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# Transport of Bodies Small Compared to the Mean Free Path in Non-uniform Gases

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## Preface

This thesis consists of one introduction to the subject and two papers.

**Paper 1.** Borg, K. I. & Söderholm, L. H. 2000 Thermophoresis of axially symmetric bodies. *Rarefied Gas Dynamics*. Submitted.

**Paper 2.** Söderholm, L. H. & Borg, K. I. 2001 Shearing Phoresis. To be submitted.

### Division of work between authors

Paper 1: Karl Borg had the idea for paper 1. Karl Borg made all the calculations, theoretical and numerical, under Lars Söderholm's supervision.

Paper 2: Lars Söderholm had the idea in paper 2 and made a preliminary calculation of the force and the torque. Karl Borg made the final calculation of force and the torque, calculated the asymptotics, and made the stability analysis, under Lars Söderholm's supervision. Karl Borg also made the numerical calculations.



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## CHAPTER 1

### Introduction

Thermophoresis and Shearing Phoresis belong to a class of interesting phenomena that appear in a gas with gradients in the temperature or the velocity in the limit where a continuum mechanical description breaks down.

If the gas is heat conducting, we have the well-known phenomenon of *thermophoresis*: A small body suspended in the gas will be transported towards the cooler parts of the gas. Thermophoresis as a phenomenon has been known for a long time, and several authors have approached the problem. For example, Einstein calculated the final velocity of a spherical particle in a heat conducting gas using elementary kinetic theory. A recent review of the phenomenon is given in an article by Sone, [2].

Thermophoresis is used in some industrial processes. For instance, to accumulate small particles in a gas onto a solid boundary, a temperature gradient is applied normal to the boundary. As a consequence, the small particles start to drift towards the cooler part of the gas, and finally wind up on the solid boundary. These applications are studied in [11] and in [12].

The first systematic attempt to describe the thermophoresis phenomenon using kinetic theory is found in an article by Waldmann from 1959, see [3]. That work was made under the assumption that the mean free path of the gas is much larger than the dimension of the body. Further results are found in a variety of articles, but these results mostly apply to bodies large compared to the mean free path. Some of these works deal with corrections to the Navier-Stokes equations. It should be mentioned that in this region, negative thermophoresis appears for bodies with high thermal conductivity, cf [4].

In paper (I) thermophoresis of an axially symmetric body is studied in the limit where the typical length of the body is much smaller than the mean free path of the gas.

If the gas is subject to a gradient in the velocity, and if the suspended body is small compared to the mean free path in the gas, *Shearing Phoresis* occurs. This means that the small body will be transported along the eigendirections of the symmetric and traceless part of the velocity gradient.

Shearing phoresis is, to the best of the knowledge of the authors, a new phenomenon. Bell and Schaaf, [6], calculated the aerodynamic forces on a cylinder in shearing gas flow in 1953. Their result for the force from the shearing is valid for a finite  $Ma$ -number. In paper (II) it is shown that the shearing will give rise to a force of first order in the global  $Ma$ -number. This force does not appear in the results in [6], because the cylinder is mirror symmetric. It is however apparent from their results that there exists a second-order force from the shearing acting on the cylinder.

Several authors, however, have studied the dynamics of larger bodies in a shearing gas, cf. [8],[9],[10]. These works deal with the problem in the continuum limit, and in particular corrections to the Navier-Stokes equations. A phenomenon of transverse diffusion of spherical particles in a shear layer is described in [13]. This phenomenon occurs in a dilute suspension of particles as a result of the collisions of the particles. Further results on this topic are found in [14] and [15].

## CHAPTER 2

### General theory

This thesis deals with a transport of small bodies that occurs in nonuniform gases. In particular, gases with gradients in the temperature and in the velocity are considered. The surrounding gas is assumed to be moderately rarefied. This means that the mean free path,  $\lambda$ , of the gas (that is, the typical distance a gas molecule travels between two collisions) is much smaller than the typical length  $L$  over which the temperature and the velocity varies. This assumption defines the region of the kinetic theory of gases called *the continuum limit*. It is in this limit the Navier-Stokes equations provide a satisfactory description of the non-uniform gas.

The typical length of the small body,  $R$ , is in turn taken to be much smaller than the mean free path of the gas. In this region, the Navier-Stokes equations cannot be applied. Thus the complete description of the problem of the small body in the inhomogeneous gas involves three different length scales:  $R$ ,  $\lambda$  and  $L$ , and for these the assumption  $R \ll \lambda \ll L$  is made.

