a motion. Apart from that, gravity is the only active force. Under these conditions the equations of motion for the system have been calculated .

Two subjects took part in the experiment – a professional trombone player and a student. They played three types of musical note sequences: 1) different movements between the seven possible positions, 2) a short musical excerpt, and 3) randomly generated notes. The 3D trajectories of the six measured points (LED), placed on the trombone and the right arm, where recorded by an Optotrak system. The experiments where simultaneously recorded on video.

When comparing the models and the subjects motions, the hands displacement along the trombone was chosen as the best suited variable to examine. The agreement turned out to be good, especially for slow motions.

The results imply that gravity provides the main force and control mechanism used in trombone playing. Skilled trombonist use less energy than less skilled, which can be assumed to depend on that they have learned to optimize their own force input and take more advantage of the force supplied by gravity.

Finally, different ways to expand the present study, are discussed.

Sammanfattning

I syfte att studera allmän mänsklig rörelse, har armrörelsen hos en person som spelar trombon betraktats. Detta möjliggör en enkel mekanisk modell, vilket ger mer lättolkade resultat. Beräkningar på modellen har jämförts med mätningar från experiment.

Arm-trombonsystemet består av en stång, som lutad en fix vinkel, representerar trombonen och armen utgörs av två stela stavar, ihopkopplade med ett gångjärn. Axeln är placerad på samma höjd som trombonen och utgörs också den av ett gångjärn. Handen tillåts glida friktionsfritt längs trombonen. Systemet har bara en frihetgrad och beteendet liknar en pendels. I början av en förflyttning kan energi tillföras systemet genom en impuls, i övrigt verkar endast gravitation. Utifrån dessa förutsättningar har systemets rörelseekvationer beräknats.

I experimentet deltog två subjekt – en professionell trombonist och en student. De spelade tre typer av notföljder: 1) olika förflyttningar mellan de sju

möjliga positionerna, 2) kort musikaliskt utdrag, samt 3) slumpvis vald sekvens av positioner. 3D-kurvorna för sex olika mätpunkter (LED), placerade på trombonen och höger arm, registrerades med ett Optotrak-system. Experimenten videofilmades samtidigt.

I jämförelsen mellan modellens och subjektens rörelser har handens förflyttning längs trombonen valts som lämplig variabel att studera. Överensstämmelsen har visat sig vara god, särskilt för långsamma rörelser.

Resultaten troliggör att gravitation är den huvudsakliga kraften som inverkar under trombonspelande. Att skickliga trombonister använder mindre energi än mindre skickliga kan antas bero på att de har lärt sig att minimera den kraft de själva tillför och låter gravitionskraften ta en större del.

Slutligen diskuteras möjliga sätt att utveckla studien vidare.

Contents

Chapter 1

Introduction

Understanding human motion is both interesting and important. Interesting because moving is something we do daily and thereby is of common concern. Important because a complete understanding of human motion would be a great help in the rehabilitation of people with injuries and in the development of prothesis. It could also be applied to robotics. More specifically, the results of this thesis, may assist in the training of trombone players. Evolution has shaped human motion into a system well adapted to the conditions on the planet. If we were able to understand and apply that kind of motion to robots a lot of complicated control theory might even be avoided.

Human motion is in many ways both complex and hard to model. This is due to the large number of parts the body consists of and the fact that most of these parts can be moved relative to each other. To model the whole human body, moving about during daily activity, would be near impossible and result in an unbearable amount of calculation. For this reason it is useful to study simpler movements, that help our insight. A study of a simple human motion can thus be valuable.

In this work we study a trombone players arm movements. There were several advantages to this choice. A trombone player moves only the arm and the shoulder, the rest of the body is relatively fixed. The number of possible motions are strictly limited - the hand is allowed to move only along the straight line that is the trombone. Simple observations suggest that the motion is almost planar, which made it plausible to study the motion in a two dimensional model.

A theoretical, mechanical model of the arm and trombone was thus developed. The aim was to construct a model that was as simple as possible, but still with a behavior matching reality. The simplicity of the model allowed analytical calculations and simplified numerics.

As can be seen in figure 1.1, a rod, tilted at a fixed angle, represented the trombone and two bars, connected by a hinge, represented the arm. The 'shoulder' was a hinge, level with the trombone and a 'hand' was allowed to slide without friction along the trombone.

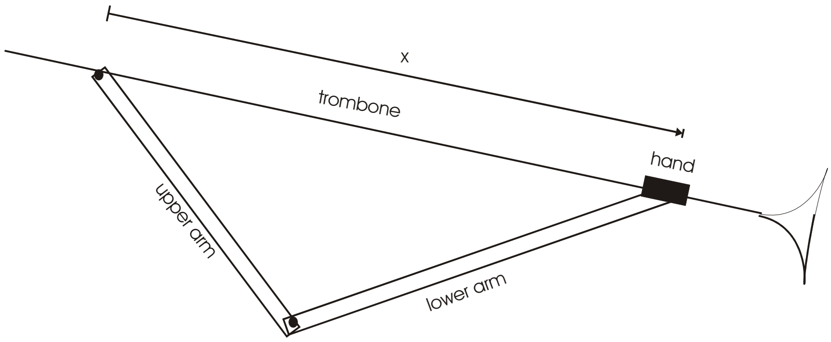

Figure 1.1: The mechanical model.

This system has many of the characteristics of a simple pendulum. Gravity and an impulse in the beginning of each transfer to a new position along the trombone provided the essential forces acting on this system.

Thanks to cooperation with Doctor Virgil Stokes, at the Department of Systems and Controls, Uppsala University, it was possible to compare the calculations with the measurements done on two trombone playing subjects.

The system, as set up, has only one degree of freedom, which made it possible to study any variable and thereby get information about the movement of the entire system. The displacement of the hand along the trombone, x, was chosen as the variable of interest. In figure 1.2 the behavior of the model system is shown, compared to the movement of a professional trombonist, playing a random sequence. For each note the equations of motion were solved with the specific initial conditions. As will be seen the result show much similarity, which implies that gravity provides an important force used in trombone playing. In the present model it is assumed that the player inserts, if necessary, an impulse in the beginning of the movement and then lets the arm move under the influence of gravity to the new desired position. The ideal must be to reach the next position at zero velocity, thereby allowing a smoother adjustment to the exact position, than would be obtained by braking. A likely implication of these results is the great importance gravity has in all motion. Humans evolved in a gravity field and were designed to be energy efficient in it. Therefore it is natural that gravitationally influenced motion is intrinsic to movement.

Parts of this material has been submitted to the International Society of Biomechanics XVIIIth Congress [12].

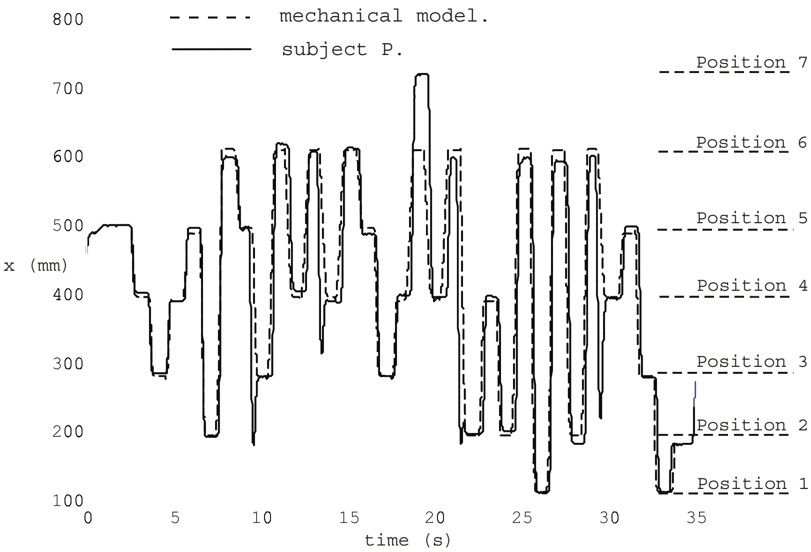

Figure 1.2: Model compared to measurements.

Chapter 2

Trombone

2.1 History

The trombone is a brass instrument with a long history. The first instruments using the principle of a slide, i.e. with a movable part changing the length of the tube in which the sound is produced, were already developed in the 15th century. This early instrument goes by the name of ‘ the sackbut’. Only a few instruments have been preserved from the 16th century. In all essential parts these instruments are almost identical with modern trombones, except for the bell being smaller.

Up to the 18th century the trombone was used mainly in the church. Often a group of trombones accompanied the choral. By the end of the 18th century the trombone became common in military bands, and as a result of this, it was given a more robust design. Sometimes the bell was even given the shape of a dragons head, with bared teeth and a flapping tongue. The trombone made its way into orchestras and in France it was used, together with other wind instruments, for dance music. In Germany the trombone became important in music for the people, as being an instrument allowed outside of the church, apart from e.g. the trumpet.

Between 1825 and 1830 a tuning-slide on the U-bend and a water key on the lower end of the slide were invented. Because of the usefulness of these improvements they became very general.

Today the trombone is widely used. It has a given place in both jazz and symphony orchestras and is also popular as a solo instrument. Although the great differences in usage no essential improvements has been, or maybe could have been, made over the last 500 years.

With this long history [1] it can be expected that in both usage and function it has been considerably optimized for human use.

2.2 Function

The trombone is a brass instrument and is based on the principle of standing waves in a tube. The tone can be changed by either changing the inflow of air trough the mouthpiece or by adjusting the length of the tube.

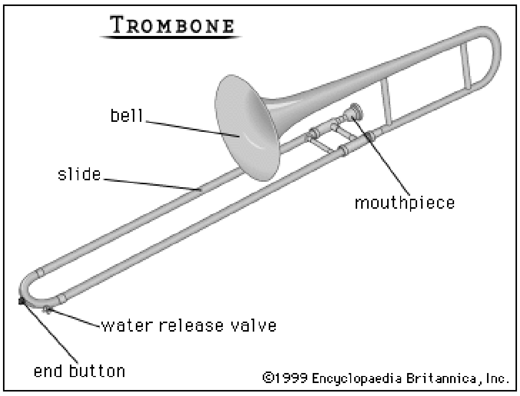

Figure 2.1: The different parts of the trombone.[3]

The essential parts of a trombone are the mouthpiece, the slide and the bell, as can be seen in figure 2.1. The slide can be moved continuously, which makes the trombone a chromatic instrument. It is most commonly played using *seven* positions, where position one is with the slide close to the mouthpiece and position seven with the slide fully extended. The distances separating the positions varies distances further out on the trombone. There are no markings for the positions and they must be memorized. A small adjustment using feedback from hearing the tone is possible and common. [6]

The trombone is held by the left hand and the slide is maneuvered by the right. (Even left-handed trombonists usually play the trombone in this way.) The right hand is best placed loosely around a stay and the trombone is often held tilted downwards, [6] The friction between the slide and the tube is low and it takes only a small force to move the slide. More important are precision and agility. To move the slide, the whole arm is used, and to some extent the wrist and shoulder. According to Kruger et. al. [8] the wrist is used more in

[1] The trombones history, as described here, is a summary of Adam Carse's more thorough treatment of the subject [4].

moving between the lower positions and at position seven. Position seven can be a bit of a problem for many trombonists. To fully extend the slide, in a comfortable way, long arms are required. All players extend the shoulder and wrist when playing position seven. Players with shorter arms also have to make use of their fingers to move the slide the last centimeters, which of course is a bit more complicated as can be seen in figure 2.2. Position seven is not used very often, though, and is not stressed for learners.

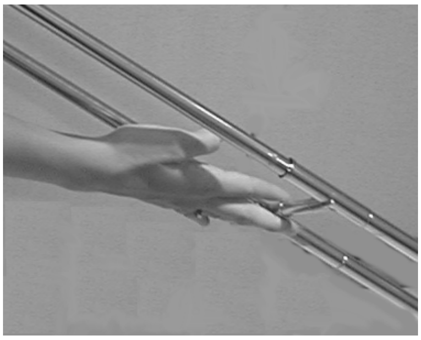

Figure 2.2: Position seven can be hard to reach.

From this we can expect a trombone players motions to be energy saving and the force used moderate. As precision is important, a difference between skilled and less skilled players can be expected in finding the exact positions. The mechanical model used, does not include a movable wrist and shoulder and a disagreement between model and measurements is likely, especially for position seven.

Chapter 3

The human arm and shoulder

The human arm and shoulder is a very complex system consisting of a large number of joints and muscles, that all take some part in the motion required to play the trombone. This chapter is to give the reader a possibility to compare the mechanical model to what it aims to be a model of. For a more thorough description see Susan Hall [5].

3.1 Shoulder

Full mobility in the shoulder means that it can be moved both up and down and forwards and backwards. The scapula can also be rotated to enable lifting of the arm. This makes the shoulder joint, which actually consists of four joints, the most complex in the human body. The sternoclavicular joint is a ball and socket joint connecting the clavicle to the sternum, that enables rotation of the clavicle. The acromioclavicular joint and the coracoclavicular joint are the contact between the clavicle and the scapula. These two joints are not very mobile. The glenohumeral joint, connecting the scapula to humerus, is what is commonly called the shoulder joint. It is a ball and socket joint, but very loosely fitted, which allows the ball to glide in the socket. This makes the shoulder very mobile, but also gives minimal stability, i.e. it is more easily displaced than other joints. The instability is to some extent compensated by a capsule of muscles that surrounds the joint. Many muscles in the shoulder have an antagonist to help keeping the joint in its place during stress. The muscles takes part in several motions, and their action can also depend on the orientation of the joint. For most shoulder motions, all shoulder joints are included.

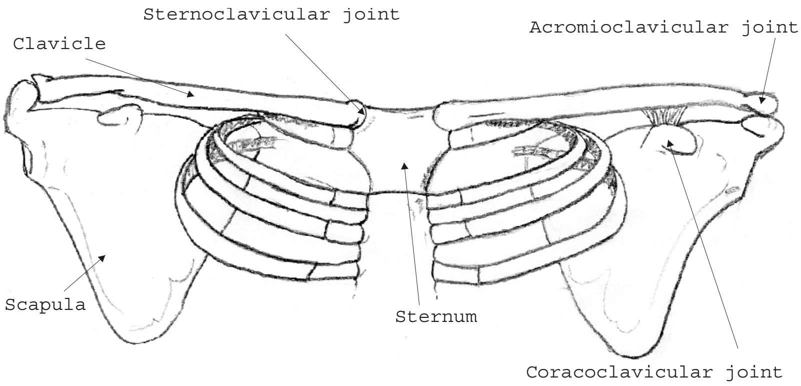

Figure 3.1: Joints connecting the sternum, clavicle and scapula.

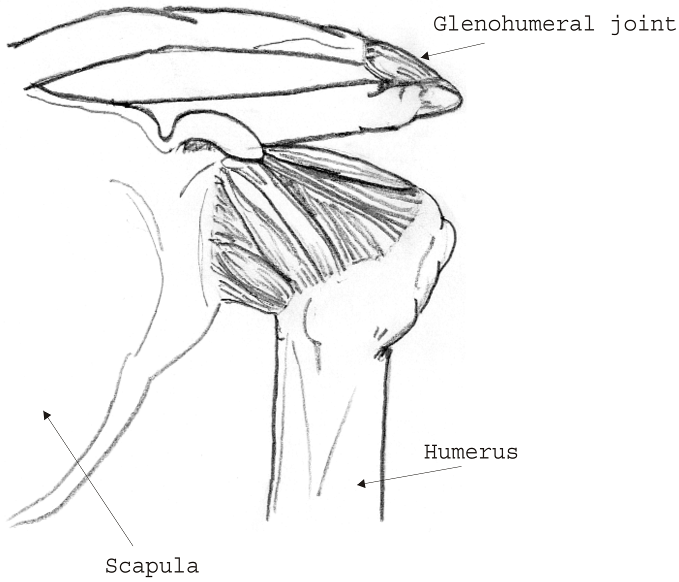

Figure 3.2: The glenohumeral joint.

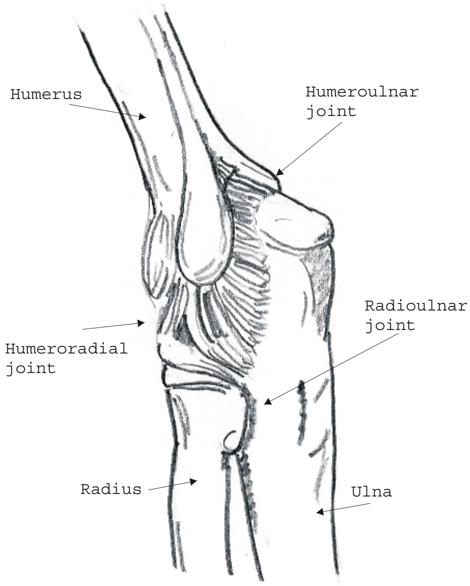

Figure 3.3: The elbow joints.

3.2 Elbow

The elbow consists of three joints in one joint capsule. The humeroulnar joint is considered the main elbow joint. It's a hinge joint between the humerus and the ulna and is used for flexion and extension of the arm. The humeroradial joint is a glide joint between the humerus and the radius, placed just next to the humeroulnar joint, and allows gliding in only the sagittal plane. The radioulnar joint is a pivot joint between the radius and the ulna and it is in this joint that the radius rolls around the ulna to enable rotation of the lower arm.

Many muscles pass the elbow, several also passing the shoulder and wrist. The main, and strongest, arm flexor is the brachialis, that goes between the humerus and the ulna. The biceps brachii, between the scapula and the radius, also flexes the arm, but only when the face of the hand is pointing upwards. The brachioradialis connects the humerus and the styloid process. It is most effective in flexing the arm, when the lower arm is in a neutral position, i.e. the face of the hand is between pointing upwards and downwards. The triceps, between the humerus and the ulna, is the main arm extender.

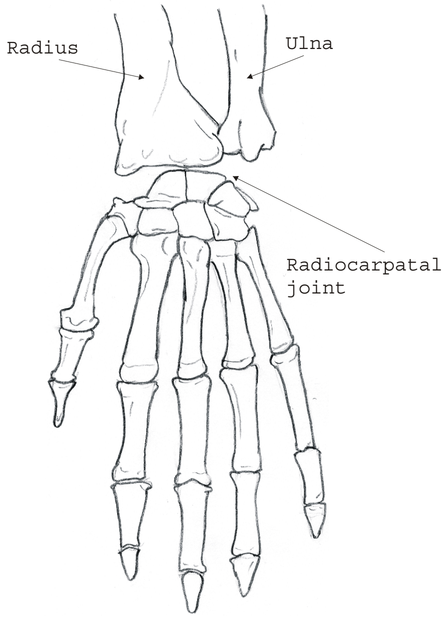

Figure 3.4: The radiocarpatal joint.

3.3 Wrist

The wrist consist of the radiocarpal joint, including the radius and the three carpal bones in the hand. Muscles connect bones in the hand with the humerus, the ulna and the radius. These muscles cooperate to enable flexion, extension, radial flexion and ulnar flexion.

Chapter 4

Models and Methods

4.1 The model

In chapter 2 the way a trombone is played is described. The aim has been to construct a mechanical model, that, although simple, has the same characteristics as a trombone player.

The trombone is modelled by a straight rod, tilted an angle α. The trombone is considered fixed in space. The arm is modelled by two bars, A for the upper arm and B for the lower arm and hand. As the lower arm does not rotate around the axis going from the elbow to the hand, the radius and the ulna does not move relative to each other, see chapter 3, which justifies modelling the lower arm as a bar. The wrist is assumed to be fixed during the motion. This is not entirely true, but the wrist motions are mostly small, and therefore the hand and the lower arm are considered to be a unity. The bars A and B are connected by a hinge. As the major elbow joint is a hinge joint, this is a good approximation. A and B have respectively mass m_A and m_B, length l_A and l_B and radius of inertia (measured from the shoulder and elbow respectively) $r_A^G$and r_B^G. Bar A is connected to the trombone-rod by another hinge that is to be a model of the shoulder. The shoulder is not a hinge joint at all, but allows for motion in all directions. In this case though, the motion is almost planar and the shoulder will move much like a hinge. Only when moving to the higher positions of the trombone the whole shoulder needs to be moved forwards. It will be shown that in this case a pure hinge joint is not sufficient. The hand is allowed to slide without friction along the rod, which is a simplification of the trombone slide moving with low friction on the bell part and the hand being fixed to the slide. See figure 4.2. Masses, lengths and radii of inertia for the upper and lower arm and hand is adjusted to be as for a human arm. See appendix B. For the hand approximate values had to be used for these parameters, as they are valid only when the hand is fully stretched, which normally is not the case in trombone playing. The model has the behavior of a pendulum.

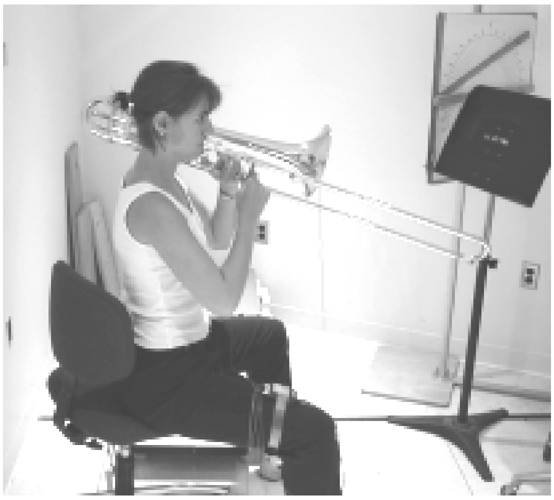

Figure 4.1: Side view of a trombone player.

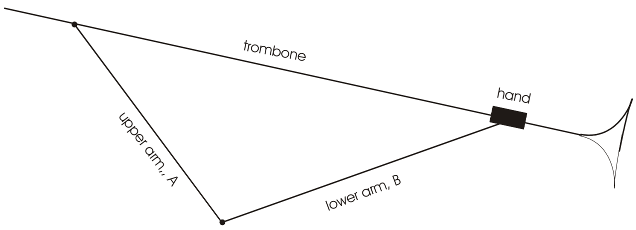

Figure 4.2: Sketch of the model.

4.2 Equation of motion

The equations of motion can, according to *d'Alemberts principle* [9], be written as inertia forces equal to active forces and constraint forces,

$$\dot{\mathbf{p}} = \mathbf{F}^a + \mathbf{F}^c. \tag{4.1}$$

More explicitly:

$$\begin{pmatrix} m_A \dot{\mathbf{v}}_A \\ \mathbf{I}_A \cdot \boldsymbol{\omega}_A \\ m_B \dot{\mathbf{v}}_B \\ \mathbf{I}_B \cdot \boldsymbol{\omega}_B \end{pmatrix} = \begin{pmatrix} \mathbf{F}_A \\ \mathbf{M}_A \\ \mathbf{F}_B \\ \mathbf{M}_B \end{pmatrix} + \mathbf{F}^c. \tag{4.2}$$

As can be seen in figure 4.3 the system consists of two rigid bodies and it has one degree of freedom. The only active forces are those provided by gravity. The angle between the upper arm (body A) and the trombone is q_A and the angle between the lower arm (body B) and the trombone is q_B.

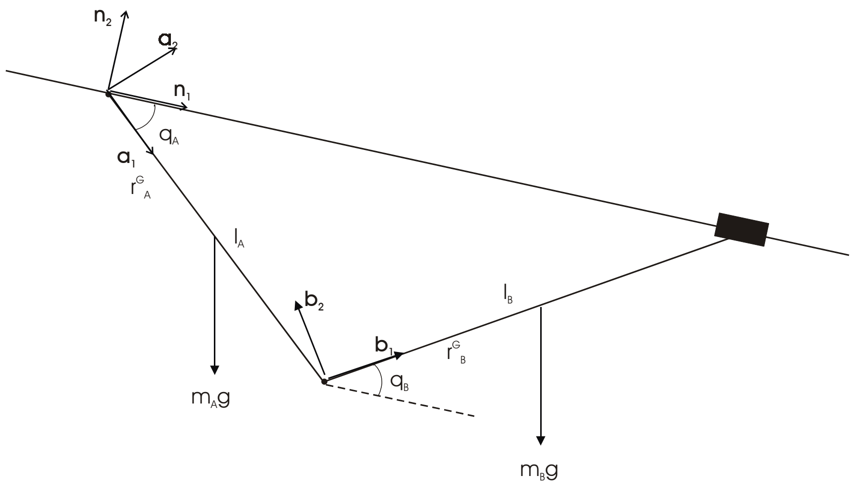

Figure 4.3: The mechanical model.