#### 2.1. Kinetic Theory and The Boltzmann equation

Consider first an ensemble of  $N$  identical mono-atomic interacting molecules. To describe the evolution of the entire system, we introduce the  $N$ -particle distribution function  $F_N$ , defined on the  $6N$ -dimensional phase space spanned by the resp. positions and velocities of the  $N$  molecules. We call this space  $\Gamma$ -space. The 6-dimensional phase space of a single gas molecule we denote by  $\gamma$ -space. The interpretation of  $F_N$  is that  $F_N d^3 x_1 d^3 p_1 d^3 x_2 d^3 p_2 \dots d^3 x_N d^3 p_N$  is the probability that the position and momentum of molecule no 1 lie within  $d^3 x_1 d^3 p_1$ ; the position and momentum of molecule 2 lie within  $d^3 x_1 d^3 p_1$  etc. The interaction between the molecules is given by Newton's laws, and consequently the  $N$ -particle distribution function  $F_N$  satisfies the Liouville equation,

which in the absence of external forces is given by

$$\frac{\partial F_N}{\partial t} + \sum_{i=1}^N \mathbf{c}_i \cdot \frac{\partial F_N}{\partial \mathbf{x}_i} = 0 \quad (2.1)$$

The first assumption to be made is that the mono-atomic gas molecules interact via a spherically symmetric potential, that is, the potential satisfies

$$V(\mathbf{r}) = V(r).$$

We assume that the gas is dilute. This means that the typical diameter of the cross-section of the interaction between the molecules,  $d$ , is small compared to  $\delta_0$ , the average distance between gas molecules. For air under standard conditions,  $\frac{d}{\delta_0} \sim \frac{1}{7}$ .

Since the gas molecules are identical, the state of the system is invariant under interchange of position of the molecules in  $\gamma$ -space. Therefore, to an assembly of  $N$  molecules in  $\gamma$ -space, there corresponds  $N!$  points in  $\Gamma$ -space. We now introduce the function  $f_N = f_N(t, \mathbf{x}_1, \mathbf{p}_1, \dots, \mathbf{x}_N, \mathbf{p}_N)$  by writing the probability of finding the system at the points  $(\mathbf{x}_i, \mathbf{p}_i)$  as

$$f_N(t, \mathbf{x}_1, \mathbf{p}_1, \dots, \mathbf{x}_N, \mathbf{p}_N) d^3 x_1 d^3 p_1 \dots d^3 x_N d^3 p_N;$$

Then we must have  $f_N = N! F_N$ .

Further, we impose the condition of molecular chaos. This is a good approximation if the correlations vanish. This condition admits factorizing the distribution function into a product of identical one-particle distribution functions, each describing a 'typical' gas molecule, that is,

$$f_N = f(\mathbf{x}_1, \mathbf{p}_1) \cdot f(\mathbf{x}_2, \mathbf{p}_2) \dots f(\mathbf{x}_N, \mathbf{p}_N). \quad (2.2)$$

Then the Liouville equation is reduced to an integro-differential equation for this one-particle distribution function: The Boltzmann equation. In the absence of external forces, it takes the form

$$\frac{\partial f}{\partial t} + \mathbf{c} \cdot \nabla f = \mathcal{J}(f, f), \quad (2.3)$$

where the *collision operator*  $\mathcal{J}(f, f)$  is given by

$$\mathcal{J}(f, f) = \int \int \int (f' f'_1 - f f_1) g b d b d \epsilon d \mathbf{c}_1, \quad (2.4)$$

see [1]. In the expression for  $\mathcal{J}(f, f)$ ,  $b$  is the impact parameter in the binary collision,  $\epsilon$  is the angle of impact,  $g$  is the relative velocity of the colliding molecules,  $\mathbf{c}$  is the velocity of the typical gas molecule,  $\mathbf{c}'$

is the velocity of the molecule it collides with.  $\mathbf{c}_1$  and  $\mathbf{c}'_1$  are the corresponding velocities after the collision. Further,  $f_1 = f(\mathbf{c}_1)$ ,  $f'_1 = f(\mathbf{c}'_1)$  and  $f'_1 = f(\mathbf{c}'_1)$ . Note that the collision operator is bilinear in  $f$ . On the left-hand-side of the Boltzmann equation we have the material time derivative of the one-particle distribution function  $f$ . On the right-hand-side, the collision operator  $\mathcal{J}(f, f)$  determines the rate of change in the one-particle distribution function due to collisions between the gas molecules. This equation is much simpler to handle than the Liouville equation. The Boltzmann equation lacks the time-reversal symmetry exposed by the Liouville equation.