The triad $\{\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3\}$ is aligned with the trombone, which is assumed to be fixed in space. The triad $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ is fixed in body A and has an angle q_A to the inertial frame,

$$\mathbf{a}_1 = \cos(q_A)\mathbf{n}_1 - \sin(q_A)\mathbf{n}_2, \tag{4.3}$$
$$\mathbf{a}_2 = \sin(q_A)\mathbf{n}_1 + \cos(q_A)\mathbf{n}_2, \tag{4.4}$$
$$\mathbf{a}_3 = \mathbf{n}_3. \tag{4.5}$$

The triad $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is fixed in body B and has an angle q_B to the inertial frame,

$$\begin{aligned}
\mathbf{b}_1 &= \cos(q_B)\mathbf{n}_1 + \sin(q_B)\,\mathbf{n}_2, && (4.6)\\
\mathbf{b}_2 &= -\sin(q_B)\mathbf{n}_1 + \cos(q_B)\,\mathbf{n}_2, && (4.7)\\
\mathbf{b}_3 &= \mathbf{n}_3. && (4.8)
\end{aligned}$$

Vectors to the center of mass for body A and B, under the constraint that the bodies are connected at the elbow:

$$\begin{aligned}
\mathbf{r}_A &= r_A^G\mathbf{a}_1 = r_A^G(\cos(q_A)\mathbf{n}_1 - \sin(q_A)\mathbf{n}_2), && (4.9)\\
\mathbf{r}_B &= l_A\mathbf{a}_1 + r_B^G\mathbf{b}_1 = (l_A\cos(q_A) + r_B^G\cos(q_B))\mathbf{n}_1 + \\
&\quad +(-l_A\sin(q_A) + r_B^G\sin(q_B))\mathbf{n}_2. && (4.10)
\end{aligned}$$

The distances and angles are defined in figure 4.3. The time derivative of the vectors $\mathbf{r}_A$ and $\mathbf{r}_B$, with respect to the inertial frame, gives the center of mass velocities,

$$\begin{aligned}
\mathbf{v}_A &= r_A^G\dot{q}_A(-\sin(q_A)\mathbf{n}_1 - \cos(q_A)\mathbf{n}_2), && (4.11)\\
\mathbf{v}_B &= \left(-l_A\dot{q}_A\sin(q_A) - r_B^G\dot{q}_B\sin(q_B)\right)\mathbf{n}_1 + \\
&\quad + \left(-l_A\dot{q}_A\cos(q_A) + r_B^G\dot{q}_B\cos(q_B)\right)\mathbf{n}_2. && (4.12)
\end{aligned}$$

The angular velocities for body A and B are:

$$\begin{aligned}
\boldsymbol{\omega}_A &= -\dot{q}_A\mathbf{n}_3, && (4.13)\\
\boldsymbol{\omega}_B &= \dot{q}_B\mathbf{n}_3. && (4.14)
\end{aligned}$$

This is a one degree of freedom system as the two coordinates $\dot{q}_A$ and $\dot{q}_B$ are dependent. Their relation is given by the *coordinate constraint equation*

$$l_A\sin(q_A) = l_B\sin(q_B). \qquad (4.15)$$

By deriving equation 4.15 We obtain the *velocity constraint equation*

$$l_A\dot{q}_A\cos(q_A) - l_B\dot{q}_B\cos(q_B) = 0. \qquad (4.16)$$

Writing this in matrix form as $A_i^\alpha v_i = 0$ gives:

$$\begin{pmatrix} l_A\cos(q_A) & -l_B\cos(q_B) \end{pmatrix}\begin{pmatrix} \dot{q}_A \\ \dot{q}_B \end{pmatrix} = 0. \qquad (4.17)$$

Any vector β^i tangent to the motion will satisfy the equation $A_i^\alpha \beta^i = 0$. Choose the tangent vector to be:

$$\boldsymbol{\beta} = \begin{pmatrix} 1 \\ \frac{l_A \cos(q_A)}{l_B \cos(q_B)} \end{pmatrix}. \tag{4.18}$$

From this choose the *generalized angular velocity* w_A:

$$\begin{pmatrix} \dot{q}_A \\ \dot{q}_B \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{l_A \cos(q_A)}{l_B \cos(q_B)} \end{pmatrix} w_A. \tag{4.19}$$

Now use w_A to rewrite equations 4.11 to 4.14 as:

$$\mathbf{v}_A = r_A^G w_A(-\sin(q_A)\mathbf{n}_1 - \cos(q_A)\mathbf{n}_2), \tag{4.20}$$

$$\mathbf{v}_B = l_A w_A \begin{pmatrix} \left(-\sin(q_A) - \frac{r_B^G}{l_B}\frac{\cos(q_A)}{\cos(q_B)}\sin(q_B)\right)\mathbf{n}_1 + \\ + \left(-1 + \frac{r_B^G}{l_B}\right)\cos(q_A)\mathbf{n}_2 \end{pmatrix}, \tag{4.21}$$

$$\boldsymbol{\omega}_A = -w_A \mathbf{n}_3, \tag{4.22}$$

$$\boldsymbol{\omega}_B = \frac{l_A}{l_B}\frac{\cos(q_A)}{\cos(q_B)} w_A \mathbf{n}_3. \tag{4.23}$$

In stating the moments of inertia for A and B we use the fact that rotation does only take place around the $\mathbf{n}_3$-axis and therefore J_A and J_B are the only components in the inertia matrices that need be considered. The following moments of inertia are used:

$$\mathbf{I}_A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & J_A \end{pmatrix}, \tag{4.24}$$

$$\mathbf{I}_B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & J_B \end{pmatrix}. \tag{4.25}$$

Thus the momentum and angular momentum for the two bodies are given by

$$
\begin{aligned}
\mathbf{p}_A &= m_A\mathbf{v}_A = -m_A r_A^G w_A(\sin(q_A)\mathbf{n}_1 + \cos(q_A)\mathbf{n}_2), \\
\mathbf{p}_B &= m_B\mathbf{v}_B = m_B l_A \begin{pmatrix} -\begin{pmatrix} \frac{r_B^G}{l_B}\frac{\cos(q_A)}{\cos(q_B)} w_A \sin(q_B) + \\ +w_A\sin(q_A) \end{pmatrix}\mathbf{n}_1 + \\ +\left(-1+\frac{r_B^G}{l_B}\right) w_A\cos(q_A)\mathbf{n}_2 \end{pmatrix}, \quad (4.26) \\
\mathbf{h}_A &= \mathbf{I}_A\cdot\boldsymbol{\omega}_A = -J_A w_A\mathbf{n}_3, \quad (4.27) \\
\mathbf{h}_B &= \mathbf{I}_B\cdot\boldsymbol{\omega}_B = -J_B\frac{l_A}{l_B}\frac{\cos(q_A)}{\cos(q_B)} w_A\mathbf{n}_3. \quad (4.28)
\end{aligned}
$$

The time derivative of the momentum and angular momentum, with respect to the inertial frame, gives the inertia force and torque:

$$
\begin{aligned}
\dot{\mathbf{p}}_A &= -m_A r_A^G \begin{pmatrix} \left((w_A)^2\cos(q_A) + \dot{w}_A\sin(q_A)\right)\mathbf{n}_1 + \\ +\left(-(w_A)^2\sin(q_A) + \dot{w}_A\cos(q_A)\right)\mathbf{n}_2 \end{pmatrix}, \quad (4.29) \\
\dot{\mathbf{p}}_B &= m_B l_A \begin{pmatrix} \begin{pmatrix} -\frac{l_A r_B^G (w_A)^2\cos^2(q_A)}{(l_B)^2\cos(q_B)} + \\ +\frac{r_B^G (w_A)^2\sin(q_B)\sin(q_A)}{l_B\cos(q_B)} - \\ -\frac{r_B^G \dot{w}_A\sin(q_B)\cos(q_A)}{l_B\cos(q_B)} - \\ -\frac{l_A r_B^G (w_A)^2\sin^2(q_B)\cos^2(q_A)}{(l_B)^2\cos^3(q_B)} - \\ -(w_A)^2\cos(q_A) - \dot{w}_A\sin(q_A) \end{pmatrix}\mathbf{n}_1 + \\ +\begin{pmatrix} (w_A)^2\sin(q_A) - \dot{w}_A\cos(q1) - \\ -\frac{r_B^G (w_A)^2\sin(q_A)}{l_B} + \frac{r_B^G \dot{w}_A\cos(q_A)}{l_B} \end{pmatrix}\mathbf{n}_2 \end{pmatrix}, \quad (4.30) \\
\dot{\mathbf{h}}_A &= -J_A\dot{w}_A\mathbf{n}_3, \quad (4.31) \\
\dot{\mathbf{h}}_B &= J_B\frac{l_A}{l_B}\begin{pmatrix} \frac{-(w_A)^2\sin(q_A)}{\cos(q_B)} + \frac{\dot{w}_A\cos(q_A)}{\cos(q_B)} + \\ +\frac{l_A (w_A)^2\cos^2(q_A)\sin(q_B)}{l_B\cos^3(q_B)} \end{pmatrix}\mathbf{n}_3. \quad (4.32)
\end{aligned}
$$

The dot product between the inertia force and the tangent planes results in the negative of *the generalized inertia force*. The constraint forces have no projection in the tangent space to the allowed motions and are thus eliminated from the resulting equations. The inertia forces are given by

$$\dot{\mathbf{p}} \cdot \boldsymbol{\beta} = \left(\begin{array}{c} -(l_A r_B^G w_A)^2 m_B l_B \cos(q_A) \sin(q_A) \cos^2(q_B) + \\ +2(l_A l_B w_A)^2 m_B r_B^G \cos(q_A) \cos^4(q_B) \sin(q_A) - \\ -2(l_A l_B)^2 m_B r_B^G \cos^2(q_A) \cos^4(q_B) \dot{w}_A - \\ -(l_A w_A)^2 l_B J_B \cos(q_A) \sin(q_A) \cos^2(q_B) + \\ +(l_A)^2 l_B J_B \cos^2(q_A) JB \cos^2(q_B) \dot{w}_A + \\ +2(l_A l_B w_A)^2 m_B r_B^G \sin(q_B) \cos^2(q_A) \cos^3(q_B) + \\ +2(l_A l_B)^2 m_B r_B^G \sin(q_B) \cos(q_A) \sin(q_A) \dot{w}_A \cos^3(q_B) + \\ +(l_A)^3 (w_A)^2 J_B \cos^3(q_A) \sin(q_B) + \\ +(l_B)^3 J_A \cos^4(q_B) \dot{w}_A + \\ +(l_A)^2 (l_B)^3 m_B \cos^4(q_B) \dot{w}_A - \\ -(l_A l_B w_A)^2 m_B r_B^G \cos^3(q_B) \sin(q_B) + \\ +(l_A)^3 (r_B^G w_A)^2 m_B \cos^3(q_A) \sin(q_B) + \\ +(l_A r_B^G)^2 m_B \cos^2(q_A) \dot{w}_A l_B \cos^2(q_B) + \\ +(l_A)^3 (w_A)^2 l_B m_B r_B^G \sin(q_A) \cos(q_B) \cos^2(q_A) + \\ +(r_A^G)^2 (l_B)^3 m_A \cos^4(q_B) \dot{w}_A) \end{array} \right) \frac{1}{(l_B)^3 \cos^4(q_B)}, \tag{4.33}$$

and the applied forces and torques by

$$\begin{aligned} \mathbf{R}_A &= m_A g(\sin(\alpha)\mathbf{n}_1 - \cos(\alpha))\mathbf{n}_2, & (4.34) \\ \mathbf{R}_B &= m_B g(\sin(\alpha)\mathbf{n}_1 - \cos(\alpha))\mathbf{n}_2, & (4.35) \\ \mathbf{T}_A &= \mathbf{0}, & (4.36) \\ \mathbf{T}_B &= \mathbf{0}. & (4.37) \end{aligned}$$

The dot product between the applied forces and the tangent vectors gives the generalized active force:

$$\mathbf{F}^a \cdot \boldsymbol{\beta} = \frac{g}{l_B \cos(q_B)} \left(\begin{array}{c} -m_A l_B r_A^G \sin(q_A) \cos(q_B) \sin(\alpha) + \\ +m_A l_B r_A^G \cos(q_A) \cos(\alpha) \cos(q_B) - \\ -m_B l_A r_B^G \sin(q_B) \cos(q_A) \sin(\alpha) - \\ -m_B l_A l_B \sin(q_A) \cos(q_B) \sin(\alpha) + \\ +m_B l_A l_B \cos(q_A) \cos(q_B) \cos(\alpha) - \\ -m_B l_A r_B^G \cos(q_A) \cos(q_B) \cos(\alpha) \end{array} \right). \tag{4.38}$$

According to *D'Alemberts principle* the equations of motion are

$$\dot{\mathbf{p}} \cdot \boldsymbol{\beta} = \mathbf{F}^a \cdot \boldsymbol{\beta}. \tag{4.39}$$

4.3 Work

In this section the work it takes to play the trombone is calculated. Chemical energy in the muscles and loss of energy due to friction in arm and trombone has not been accounted for.

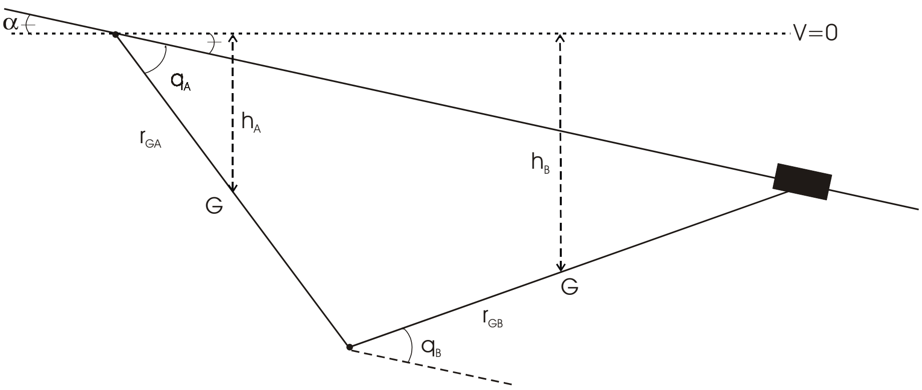

Figure 4.4: The model, with parameters used to calculate the kinetic and potential energy.