If we denote the length scale of the problem for which we want to solve Boltzmann's equation by  $\Lambda$ , an order of magnitude estimate of the Boltzmann equation shows that the left-hand side is of the order of  $\frac{\lambda}{\Lambda}$  times smaller than the collision term on the left hand side (we recall that  $\lambda$  denotes the mean free path of the gas. That is, in non-dimensional variables the Boltzmann equation looks like

$$\frac{\lambda}{\Lambda} \left( \frac{\partial f^*}{\partial t^*} + \mathbf{c}^* \cdot \nabla^* f^* \right) = \mathcal{J}^*(f^*, f^*). \quad (2.5)$$

(the \*-superscript will be dropped in what follows.) For our purposes, there are two separate length scales for which we want to solve Boltzmann's equation: Firstly, we need a description of the gas subjected to the macroscopical gradients, that is,  $\Lambda = L$ . Secondly, we want to examine the situation for the small body, that is,  $\Lambda = R$ . Since by assumption  $\frac{\lambda}{L} \ll 1$  and  $\frac{\lambda}{R} \gg 1$ , the corresponding investigations turns out to be quite different. For  $\frac{\lambda}{L} \ll 1$  we use the so-called *Chapman-Enskog solution*, [5], to the Boltzmann equation. For  $\frac{\lambda}{R} \gg 1$  we end up in the equations of *Free Molecular Flow*.

### 2.1.1. The Chapman-Enskog Solution

In the limit where  $\lambda \ll L$  the solution to the Boltzmann equation can be expanded in the small parameter  $\frac{\lambda}{L} \equiv Kn$ , where the *Knudsen number*  $Kn$  measures the rarefaction of the gas. To zeroth order in  $Kn$ , we acquire the solution for a gas in local equilibrium: the Maxwellian,  $f^{(0)}$ , given by

$$f^{(0)} = n \left( \frac{2\pi k_B T}{m} \right)^{-3/2} e^{-m\mathbf{c}^2/2k_B T}. \quad (2.6)$$

In this expression,  $n$  is the number of molecules per unit area,  $k_B$  is Boltzmann's constant,  $T$  is the temperature,  $m$  is the mass of a gas

molecule and  $\mathbf{c}$  is the velocity of a gas molecules. The Maxwellian is thus the solution to  $\mathcal{J}(f, f) = 0$ . This solution describes a gas in local equilibrium, with no stresses and no heat currents.

To take into account the influence of the non-uniformity of the distribution function, we must include deviations from the Maxwellian in the distribution function. Therefore, we seek the solution to first order in  $Kn$ . This solution can be found through the Chapman-Enskog expansion. In case of a gas subject to gradients in the temperature and the velocity  $\mathbf{v}$ , this solution takes the form

$$f = f^{(0)} \left[ 1 - \frac{1}{nT} \sqrt{\frac{2k_B T}{m}} A(\mathcal{C}^2) \mathcal{C}_i T_{,i} - \frac{2}{n} B(\mathcal{C}^2) \mathcal{C}_{\langle i} \mathcal{C}_{j \rangle} v_{i,j} \right]. \quad (2.7)$$

Here,  $\mathcal{C}_i \equiv \sqrt{m/2k_B T} c_i$  is a non-dimensional molecular velocity. The function  $A$  is related to the heat conduction of the gas and  $B$  to the shearing.  $A$  and  $B$  both depend on the intra-molecular potential and has the order of magnitude of  $Kn$ . Further,  $\langle \dots \rangle$  denotes symmetric and traceless part.

This solution is well-established and gives correct values of the coefficients of heat conductivity and viscosity. This completes the macroscopic description of the gas.