The relationship between work and kinetic and potential energy is

$$W_{ab} = \Delta T + \Delta V_g. \tag{4.40}$$

W_{ab} is the work performed by external forces, gravity excluded. ΔT is the change in kinetic energy and can be divided into two parts, translation, $\frac{mv^2}{2}$, and rotation, $\frac{I\omega^2}{2}$. Body A rotates around the fixed axis O and its kinetic energy can therefore be described as rotation only, with I_A^O as the moment of inertia for body A around the axis O, thus A's kinetic energy can be expressed as

$$T_A = \frac{I_A^O \dot{q}_A^2}{2}. \tag{4.41}$$

Body B performs both rotation and translation and with v_B as the velocity for body B's center of mass, G, and I_B^G as the moment of inertia for body B around G, the kinetic energy for B is

$$T_B = \frac{m_B v_B^2}{2} + \frac{I_B^G \dot{q}_B^2}{2}. \tag{4.42}$$

When travelling from a point a to a point b in the phaseplane, $(q_i, \dot{q}_i)$, the total change in kinetic energy is:

$$\begin{aligned}\Delta T &= T_A^b + T_B^b - T_A^a - T_B^a = \\ &= \frac{I_A^O \left(\dot{q}_A^b\right)^2}{2} + \frac{m_B \left(v_B^b\right)^2}{2} + \frac{I_B^G \left(\dot{q}_B^b\right)^2}{2} - \\ &\quad -\frac{I_A^O \left(\dot{q}_A^a\right)^2}{2} - \frac{m_B \left(v_B^a\right)^2}{2} - \frac{I_B^G \left(\dot{q}_B^a\right)^2}{2}. \end{aligned} \tag{4.43}$$

The change in potential energy is

$$\Delta V_g = m_A g \left(h_A^b - h_A^a\right) + m_B g \left(h_B^b - h_B^a\right), \tag{4.44}$$

where h_A and h_B are the shortest distances between the centers of mass and the line defining zero potential, according to figure 4.4.

In the work performed by the trombone player the work performed by gravity is not included, but only the work to get a starting velocity and the work to stop the motion. This work is the change in kinetic energy, that is not caused by the difference in potential energy,

$$W = \Delta T - \Delta V_g. \tag{4.45}$$

If the potential energy in the starting position is exceeds that in the target position, no input of energy is necessary. The arm performs the motion under the influence of gravity alone, and the work of the player is zero. In the opposite situation, that the target position is the one with higher potential energy, an impulse in the beginning of the motion compensates for the lack of energy. A force is applied in the short interval q_A^a to q_A^b to accelerate the system. Using the difference in potential energy, the velocity $\dot{q}_A^b$ can be calculated so that the next position is reached with zero velocity. The work needed is:

$$\begin{aligned} W^{acc.} &= \frac{I_A^O \left(\dot{q}_A^b\right)^2}{2} + \frac{m_B \left(v_B^b\right)^2}{2} + \frac{I_B^G \left(\dot{q}_B^b\right)^2}{2} - \\ &\quad - m_A g \left(h_A^b - h_A^a\right) - m_B g \left(h_B^b - h_B^a\right). \end{aligned} \tag{4.46}$$

After reaching the velocity $\dot{q}_A^b$ the arm is left to do a "falling" motion without external forces and the system will stop at the target position without an external force.

If the motion goes from a higher to a lower potential, braking is necessary, to remove the surplus energy. The braking starts at a small distance before the final position, q_A^d, at the angle q_A^c, to make a smooth stop. The work is:

$$W^{ret.} = -\frac{I_A^O\left(\dot{q}_A^c\right)^2}{2} - \frac{m_B\left(v_B^c\right)^2}{2} - \frac{I_B^G\left(\dot{q}_B^c\right)^2}{2} - -m_A g\left(h_A^d - h_A^c\right) - m_B g\left(h_B^d - h_B^c\right). \tag{4.47}$$

The models movements, as described above, is the most energy efficient way to move between the different positions. The start and stop values – q_A^a, $\dot{q}_A^a = 0$, q_A^d and $\dot{q}_A^d = 0$ – are given and can not be altered. Left to deal with is then q_A^b, $\dot{q}_A^b$, q_A^c and $\dot{q}_A^c$. The force applied in the interval q_A^a to q_A^b accelerates the system to exactly the velocity $\dot{q}_A^b$ that will make it reach the target position at zero velocity. The choice of q_A^b, i.e. how long the force is applied, will of course alter the velocity $\dot{q}_A^b$, but the resulting energy will still be the minimum. The same argument is valid for the choice of q_A^c. If the braking interval is made longer, less force is needed, and the energy used for braking will be the same minimum energy. Between q_A^b and q_A^c no force is applied and as the system is conservative, the energy is constant throughout the interval.

4.4 The experiment

We now compare calculations with measurements on two trombone players, a student (subject S) and a professional (subject P). The measurements were performed in collaboration with Doctor Virgil Stokes, at NMRC, Boston University and the data analysed at the mechanics department at KTH.

The OptoTrak System (Northern Digital, Inc.), model 3010 (version 10 of ODAU) was used for all kinematic measurements with a sample rate of 200 samples per second. The experiment was also recorded on video and photos were taken with a digital camera.

During the experiment 6 LED's (Light Emitting Diode) were used. Two were attached to the trombone and four to the subject as follows.

#1	the fixed part (bell structure)
#2	the moving slide
#3	shoulder
#4	elbow
#5	wrist
#6	knuckle of index finger

First reference trials were taken. Data was collected while the subject tilted the trombone (from horizontal) until the slide moved (2 trials) and the seven trombone positions were recorded, in turn. Then data was collected while the subject performed the following seven different random sequences.

$$
\begin{array}{l}
5 \to 4 \to 5,\ 5 \to 7 \to 5,\ 5 \to 6 \to 5,\ 5 \to 3 \to 5,\ 5 \to 2 \to 5,\ 5 \to 1 \to 5 \\
2 \to 4 \to 2,\ 2 \to 7 \to 2,\ 2 \to 5 \to 2,\ 2 \to 3 \to 2,\ 2 \to 6 \to 2,\ 2 \to 1 \to 2 \\
6 \to 7 \to 6,\ 6 \to 1 \to 6,\ 6 \to 4 \to 6,\ 6 \to 3 \to 6,\ 6 \to 2 \to 6,\ 6 \to 5 \to 6 \\
3 \to 6 \to 3,\ 3 \to 4 \to 3,\ 3 \to 1 \to 3,\ 3 \to 2 \to 3,\ 3 \to 5 \to 3,\ 3 \to 4 \to 3 \\
4 \to 7 \to 4,\ 4 \to 5 \to 4,\ 4 \to 2 \to 4,\ 4 \to 6 \to 4,\ 4 \to 3 \to 4,\ 4 \to 1 \to 4 \\
1 \to 2 \to 1,\ 1 \to 7 \to 1,\ 1 \to 4 \to 1,\ 1 \to 5 \to 1,\ 1 \to 3 \to 1,\ 1 \to 6 \to 1 \\
7 \to 1 \to 7,\ 7 \to 2 \to 7,\ 7 \to 5 \to 7,\ 7 \to 3 \to 7,\ 7 \to 6 \to 7,\ 7 \to 4 \to 7
\end{array}
$$

The subject played the sequences at 60 and 125 bpm, with and without blindfold. The tempo was controlled by a metronome.

After the reference trial the subject played a musical excerpt, No. 6 from Rochut book one, see appendix C, at 60 bpm and then, at the same tempo, a random sequence generated by a pseudo random number generator, see appendix D.

All the trials were repeated several times to obtain the raw data.

Each measured sequence was saved as a file, in ASCII format, with information on the experiment on the top, then the raw data and in the end errors that occurred during the measurement. The data was organised in 20 columns, where the first column was the sample number, the second the time and then the $\{x, y, z\}$-coordinates for each of the six LED's. To be able to read the data into Matlab 5.3.1, it was necessary to first remove the experiment information and the error reports. After that the data could be read into Matlab with the command `load file.txt`.

As the model is a simplified version of a real trombonist we chose not to use data from the LED's that was attached to the subjects body. From these we could expect errors coming from the motion of the wrist and the fact that the mouthpiece of the trombone is not being held fixed related to the shoulder. Instead the two LED's attached to the trombone were used. The distance between LED-1, attached to the bell part, and LED-2, attached to the slide, is affected only by the motion of the slide and can easily be compared to the motion of the hand in the model. If we name the coordinates of the two LED's $\{x_1, y_1, z_1\}$ and $\{x_2, y_2, z_2\}$ the distance, x, between the LED's can be written:

$$x = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \tag{4.48}$$

For comparisons of phaseplots the velocity was calculated from the distance and time vectors as

$$\dot{x} = \frac{x_{j+1} - x_j}{t_{j+1} - t_j}. \tag{4.49}$$

The distance and the velocity was calculated for each sample in a sequence and put in a vector.

As the LED's was placed slightly arbitrary and not at the exact locations of the mouthpiece and hand, the data needs adjusting before the comparison with the model. The data is also expressed in millimeters, where the model uses meters, which makes a factor 1000 necessary. For subject P the relation is approximately

$$x_{Model} = \frac{x_P}{1000} - 0.1, \tag{4.50}$$

and for subject S

$$x_{Model} = \frac{x_S}{1000} - 0.03. \tag{4.51}$$

The distances between the trombones positions, that are used by the model, have also been taken from the data and the approximate distances, measured from the models mouthpiece, are

Position	P	S
1	0,110	0.110
2	0.190	0.200
3	0.275	0.285
4	0.390	0.375
5	0.490	0.480
6	0.610	0.605
7	0.720	0.720

In some of the measured sequences the data collection started before the subject had started playing. This makes it necessary to also adjust the time between the model and the measured data in order to make comparisons.

All calculations for the model were performed in Maple 5.5. The equations of motion where deduced and then solved, using the subjects parameter values, for the initial conditions of each motion. Between the motions periods of the hand being held still was added, i.e. where the note was to be played. The time for the periods were adjusted to keep the tempo of the music. Lists, containing time and position, where exported as text files, to be loaded into Matlab and compared with the data.

The trombones tilt angle was calculated, to examine its variation. The angle can be extracted from the data as

$$\alpha = \arctan\left(\left|\frac{y_1 - y_2}{x_1 - x_2}\right|\right). \tag{4.52}$$

In this study the musical excerpt and the randomly ordered notes have been examined for both subjects. From the raw data the four sequences which contained the least errors were used.

4.5 Glossary of symbols

α – Tilt angle of the trombone.

$\boldsymbol{\beta}$ – Vector tangent to the systems motion.

$\boldsymbol{\omega}_A$ – Angular velocity for body A.

$\boldsymbol{\omega}_B$ – Angular velocity for body B.

$\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ – Triad fixed in body A.

$\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ – Triad fixed in body B.

$\mathbf{F}_A$ – External forces acting on body A.

$\mathbf{F}_B$ – External forces acting on body B.

$\mathbf{F}^a$ – Active forces.

$\mathbf{F}^c$ – Constraint forces.

G – Center of mass.

g – Gravitational constant, 9.81.

$\mathbf{h}_A$ – Angular momentum for body A.

$\mathbf{h}_B$ – Angular momentum for body B.

h_A – Shortest distance between body A's center of mass and the line defining zero potential, see figure 4.4.

h_B – Shortest distance between body B's center of mass and the line defining zero potential, see figure 4.4.

$\mathbf{I}_A$ – Inertia matrix for body A.

$\mathbf{I}_B$ – Inertia matrix for body B.

I_A^O – Moment of inertia for body A, around the axis O.

I_B^G – Moment of inertia for body B, around the axis G.

J_A – The 33-component of inertia matrix $\mathbf{I}_A$.

J_B – The 33-component of inertia matrix $\mathbf{I}_B$.

l_A – Length of body A.

l_B – Length of body B.

$\mathbf{M}_A$ – External torques acting on body A.

$\mathbf{M}_B$ – External torques acting on body B.

m_A – Mass of body A.

m_B – Mass of body B.

$\{\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3\}$ – Triad fixed in the trombone and in the inertial frame.

O – Fixed axis through the shoulder.

$\mathbf{p}_A$ – Momentum of body A.

$\mathbf{p}_B$ – Momentum of body B.

q_A – Angle between the trombone (or $\mathbf{n}_1$-frame vector) and body A.

q_B – Angle between the trombone (or $\mathbf{n}_1$-frame vector) and body B.

$\mathbf{R}_A$ – Forces applied to body A.

$\mathbf{R}_B$ – Forces applied to body B.

$\mathbf{r}_A$ – Vector to body A's center of mass.

$\mathbf{r}_B$ – Vector to body B's center of mass.

r_A^G – Distance from the shoulder to body A's center of mass.

r_B^G – Distance from the elbow to body B's center of mass.

ΔT – Change in kinetic energy.

$\mathbf{T}_A$ – Torques applied to body A.

$\mathbf{T}_B$ – Torques applied to body B.

T_A – Kinetic energy of body A.

T_B – Kinetic energy of body B.

ΔV_g – Change in potential energy.

$\mathbf{v}_A$ – Center of mass velocity for body A.

$\mathbf{v}_B$ – Center of mass velocity for body B.

v_B – Center of mass speed for body B.

W_{ab} – Work performed when moving the system from a to b.

w_A – Generalized angular velocity.

Chapter 5

Results

5.1 Positions of the trombone

The trombone has seven defined positions. As was mentioned in chapter 2, to reach position seven all players have to move the shoulder and some also the hand and fingers. As the model consists of upper and lower arm only, it can reach six of the positions, but not position seven. Including a movable shoulder in the model would mean another degree of freedom and this has been avoided for simplicity and insight. Thus we do not consider position seven.

In figures 5.1 and 5.2 the models six positions for parameters for subject P (professional) and S (student), respectively, are shown. For the trombone's tilt from the horizontal the angles $\alpha_P = 0.4878$ for P and $\alpha_S = 0.5792$ for S have been used, as they were the mean angles for the subjects when they played a random sequence D, as described below. The dashed line in the figures corresponds to the systems equilibrium position.