## 2.2. Free Molecular Flow

We must now adopt the perspective of the gas of the small body. On the length scale of  $R$  the molecules of the surrounding gas do not collide with each other. Still, they will of course collide with the small body. Accordingly, in the limit where  $\frac{\lambda}{R} \gg 1$  the collision operator drops out of the Boltzmann equation, and we have

$$\frac{\partial f}{\partial t} + \mathbf{c} \cdot \nabla f = 0. \quad (2.8)$$

This is the Liouville equation for the one-particle distribution function  $f$ . As a consequence, the gas molecules move along straight lines. Further, far away from the body, the distribution function has to approach the Chapman-Enskog solution.

The body is taken to be convex. By this assumption, a gas molecule will never undergo two consecutive collisions with the small body. It is further assumed that the body is small enough for the test-body approximation to be valid, that is, the gas influences the body, but the impact of the small body on the gas is negligible.

The fate of a gas molecule that has collided with the body is not obvious. A rather well-established model that can be used is the Maxwell boundary condition. This model states that a fraction of the stream of molecules incident on the body surface is reflected specularly (or like a particle colliding with a solid wall). The remaining fraction of the incident stream of molecules reaches thermal equilibrium with the body (complete energy accommodation is assumed), and is reflected as a local Maxwellian. This fraction is said to be diffusely reflected. It is easy to see that the part specularly reflected on the surface element does not transfer any tangential momentum. Therefore, the fraction number is called, 'the accommodation coefficient of tangential momentum', and is usually denoted by  $\alpha_\tau$ . Put in mathematical terms, the reflected stream  $f^{(r)}$  fulfills

$$f^{(r)}(\mathbf{c}) = (1 - \alpha_\tau)f^{(i)}(\mathbf{c}') + \alpha_\tau \frac{n^{(r)}}{n} f^{(0)}(\mathbf{c}). \quad (2.9)$$

Here,  $f^{(i)}$  is the distribution function describing the incident stream of molecules given by the first order Chapman-Enskog solution. Further,  $c'_i = c_i - 2n_i n_j c_j$ .  $n^{(r)}$  is a number density to be determined, and  $f^{(0)}$  is a Maxwellian with the temperature of the body.

Now we can describe the particle flux incident on a surface element  $dS$  with unit normal  $\mathbf{n}$  on the body,  $N^{(M)}$ , in terms of the distribution function  $f$ :

$$N = - \int_{\mathbf{n} \cdot \mathbf{c} < 0} (\mathbf{n} \cdot \mathbf{c}) f d^3 c, \quad (2.10)$$

and the momentum flux incident on the surface element according to

$$P_i = - \int_{\mathbf{n} \cdot \mathbf{c} < 0} (\mathbf{n} \cdot \mathbf{c}) m c_i f d^3 c. \quad (2.11)$$

With the Maxwell model we can also in a similar manner formulate the out-flux of mass and momentum by the stream of molecules that leave the surface. From demanding that the surface of the body is impermeable we can determine  $n^{(r)}$ . Then we are in a position to calculate the net momentum transferred from the gas to the surface element and thus arrive at an expression for the force and the torque exerted on the body surface element by the gas. The corresponding expressions can then be integrated over the total surface of the body to yield the total force and torque acting on the body.



## CHAPTER 3

### Introduction to the papers

In the papers (I) and (II), it is shown that a body small compared to the mean free path in a nonuniform gas is set in a motion relative to the surrounding gas due to gradients in the temperature and the velocity. The typical length of the small body,  $R$ , is assumed to be much smaller than the mean free path of the gas. Further, the typical length over which the temperature and the velocity varies,  $L$ , is much larger than the mean free path of the gas. The body surface is convex. The way of finding the force and torque acting on the small body presented in preceding paragraphs is applied.

#### 3.1. Axially symmetric body

The small bodies under consideration here are all axially symmetric. This means that the body has no other geometrical direction than the axis of symmetry,  $\mathbf{N}$ .

Axially symmetric bodies can possess an additional symmetry: If there exists a plane orthogonal to the axis of symmetry in which the body is mirror symmetric, the body is said to be *equatorially symmetric*.