The model behaves in a sense like a pendulum, where an initial force impulse and gravity are the only forces acting. This means that already gained potential energy can be used for moving to other positions and the total energy cost will be lower. In figure 5.3 the motion of the system, released from position three, at zero velocity, is shown. No energy has been put in. The pendulum motion will take the arm to somewhere close to position six and then back again.

5.2 Playing a random sequence

There is no friction included in the model and thus the energy is conserved, except for the impulsive action used to achieve different positions. Each set of initial conditions $(x, \dot{x})$ gives a closed curve in the phaseplane on which the system can travel to other positions. A sequence of notes can then be played by

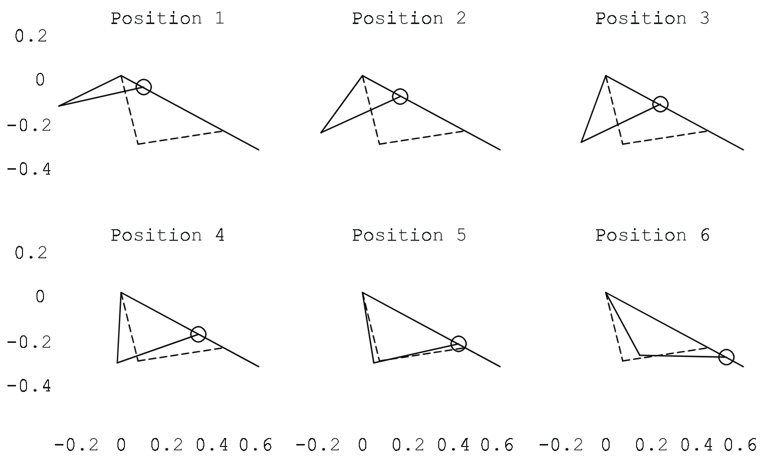

Figure 5.1: The mechanical model for the seven trombone positions. Parameters for subject P. The dashed line corresponds to the equilibrium position for the arm.

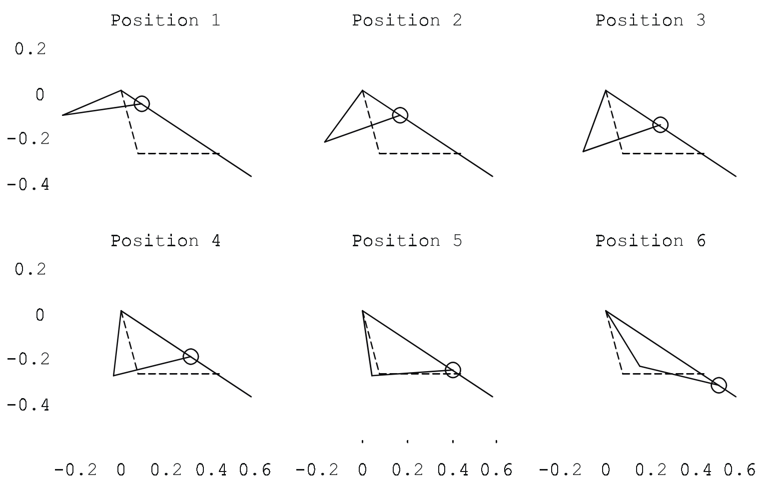

Figure 5.2: The mechanical model for six of the seven trombone positions. Parameters for subject S. The dashed line corresponds to the equilibrium position for the arm.

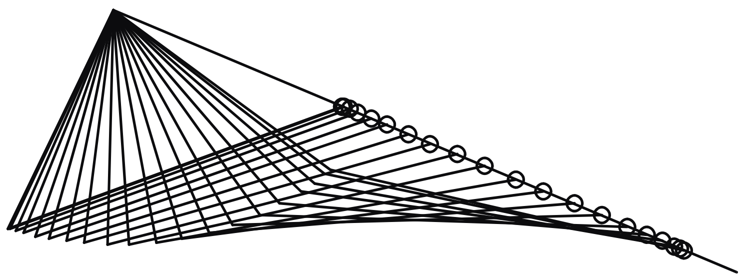

Figure 5.3: The motion of the system, when allowed to fall freely from position three.

jumping between the closed curves in the phaseplane. In figures 5.4 and 5.5 the phaseplane trajectories of the subjects playing the sequence $5-4-3-4-5-2-6-5-3-6$, compared to the model, are shown. Both subject P and S reaches zero velocity, at the appropriate position, when playing a note. The model, on the other hand, does not always come to a halt in a position. A known deficiency is the absence of braking. Instead the calculation is terminated when the correct position is reached, even if the velocity isn't zero. To make the system jump to another curve in the phaseplane, an input of energy is necessary. This is achieved by adding an impulse in the beginning of the motion, i.e. the motion is started with an initial velocity. The subjects varies a bit in hitting the exact positions. The professional player P is more adept at this than the student S. S sometimes overshoots and has to go back. The little loop at A in figure 5.5 is an example of this. The match between the model and the subjects is better for the shorter movements. One reason for this might be that it is more difficult for a human to judge the force needed and the transportation time for longer movements.

Plotting the displacement against time, gives a clearer view of the actual movement from one position to another, see figure 5.6. At this resolution the model adapts pretty well to the measured data from subject P. It can be seen from the figure that P and the model doesn't always agree on the location of a position. The aim is of course to always find the exact displacement for each position, but errors in the range of centimeters can easily be compensated by how the players uses their lips and control the flow of air. The model uses positions that are mean values of the subjects positions, when playing the random sequences, see appendix D.

How long subject P:s stay at a position also differs, while the model stays for more equal amounts of time. The reason for this is that P can chose when to move the slide, as long as he gets to the next position in time. I.e. if he reaches the position just in time to play the note, and then immediately goes

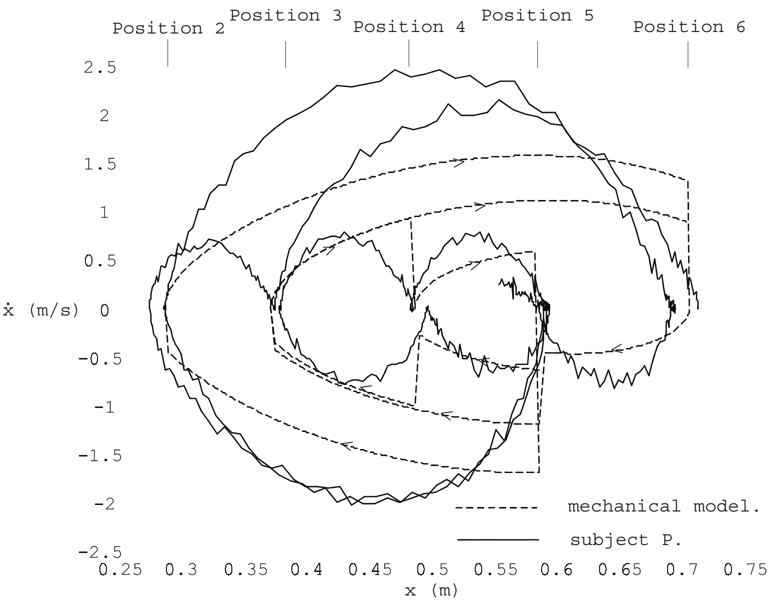

Figure 5.4: Phaseplot of subject P, playing a random sequence, compared to the model.

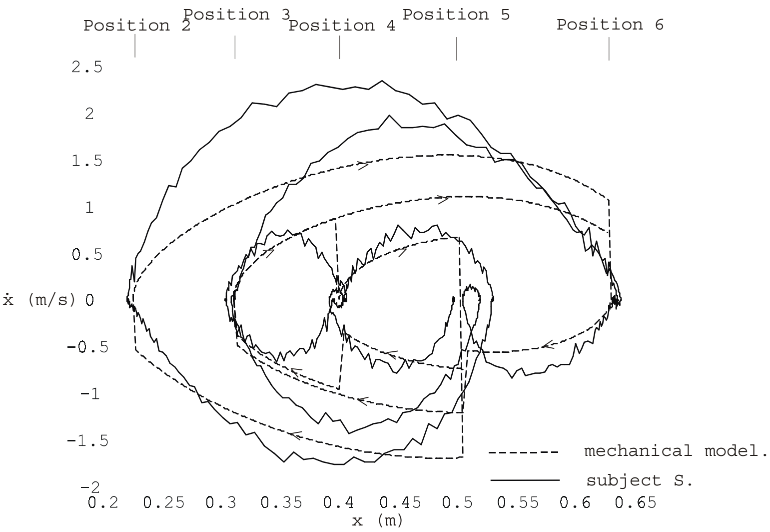

Figure 5.5: Phaseplot of subject S, playing a random sequence, compared to the model.

to the next position it results in a very short pause. An example can be seen at A in figure 5.6.

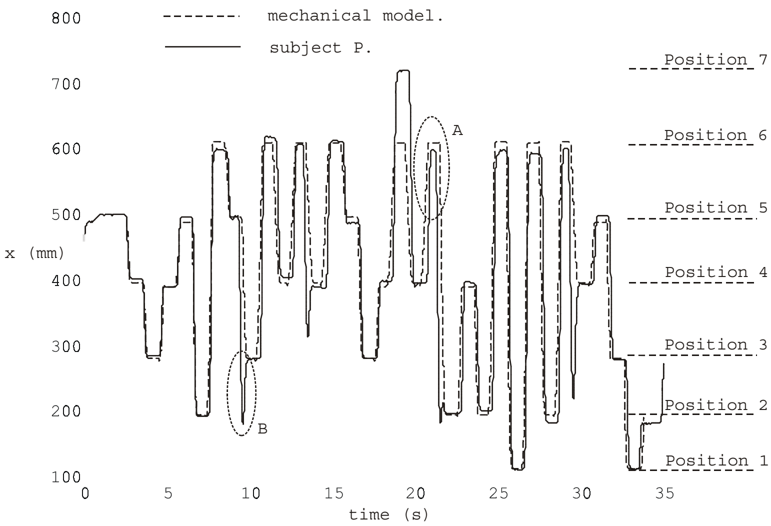

Figure 5.6: Subject P playing a random sequence, compared to the mechanical model.

The model, on the other hand, follows a more exact scheme. The sequence was played in 60 bpm, meaning one note per second. Therefore the model adapts to *transportation time + pause = one second*. The time of the displacement depends entirely on mechanical properties of the system, thus only the pause time can be adjusted.

Some major differences between model and measurement are worth considering. One is that the model never reaches position seven, for reasons mentioned above. The other is that P occasionally pulls back the slide, past the target position, then going back at once, see B in figure 5.6. This is hardly a mistake, but more likely depends on the fact that the sequence is simple and played at a very slow tempo, which probably is tedious for a skilled player. It is conjectured that he has simply moved the hand back for a short rest before heading for the next position.

In figure 5.7 a random sequence, played by subject S, is compared to the model. Just as in figure 5.6 the model and the measurement show the same behavior in this resolution. It is noticeable that S has to adjust the position more than P and that the displacements around the same position differs a bit more. In other words, this is a difference that might be expected between a student and a professional musician.

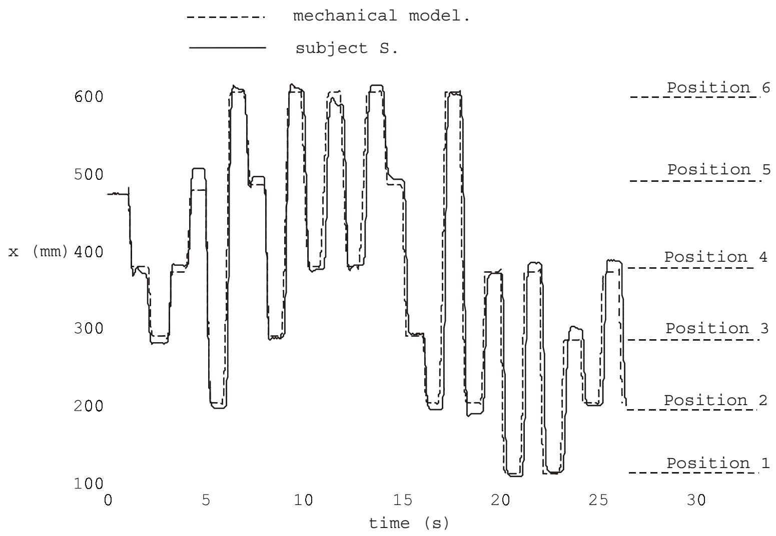

Figure 5.7: Subject S playing a random sequense, compared to the model.

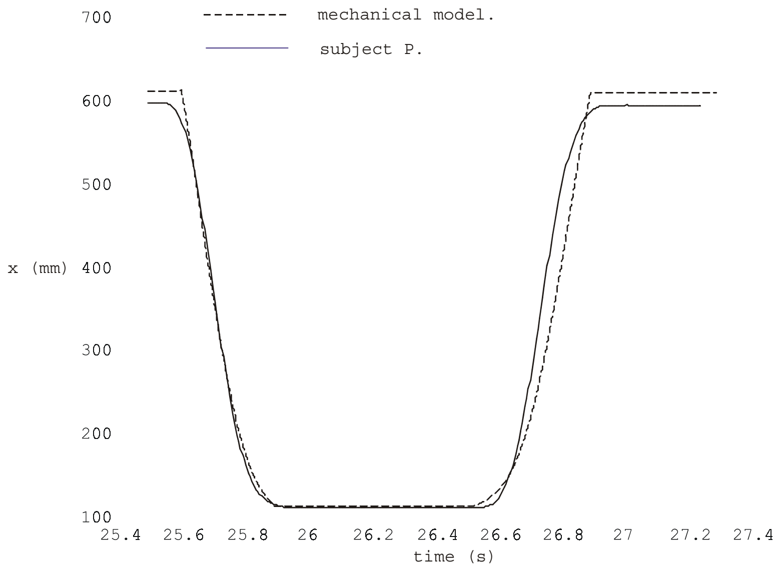

Figure 5.8: Detail of figure 5.6.

Figure 5.8, which is a detail of figure 5.6, shows the differences between model and P:s data more clearly. Moving from position six to position one means an increase of the potential energy. To achieve this it is necessary to apply an impulse in the beginning of the movement. The impulse has been calculated to give the system just enough energy to reach the next position with zero velocity. As can be seen the model follows the measured curve closely. Subject P applies a more continuous force in the beginning, but then lets the arm pendel, just like the model. The likeness is not as good in moving from position one to position six. The model being on a higher potential energy level and then falling to a lower one, then reaches the next position with a non-zero velocity. Subject P, on the other hand, doesn't take full advantage of the difference in potential energy, but accelerates a bit in the beginning of the movement and then brakes to come to a halt in position six.

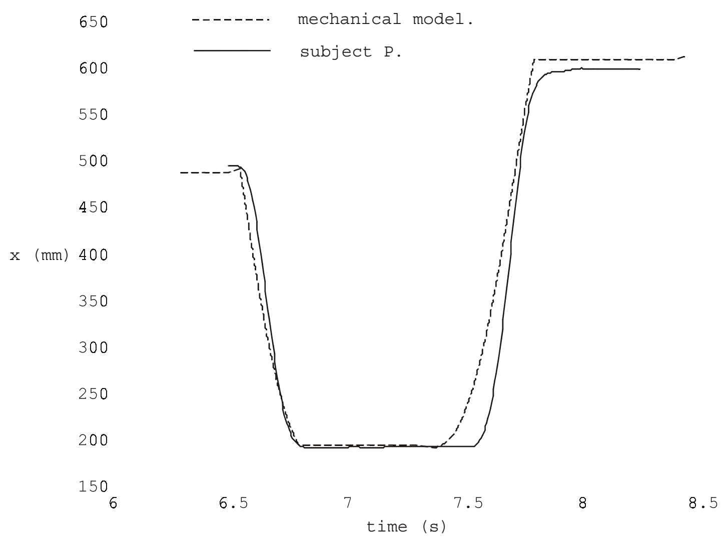

Figure 5.9: Detail of figure 5.6.

In figure 5.9 another detail of figure 5.6 is shown. Here too, the agreement is better, when going from a lower potential energy level to a higher, and an initial impulse is needed. The movement from position two to position six shows large differences between model and data. As the model is falling, P chooses to stay a little longer in position two and then accelerates and catches up with the model in position six.

5.3 Parameter variations

The model is adapted to subjects P and S by length and mass parameters and the tilting angle α of the trombone. The mass of single body parts, like upper and lower arm, is difficult to measure, as is the radius of inertia. Instead statistical relations have been used, based on the subjects total mass and length, see [5]. For simplicity also the relations for the more easily measured length parameters are used. For α the mean values from the measurements are taken. To see if these choices of parameters gives an acceptable accuracy, the models behavior when changing the different parameters, has been examined.

5.3.1 Arm length

In figure 5.10 the effect of varying the height of the subject is shown. The solid line is the model adapted to subject S, and the dashed line is the model using the same parameters except for the total height of the subject being increased by 19%. The difference between the two curves turns out to be minor, which can be seen more clearly in a detailed view, in figure 5.11. The long arm is a bit slow in the start, but catches up, as can be seen when going from position two to position six. The longer arm is able to make a longer pause, which implies that it is overall faster.

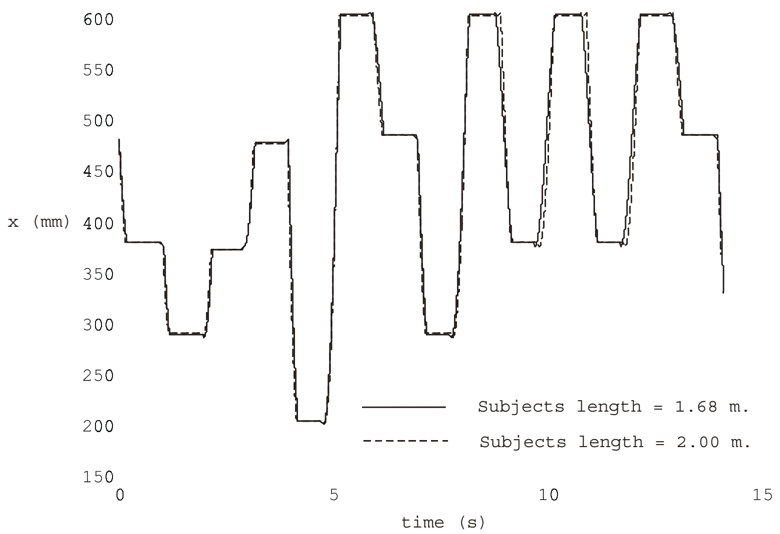

Figure 5.10: The effect of varying the height of the subject.

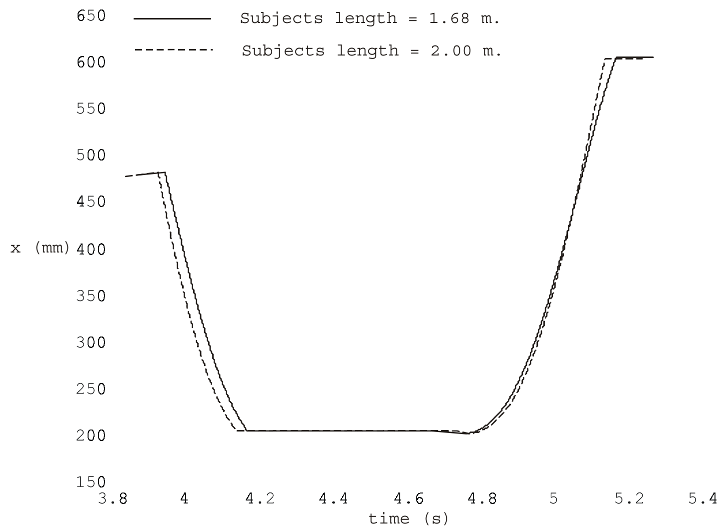

Figure 5.11: Detail of figure 5.10.

5.3.2 Mass

Varying the subjects total mass, will make no difference, as the mass cancels out in the equations of motion, but a change in the mass ratio between the upper and the lower arm gives the model a slightly different behavior. It is hard to see in a larger scale, but in figure 5.12 a detail is shown. Here the mass of the upper arm has been lessened 50% related to the lower arm. It shows that the arm with less mass on the upper arm moves a little bit faster than the normal arm. That this relatively major change in the mass relations has such little effect, suggests that normal variations in the arm mass distributions has no influence on trombone playing, and the model is robust with respect to this parameter.

5.3.3 Tilt angle

A trombone player almost never sits completely still, but performs a slight rocking motion. As a result of this the trombone isn't held exactly still either, but the tilt angle α is varying. The most important effect of a changing α is that it changes the equilibrium position. Displacements that earlier were 'up' can be changed to 'down' and the opposite. In figure 5.13 the models behavior is plotted for subject P:s parameters, with $\alpha = 0$ (trombone held horizontal) and $\alpha = 0.4878$ (mean value for subject P playing the random

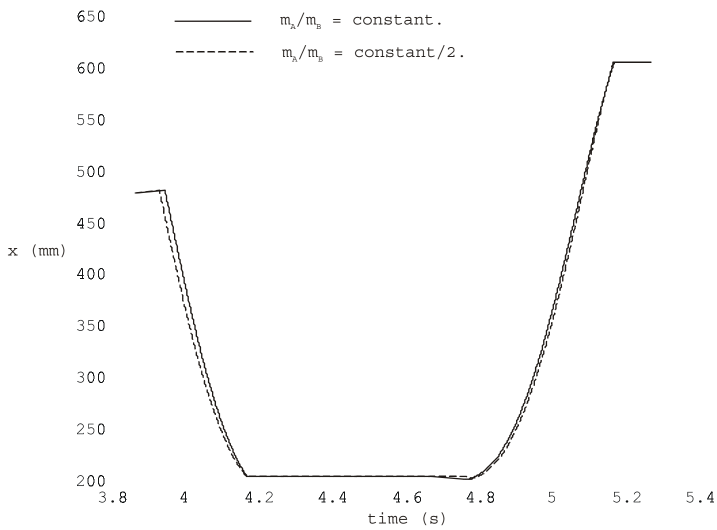

Figure 5.12: The effect of varying the mass ratio between the upper and lower arm.

sequence) respectively. This is a very large variation, larger than a trombone player normally would have. The two subjects in the study didn't vary α more than that the equilibrium position was placed between position five and six throughout the sequences played. This is valid for both the random sequences and the musical excerpts. From this it is concluded that varying the models α according to the measurements wouldn't make a significant difference and for practical reasons α is kept fixed.

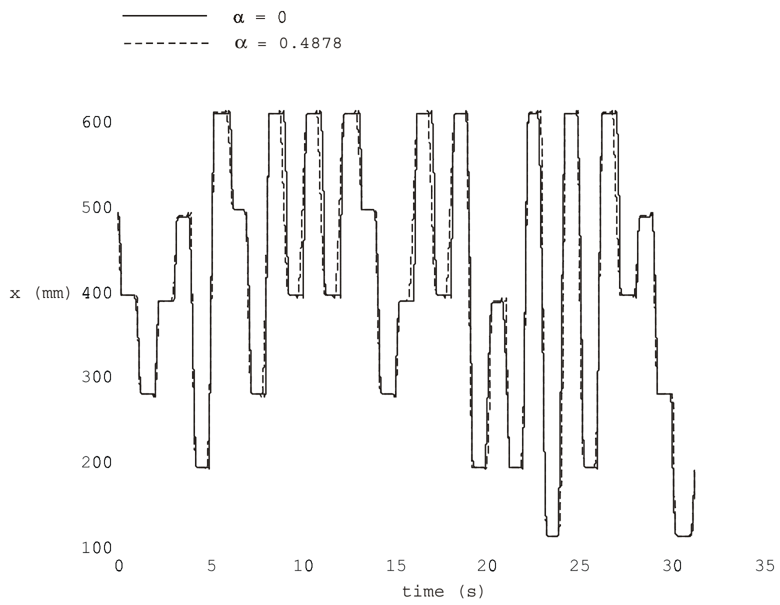

Figure 5.13: The effect of changing the trombone angle α.

5.4 Musical excerpt

Both subjects played a musical excerpt from Rochut book 1, [10], which is used for advanced training. To study this data is of considerable interest, as it shows a 'real' piece of music and, in contrast to the random sequences described above, includes several fast motions.

In figure 5.14 it can be seen that subject P doesn't get to the positions with the same exactness as in the random sequence, which might depend on this sequence being more complex. The model follows the slower motions better than the faster ones. It is likely that the faster movements need more acceleration and braking, that are specific control actions. The two notes at A and B are not included in the model. In the music, the time to play them are not included in the measurement, and it's up to the musician to fit them in without altering the total time. This is beyond the scope of the present model. The interpretation a musician makes of a piece of music can be a problem in the comparisons with the model. Some of the differences that can be seen are likely to depend more on the musicians interpretation than as being a defect in the models assumptions.

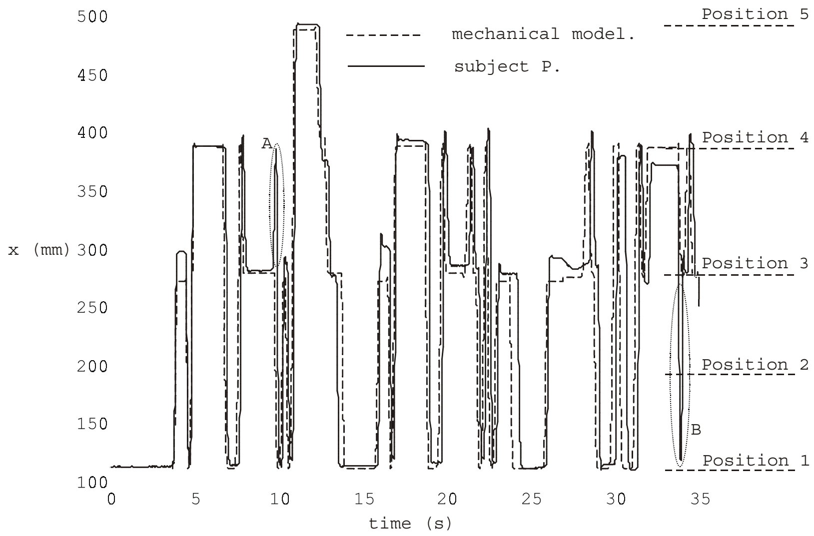

Figure 5.14: Comparison of time histories between subject P and the model.