Further, geometric integrals over the total body surface of local geometrical quantities such as the unit normal  $\mathbf{n}$  and the vector from the center of mass of the body to a point on the body surface,  $\mathbf{x}$ , must all be isotropic functions of the axis of symmetry  $\mathbf{N}$ . That is, these geometric integrals are sums of products of  $N_i$  and the Kronecker delta  $\delta_{ij}$ . An example of an integral of this type is given by  $\int_S n_i n_j dS$ , and due to isotropy it must fulfill

$$\int_S n_i n_j dS = S \left( c_1 \frac{1}{3} \delta_{ij} + c_2 N_i N_j \right).$$

Here,  $S$  is the total body surface area. The non-dimensional scalar coefficients  $c_1$  and  $c_2$  can be found from successive contractions with products of  $N_i$  and the Kronecker delta. When this is done, it is convenient to write the tensor as the sum of one isotropic part proportional to  $\delta_{ij}$  and

one symmetric traceless part proportional to  $N_{\langle i}N_{j \rangle} = N_iN_j - \frac{1}{3}\delta_{ij}$ . The tensor integral then takes the form

$$\int_S n_i n_j dS = S \left( \frac{1}{3}\delta_{ij} + aN_{\langle i}N_{j \rangle} \right).$$

One finds that  $a$  measures symmetric deviations of the body shape from a sphere: For a coin shaped body,  $a = 1$ , and for a needle shaped body,  $a = -\frac{1}{2}$ . For a sphere,  $a = 0$ .

### 3.2. Force and torque on the body

The force,  $d\mathbf{F}$ , acting on a surface element of the body can now be obtained by calculating the net transfer of momentum to the surface element from the gas. This force will in general also produce a torque acting on the body surface element according to  $d\mathbf{M} = \mathbf{x} \times d\mathbf{F}$ , where  $\mathbf{x}$  is the vector from the center of mass of the body to the surface element. As a consequence, the body will start to move and rotate, and a full rigid-body motion results. A body-fixed frame of reference is introduced, where one of the principal axis is chosen to be  $\mathbf{N}$ . The time evolution of this frame can then be related to the angular velocity by Euler's equations. The resulting motion of the body will generate additional forces and torques on the body.

Both the force and the torque acting on the surface element of the body will be tensor functions of the unit normal  $\mathbf{n}$  and the vector  $\mathbf{x}$ . To obtain the net force and torque acting on the body, we must integrate the force and the torque acting on a surface element of the body over the total body surface. This can be done using the method described above.

The total force will, to the present order of approximation, contain three different forces: One force arising from the non-uniformity of the gas, that is, from the heat currents or the stresses; One force depending on the velocity of the center of mass of the body, and one force depending on the rotation of the body. The total torque on the body can be split up in a similar manner into three corresponding parts.

### 3.3. Asymptotic solutions

Given the force, the torque and Euler's equations, the equations of motion of the rigid body can be formulated. This resulting system is a set of non-linear coupled ordinary differential equations, that contain scalar coefficients that depend on the shape of the body.

With these at hand, asymptotic solutions with no rotation and a constant velocity can be found. These differ very much in character between thermophoresis and Shearing Phoresis. This is due to the difference in symmetry between the vectorial heat current and the tensorial stress tensor.

In order to investigate the stability of the resulting asymptotic states, the equations of motion of a test body are linearized close to these states. The test body is a 'double cone'. It consists of two cones, pointing in the opposite directions and glued together at their common base. The radius of the base is denoted by  $D$ , and the total length by  $R$ .

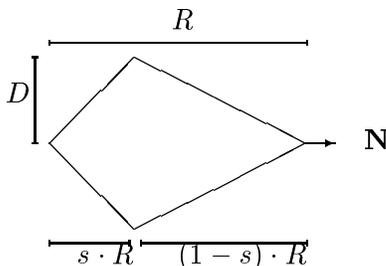


FIGURE 3.1. The 'double cone'

The base is situated a distance  $s \cdot R$  from the left cusp, where the dimensionless parameter  $s$  obeys  $0 \leq s \leq 1$ . When  $s = 0$  the double cone degenerates into a single cone with its cusp pointing in the direction of  $\mathbf{N}$ . When  $s = 1$  we recover another single cone, pointing in the direction of  $-\mathbf{N}$ . This body is in general not equatorially symmetric, except when  $s = \frac{1}{2}$ .

In the case of Shearing Phoresis, the parameter  $s$  is restricted to vary in the interval  $0 \leq s \leq \frac{1}{2}$ ; Thus, in this case,  $\mathbf{N}$  always points in the same direction as the sharpest cusp of the double cone.

Given the linearized system, the eigenvalues of the infinitesimal motion of the double cone close to the asymptotic states can be calculated numerically. The stability character of these states can in this way be obtained for the double cone.

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