A couple of details from figure 5.14 are shown in figure 5.15. As in the random sequences, the model aims for the mean values of each position. Area A shows that the variations are large, even for a skilled trombonist.

In B one of the models known deficiencies can be observed. The model doesn't use more initial impulse than necessary to reach the next position, which is the most energy effective way of moving. The drawback is that the time of transportation isn't taken into account, resulting in that following the model might not leave the player time to pause in the position, i.e. play the note in B. This however, is an infrequent occurrence. For the sequence in figure 5.14 the model is some hundreths of seconds late three times and for some more notes, the pause is very short.

The column in C is displaced compared to the model, which otherwise follows subject P fairly well, during this short time. This is a consequence of the trombonists possibility to move the slide after his own liking, as long as the pause covers the time when the note is to be played.

In D, subject P is performing a faster motion than necessary. He takes off later than the model, but reaches the target at the same time. This motion of course takes more additional energy in comparison to the model.

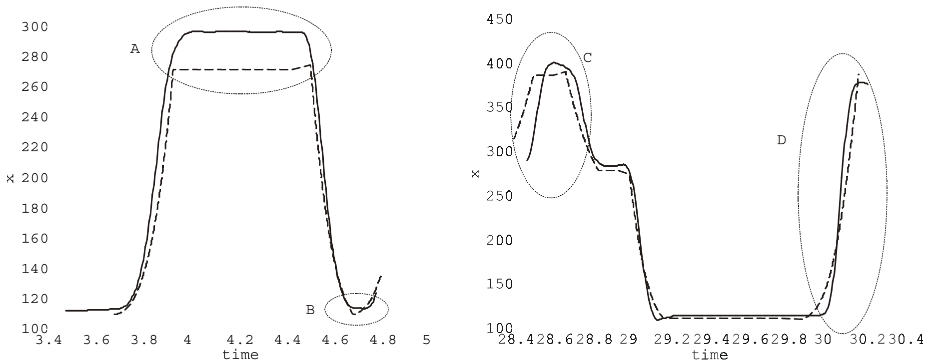

Figure 5.15: Details of portions of figure 5.14.

That the person playing the trombone has a choice of how to play is clearly visible in figure 5.16. Subject P and S are playing the same melody in the same tempo, but still there are great differences. As has been noted earlier P is more skilled than S in hitting the positions correctly. For some reason S and P do not agree about what position to play at the end of the sequence and P seems to be somewhere in between the two positions. This might be just a mistake.

A difference that is best seen in the details in figure 5.17 is that P performs all movements faster than S. P can, if desired, stay in each position longer and still get to the next position in time. This fits well with an article by Kruger et. al. [8] that shows that professional trombonists move the slide faster than less skilled players. Another reason may be that subject P has a longer arm and therefore has an advantage as shown on page 34.

5.5 Summary

In the calculations efforts has been made to make the model similar to the subjects by adjusting the parameters height, mass and the trombones tilt α. When moving from a lower energy level to a higher one, an impulse is applied in the beginning of the motion. Apart from that no forces, except gravity, are applied to the model during the motion. Moving between the trombone positions may be seen as a transportation between different energy levels. Sometimes the energy needs to be added, sometimes the existing potential energy is sufficient. The resulting motion is similar to that of a simple pendulum.

The playing of a randomly generated sequence was examined. The tempo

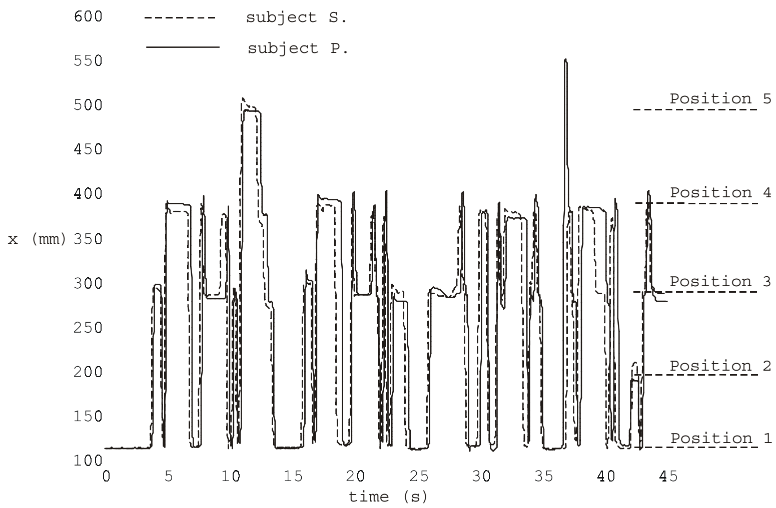

Figure 5.16: Comparison between subject P and subject S, playing the musical excerpt.

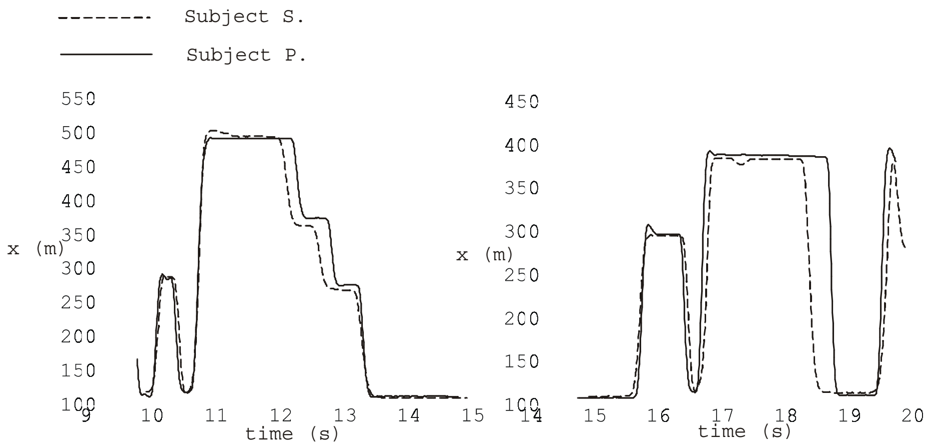

Figure 5.17: Details of portions of figure 5.16.

was slow and all the notes were of equal length. Agreement between model and measurements were good. For a musical excerpt, containing some faster movements of the trombone slide, the variation between model and measurements were larger. A possible conclusion is that faster motions needs more controlling force, i.e. accelerations and breaking, a behavior the model isn't adapted to.

The influence of the different parameters have been examined by varying them and comparing the result. From this it can be seen that all but large variations have little effect. The errors introduced by using statistical values for the subjects height and mass can be considered negligible.

Chapter 6

Discussion

6.1 Geometry

As mentioned earlier the model does not include the possibility of attaining position seven. There are two reasons for this lack of completeness. One is the assumption of a fixed shoulder. A real trombone player moves the shoulder forward to reach the higher positions and especially position seven. The other reason is that the model does not include wrist and finger motions. Players with a short arm often reach position seven by moving the slide with their fingers as well as moving the wrist. [6] Not including the wrist motion also makes a difference in the lower positions, where the player tends to use movements that include both arm and wrist.

The planar motion is another simplification in the model. The real motion is reasonably planar though, especially for skilled players [8], so this assumption is quite reasonable.

6.2 Tilt angle

Both subjects tilt the trombone downwards, choosing an angle that places the systems equilibrium configuration between position five and six, when playing. To achieve this subject S has to tilt her trombone more than P tilts his. One reason for a player to tilt the trombone is to increase the reach. The models reach is not affected by the tilt, but because the mouthpiece and the shoulder in reality are separated, the player gain in reach by tilting, as can be seen from the sketch in figure 6.1.

Why do they both choose to place the equilibrium between position five and six? It could be for pure geometric reasons. This angle gives a better reach,

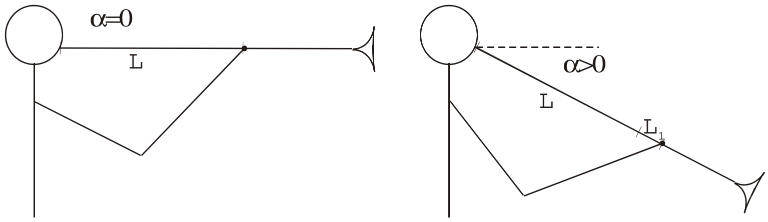

Figure 6.1: Tilting the trombone increases the players reach.

but not to the extent that playing the lower positions becomes uncomfortable. An argument against this is that if it's geometry only, subject S should tilt the trombone a lot more, to be able to move to position seven in a more comfortable way. On the other hand, when tilting too much the player might loose grip on the slide and drop it to the floor, especially if the starting grip wasn't sufficient.

Another possibility, that is worth looking in to, is that there could be an optimum angle α for which the work is minimized. Both subjects vary the angle when playing, where the model, on the other hand uses a fixed angle. The variations are fairly small though, and would not make a great change in the models behavior if included. It could be that they vary the angle to simplify the moving of the slide, but the data gives no clear indication of this. It is more likely that the player moves a bit to feel the tempo in the music, just like a singer would tap his or her foot. This is supported by the fact that the variations where greater for the musical excerpt than for the random sequence, i.e. it is more important to feel the tempo in real music than in a monotonous sequence.

6.3 Pendulum motion

The upper and lower arm have been treated as rigid bodies, rotating around the shoulder and elbow. Together with the trombone, this system is a constrained simple linkage, that can perform a pendulum like motion. Although there are some differences between model and reality, e.g. the moveable shoulder and wrist, comparison with the data confirms that the arm of a trombone player does appear to perform a pendulum motion. Letting the arm pendel is an energy efficient way of moving, as it uses gravity. This is logical as humans evolved in a gravity field and adapted to take advantage of it, making both arms and legs double pendulums. The same pendulum motion is most likely used by all larger animals on earth, that have limbs constructed as double pendulums, as an energy saving way of moving about. The muscles don't even have to be active during the whole movement of a limb. A force to start the motion and another force to stop it is sufficient. Basmajian and Tuttle [2] has shown that most of the muscles aren't active when gravity can supply the motive force for movement, as is the case in a pendulum phase.

6.4 Force

The results show that both subjects apply a force in the beginning and in the end of each motion, even when, according to the model, no force should be necessary. To some extent this is due to friction, which is not included in the model, but less force would still be sufficient. It is likely that the extra force is the cost of precision. If it is essential exactly where the motion stops it is better to apply some extra force in the beginning and then controlling where to stop by braking. The distance is also an important factor. It is harder to judge the forces needed for a longer motion than for a shorter. One example of this is that it is a lot simpler hitting the bull's-eye with the dart if you are close to the target. Total absence of force in either beginning or end of the motion is not possible, as a static force is needed to keep the hand in position, while playing the note.

The use of force may also be influenced by how trombone playing is taught. In teaching it is often stressed that the slide is to be moved as fast as possible at all times. This, of course, results in great accelerations and decelerations and it is very probable that this effects the playing also at higher levels. An implication of this is that, with other training methods, it could be possible to play the trombone in a more harmonic and energy efficient way.

In the model an impulse is applied in the beginning of the movement, to give the system enough energy to reach the next position. The force used by the subjects is of course more continuous. A braking force has not been used in the model, which is a deficiency when going from a higher to a lower potential energy. In that case no initial impulse is needed and the system is merely released and without braking it will fall past the target position. Being able to reach zero velocity at the positions is essential for the player. Apart from this, the model performs its task well, i.e. it moves between the positions at a given tempo, with minimum energy expenditure.

6.5 Friction

In the model, friction is not included. There is friction in the slide and in muscles and joints and energy is lost due to dissipation.

According to Williams and Lissner [11], the coefficient of friction between articular surfaces, e.g. the elbow, is about 0.015. The friction in the trombone is a bit bigger. For greased metal on metal the coefficient of friction ranges between 0.03 and 0.05. The largest loss of energy no doubt takes place, when moving the slide of the trombone. An estimate of the maximum energy loss is the work needed to move the slide between position one and seven.

$$W_{\max} = \mu M_{slide} g L_{1\to 7} \approx 0.05 \cdot 0.737 \cdot 9.81 \cdot 0.6 \approx 0.22\ [J] \tag{6.1}$$

Even this largest possible energy loss is clearly negligeable as it is less than a tenth of the work required to move from one position to another.

6.6 Random sequence and musical excerpt

The difference between the random sequence and the musical excerpt is interesting, as the two pieces are of different character for the player. The random sequence is very simple to play – almost boring – and it is very monotonic as all the notes have the same length. It can hardly be called music and the result is that the subjects move in a more unnatural way, without adding any musical touch of their own. As could be expected, the model follows this pattern well.

The model has more trouble with the sequence that is 'real music' – the musical excerpt. The subjects have the opportunity to make their own interpretations of the music, which, naturally, the model cannot follow. That this piece of music has more 'feeling' than the random sequence can also be seen by the fact that the two subjects move with the beat in different ways, thus varying the tilt angle of the trombone more.

The shorter notes sometimes demands faster movements than what is possible with the energy efficient method implied by the model. A way to overcome this is to apply a larger impulse, when in a hurry. It has been shown [7] that fast movements require a different use of the muscles, with larger accelerations and decelerations.

6.7 Subject P and subject S

The individual differences between trombone players are quite large. That this study only includes two subjects of course makes it difficult to draw any general conclusions about different people's trombone playing. Some behaviors matched the expectations fairly well though. Subject P, being a professional trombonist, was generally better at finding the positions than subject S. P also moved the slide faster than S, which goes well with the study made by Kruger et. al. [8]. An interesting question in this context, is what advantage P's longer arms gives him. From a mechanics point of view, longer arms makes the pendulum motion faster. How it affects his playing is impossible to say though, as the effect cannot be distinguished from the fact that he is a more skilled player than S. But can it be so, that the more skilled players in general have longer arms than the less skilled?

P's longer arms also have drawbacks as longer arms normally are heavier. This does not effect the velocity of the movement – the mass cancels out in the equations of motion – but moving a heavier arm requires more work. The risk of exhausting oneself by playing the trombone is not very big though, and if the extra weight consists of muscles it's even less. The obvious conclusion of this argument is that long and slender arms are better optimized for trombone playing.

6.8 Generalities

A brief description of the motion when playing a trombone is that it is a pendulum motion, that makes use of gravity to save energy. Pure pendulum motions are often described as simple and harmonic – not strenuous. For instance to walk with stiff knees is unnatural and tiring, but let the leg pendel and the motion feels simple and natural. Walking on ice is more tiring than walking on a surface with good friction. To prevent slipping the center of mass must be placed above the foot in each step and it is not possible to make use of the whole pendulum motion of the leg, but instead a large amount of control is necessary.

More complex motions are often harder to do and more energy consuming before we have learned them. After achieving the skill to perform the motion it feels both better and easier. The difference is that the motion has been mechanically optimized. When moving the slide of a trombone from one position to another for the first time, you are uncertain of exactly where to stop and how fast you have to move to get there. A skilled player will know, by experience, the exact amount of initial force needed to go from e.g. position three to six to arrive there just in time to play the next note. It can be done without conscious control.

Another example is a javelin throw, see figure 6.2. A mechanically good throw consists of the athlete first gaining speed by running. This speed is then transferred to the javelin by the thrower braking with the feet to the ground (A), which makes the upper body rotate forward, around a horizontal axis. Simultaneously the heel is twisted outwards, which makes the hip rotate forward, around a vertical axis (B). Directly after follows the shoulder, then the elbow (C) and finally the hand with the javelin (D). The motion is performed like a whiplash, where the whip consists of a number of connected, approximately rigid bodies. To perform this motion is of course difficult and needs practice. Not only every part of the motion must be correct, but the parts have to follow each other smoothly. But practise makes perfect, and finally, after learning the motion in the gravitational field the athlete will reach an optimal level of performance.

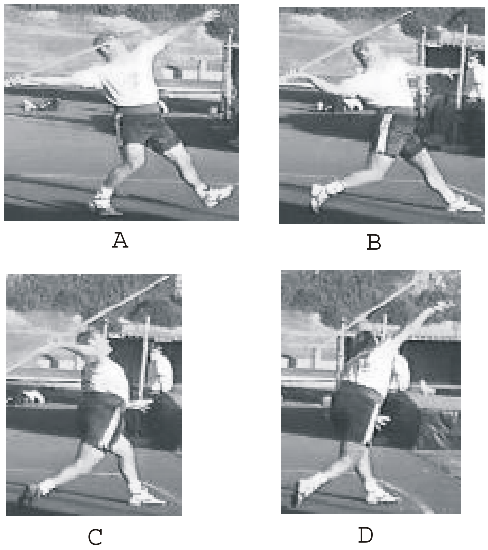

Figure 6.2: Phases of a javelin throw. [1]

A purer motion, that in the beginning does not feel natural, is the motion the lower leg performs when dancing the Charleston, see figure 6.3. As the body weight is put on one leg, the heels simultaneously are twisted outwards, and as a result of this the lower leg is thrown sideways backwards in a pendulum motion around the knee. The first hours of practising the Charleston are painful. The movement feels unnatural and all leg muscles are strained to perform and control the motion, resulting in a considerable amount of lactic acid production in the muscles. After diligent training, though, a natural way of using the pendulum motion for resting the muscles can be found. Suddenly it's hard to realize what was so tiring earlier.

As a example, consider a fairly advanced, but energy saving, technique for down-hill skiing, that has been made a lot easier by the development of carving skis. With carving skis it is easier to ski on the edges of the skis, instead of making a skidding turn, which is the common way to ski for a less experienced skier.

The older type of skis had to be long to give the skier stability. This had the disadvantage of a larger area of contact between the ski and the snow, which gave larger friction. The technique to overcome the friction when turning is called vertical movement. To turn the skier relieves the pressure on the skis by making a small jump, thus lifting the body weight. The result is a motion where the knees are straightened just before the turn and bent otherwise. For the more

Figure 6.3: Dancing the Charlston is making use of a pendulum motion.

advanced technique the relationship is the opposite – the knees are bent when initializing a turn. The ski is let to do much of the work. In a turn, the legs are stretched, pressing the edge of the skis into the snow. The edge gives the stability, thus allowing for shorter skis, with less friction. When turning, the legs are relaxed and the pressure is taken off the skis. The skis are then, by the pressure from snow and the potential energy in the ski, thrown to the opposite side, where the legs are stretched again, pressing the other edge of the skis into the snow. Using the dynamic forces in this way allows the legs to perform a pendulum motion under the body. Not that it's not tiring to ski in this way, but still it's more energy effective. Before the carving ski, that makes it a lot easier to lie on the skis edges, very few were able to perform the technique of levelling. Now it's possible for all skiers with sufficient training.

The conclusion to draw from this is that human motion is often similar to a pendulum motion, governed by gravity. When a motion is practised it is at the same time mechanically optimized to use less energy. Some people are better at optimizing than others and also have advantageous anthropometric parameters for certain activities. Those are the ones that turn out to be the best runners, swimmers etc.

6.9 Possible extensions

The model, in it's present state, is very simple and there are a number of possibilities to extend it. As mentioned earlier in this chapter the reach would improve by separating the mouthpiece from the shoulder. The model would still only have one degree of freedom. Introducing a wrist and a movable shoulder would take the model closer to reality, but also increase the complexity of the system by adding more degrees of freedom. There might be a way around this though. The shoulder can be assumed to move very little except for when reaching for the higher positions. Letting the motion of the shoulder be some function of e.g. the angle between the upper arm and the trombone would probably be a good approximation. The angle of the wrist is a bit more complicated as it not only depends on the present position, but also on the previous. Moving from position two to three can be done by almost only moving the wrist, if desired. When going from position six to three the whole arm has to be moved, and the final wrist angle will probably be a little different compared to the first case. Still it is worth a try, to see if it's possible to let also the wrist be a function of another system variable.

A more thorough examination of the friction in the system and how it affects the motion would be interesting, but the results seem to show that the importance of friction is secondary.

To better validate the results from this study, more extensive experiments are necessary. Comparisons with two subjects can give good indications of correlations, but it is hard to draw any more general conclusions. Evaluating the measurements so far, would be a lot of help in constructing a new set of experiments.

The results show that gravity plays an important role in trombone playing. Would it be possible to play the trombone in a state of weightlessness? It can probably be done, but it would require new technique. This is made plausible by the tiring nature of work as reported by astronauts operating in free fall. The potential energy of gravity will not be present and apart from the small loss due to friction, all kinetic energy put into the system at one position will still be there at the next. Unfortunately it hardly lies within the economic limits to send a trombone player into space, but the study of how astronauts move in space might provide some kind of answer.

6.10 Conclusions

We have seen in this work that the playing of the trombone provides an interesting, yet relatively simple example of human movement. A simple model of this motion has been proposed. It should be noted that despite its simplicity the model is very insightful, and for this reason can be of more interest than an more complex and hard to understand model. It of course also simplifies the numerical work.

Comparison of the calculations made using this model and the measurements has shown the agreement is good. In playing the trombone the player makes use of gravity to save energy. A force is applied in the beginning and end of the motion, but between that the arm is let to pendle. The models behavior is stable for variations in parameters, like tilt angle, arm length and arm mass.

The trombonists interpretation of the music is of great importance. The difference between subject P and S is clearly visible and shows that it is unlikely that even the best model would ever would be able to exact copy the behavior of human player.

This study also allows us to draw some conclusions in regard to the training effects in human movement. A skilled player uses less energy, by using the earths gravity and learning the precise amount of force needed for the initial impulse for going from one position to another. A beginner is likely to use force during a larger portion of the movement in order to control the motion. This relation is valid for several human motions, as has been discussed above. One of the most energy efficient human motions is walking, but then most people have had a lot of training since their early years.

Chapter 7

Acknowledgments

I'd like to thank my supervisor, Professor Martin Lesser, for his help and time. Thanks to Doctor Virgil Stokes, for providing real data to work with. He has also been very helpful in discussing interpretations of the results.

I'm grateful to David Hedfors for discussing the art trombone playing with me and volunteering as an experimental subject. Thanks also to Jesper Höjeberg for help with human anatomy.

My colleagues have all been very helpful and have shared their knowledge with me, ranging from the mechanics of my subject to computer related problems.

And finally thanks to Måns Elenius for discussing, reading and commenting on my thesis.

Appendix A

Subject data

Subject S has played the trombone for 11 years and is now a student at the Department of Music at Boston University. Subject P is a professional trombone player and plays in a jazz orchestra.

Subject S' trombone was a type Bach-42B (tenor), with a Schilke 51 mouthpiece. Subject P's trombone was a type Shires Custom (tenor), with a Stork 5 mouthpiece.

	Subject S	Subject P
Weight (kg)	61.2	100
Height (m)	1.68	1.83
Upper right arm length (m)	0.31	0.33
Lower right arm length (m)	0.25	0.28
Right hand length (m)	0.195	0.20
Birthdate	April 5, 1979	October 11, 1964
Gender	female	male

Appendix B

Anthropometric Parameters for the Human Body

These are the parameters [5] used in the calculations in the chapter Models and Methods.

B.1 Segment lengths

The lengths are expressed in percentages of total body height.

Segment	Males	Females
Upper arm	17.20	17.30
Forearm	15.70	16.00
Hand	5.75	5.75

B.2 Segments weights

The weights are expressed in percentages of total body weight.

Segment	Males	Females
Upper arm	3.25	2.90
Forearm	1.87	1.57
Hand	0.65	0.50

B.3 Segmental center of gravity location

The locations are expressed in percentages of segment length, measured from the proximal end of the segment.

Segment	Males	Females
Upper arm	43.6	45.8
Forearm	43.0	43.4
Hand	46.8	46.8

B.4 Segmental radii of gyration

The radii of gyration is expressed in percentages of segment length, measured from the proximal end of the segment.

Segment	Males	Females
Upper arm	54.2	56.4
Forearm	52.6	53.0
Hand	54.9	54.9

Appendix C

Musical excerpt

This is the musical excerpt, No. 6 from Rochut book one[10], that was used in the experiments.

Figure C.1:

Appendix D

Random sequences

The two subjects each played a random sequence in the experiment.

D.1 Subject P

Subject P played a sequence consisting of the following positions.
$5 \rightharpoonup 4 \rightharpoonup 3 \rightharpoonup 4 \rightharpoonup 5 \rightharpoonup 2 \rightharpoonup 6 \rightharpoonup 5 \rightharpoonup 3 \rightharpoonup 6 \rightharpoonup 4 \rightharpoonup 6 \rightharpoonup 4 \rightharpoonup 6 \rightharpoonup 5 \rightharpoonup 3 \rightharpoonup 4 \rightharpoonup 7 \rightharpoonup 4 \rightharpoonup 6 \rightharpoonup 2 \rightharpoonup 4 \rightharpoonup 2 \rightharpoonup 6 \rightharpoonup 1 \rightharpoonup 6 \rightharpoonup 2 \rightharpoonup 6 \rightharpoonup 4 \rightharpoonup 5 \rightharpoonup 3 \rightharpoonup 1 \rightharpoonup 2 \rightharpoonup 3 \rightharpoonup 6 \rightharpoonup 3 \rightharpoonup 6 \rightharpoonup 3 \rightharpoonup 5 \rightharpoonup 2 \rightharpoonup 3 \rightharpoonup 7 \rightharpoonup 6 \rightharpoonup 1$

D.2 Subject S

Subject S played a sequence consisting of the following positions.
$5 \rightharpoonup 4 \rightharpoonup 3 \rightharpoonup 4 \rightharpoonup 5 \rightharpoonup 2 \rightharpoonup 6 \rightharpoonup 5 \rightharpoonup 3 \rightharpoonup 6 \rightharpoonup 4 \rightharpoonup 6 \rightharpoonup 4 \rightharpoonup 6 \rightharpoonup 5 \rightharpoonup 3 \rightharpoonup 2 \rightharpoonup 6 \rightharpoonup 2 \rightharpoonup 4 \rightharpoonup 1 \rightharpoonup 4 \rightharpoonup 1 \rightharpoonup 3 \rightharpoonup 2 \rightharpoonup 4 \rightharpoonup 2 \rightharpoonup 4 \rightharpoonup 6 \rightharpoonup 2 \rightharpoonup 4 \rightharpoonup 2 \rightharpoonup 6 \rightharpoonup 1 \rightharpoonup 6 \rightharpoonup 2 \rightharpoonup 6 \rightharpoonup 4 \rightharpoonup 5 \rightharpoonup 3 \rightharpoonup 1 \rightharpoonup 2 \rightharpoonup 3 \rightharpoonup 6 \rightharpoonup 3 \rightharpoonup 6$

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