Development and analysis of turbulence models for flows with strong curvature and rotation

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Olof Grundestam

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Royal Institute of Technology
Department of Mechanics
SE-100 44 Stockholm, Sweden

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Development and analysis of turbulence models for flows with strong curvature and rotation

Olof Grundestam
Department of Mechanics, Royal Institute of Technology
SE-100 44 Stockholm, Sweden.

Abstract

An explicit algebraic Reynolds stress model (EARSM) based on a pressure strain rate model including terms tensorially nonlinear in the mean velocity gradients is developed in order to improve predictions for flows with strong curvature and/or rotation. This work has been carried out in the context of a collaborative international project on high-lift aerodynamics. For 2D mean flows the nonlinear terms can easily be accounted for in the model formulation. This is not the case for 3D mean flows and approximations making the 2D and 3D mean flow formulations consistent are suggested. The proposed EARSM, the parent-EARSM and the corresponding differential Reynolds stress models (DRSM) are tested for spanwise rotating channel flow and axially rotating pipe flow. The model predictions are compared to experimental and DNS data. The nonlinear extensions are shown to have a significant effect on the flow predictions, somewhat less pronounced for the DRSM though. The turbulent diffusion modelling in the EARSM computations is important for the rotating pipe. It is shown that by using a Daly and Harlow diffusion model, turbulence levels in good agreement with experiments and DRSM can be achieved. However, by using a simpler effective eddy viscosity based diffusion model the turbulence kinetic energy levels are drastically overpredicted. Finally the proposed EARSM is tested on a standard high-lift configuration. The EARSM predictions are compared with experiments and the predictions made by the standard $K-\omega$ two-equation model.

Descriptors: Turbulence model, nonlinear modelling, streamline curvature, high-lift aerodynamics.

Preface

This thesis is based on and contains the following papers:

Paper 1. Olof Grundestam, Stefan Wallin & Arne Johansson 2004 An explicit algebraic Reynolds stress model based on a nonlinear pressure strain rate model. *To be submitted*

Paper 2. Olof Grundestam, Stefan Wallin & Arne Johansson 2004 Observation on predictions of fully developed rotating pipe flow using differential and explicit algebraic Reynolds stress models. *To be submitted*

Paper 3. Olof Grundestam, Gustaf Mårtensson, Stefan Wallin & Arne Johansson 2004 Modelling rotating turbulent channel flow. *To be submitted*

Paper 4. Olof Grundestam & Stefan Wallin 2004 Application of EARSM turbulence models to high-lift aerodynamics applications. *Technical report*

Division of work between authors

The work presented in this thesis has been done in collaboration with other researchers. Professor Arne Johansson has acted supervisor in paper 1-3. The first author did the major part of the work in paper 1 and 2. The work in paper 3 was performed on an equal basis with Gustaf Mårtensson. Dr Stefan Wallin has contributed with comments and discussions on the work and manuscripts. A slightly shorter version of paper 1 has been presented at TSFP-3 in Sendai, Japan, June 2003. The work on paper 4 was supervised by Dr Stefan Wallin.

Contents

CHAPTER 1

Introduction

The problem of accurately predicting the fluid flow around bodies and over surfaces is a cornerstone problem in the field of engineering sciences. Traditionally, aeronautical vehicle design has been based on experiments, experience from previous designs and a "good" bit of engineering intuition. Due to the lack of computational power, it is only during the last couple of decades that computational fluid dynamics (CFD) has offered an alternative to experiments. To numerically predict a fluid flow is a considerable challenge. It is in principle possible to solve the governing equations in a so called direct numerical simulation (DNS). But since the smallest length and timescales need to be resolved, the computational effort needed to accurately predict the flow characteristics of a turbulent flow rapidly becomes enormous. Therefore the turbulence needs to be modelled.

Cornerstone problems for turbulence models are flows affected by rotation, streamline curvature and separation. These phenomena are of high interest since they play a determining role in many engineering applications such as turbomachinery and aeronautics. High-lift aerodynamics is an application of great interest where all these effects are present. Since the maximum load of an aircraft is determined by its take-off and landing performance, it is crucial, not least from an environmental and economical point of view, to be able to make accurate predictions by means of CFD when designing new high-lift configurations. Due to the complex nature of this type of flow, standard two-equation turbulence models are in general inadequate making the development of more sophisticated models such as Reynolds stress transport models, explicit algebraic Reynolds stress models and large eddy simulations necessary.

The turbulence modelling approach dicussed here, is based on Reynolds averaging meaning that the instantaneous flow is decomposed into a mean part which is constant or varying slowly and a fluctuating part which is allowed to vary rapidly. Other approaches have been proposed, such as large eddy simulations and two-point correlation closures.

The Reynolds decomposition used, is represented by $\tilde{u}_i = U_i + u_i$ and $\tilde{p} = P + p$ for the velocity and pressure respectively. Quantities marked with $\tilde{}$ denote the corresponding instantaneous value, while capital and small letters denote the mean and fluctuating part, respectively. By taking the mean of the Navier-Stokes equations, using the decomposition, the Reynolds avergaged Navier-Stokes equations are achieved. It turns of that the governing equations of the mean flow have a dependence on the fluctuating velocity in the form of a quadratic velocity correlation $\overline{u_i u_j}$, for incompressible flow. The term $-\rho\overline{u_i u_j}$ (ρ is the density) is referred to as the Reynolds stress and normally dominates over the viscous stresses in turbulent flows. The Reynolds stress tensor adds six unknowns to the Reynolds averaged Navier-Stokes equations, which means that extra equations/assumption are needed to close the system of equations for the mean flow.

The natural way to close the system of equations would then be to derive the transport equation of the Reynolds stresses and to solve this in conjunction with the mean flow equations. The transport equation of the Reynolds stresses reads

$$\frac{D\overline{u_i u_j}}{Dt} - \mathcal{D}_{ij} = \mathcal{P}_{ij} - \varepsilon_{ij} + \Pi_{ij} \tag{1.1}$$

where the first term is the material derivative, representing the temporal derivative and the advection by the mean flow, the following terms represent diffusion, production, dissipation and pressure strain rate, respectively. However, the diffusion, dissipation and pressure strain rate tensors are dependent on unknown correlations, which in turn will have governing transport equations dependent on new unknown correlations and so on. Therefore $\mathcal{D}_{ij}$, ε_{ij} and Π_{ij} need to be modelled in terms of the known quantities. Widely used models have been proposed by for instance Launder et al. (1975), Daly and Harlow (1970) and Speziale et al. (1991). In addition, (1.1) needs to be complemented with a transport equation of some quantity used for determining the lengthscale. The dissipation of the turbulence kinetic energy, ε, and the inverse turbulence timescale, ω, are quantities frequently used for this purpose .

The model incorporating the full transport equation of the Reynolds stresses is commonly called a differential Reynolds stress model (DRSM), Reynolds stress transport model (RST) or second moment closures (SMC). Full DRSM closures have been proposed by for instance Speziale et al. (1991) and Sjögren and Johansson (2000).

A widely used type of model, that is also the lowest level of model discussed in this thesis, is the (linear) eddy-viscosity-based model. The $K-\varepsilon$ and $K-\omega$ models, where K is the turbulence kinetic energy, are typical closures of this type. The original Bousinessq hypothesis, Boussinesq (1877), related the the shear component of the Reynolds stress tensor in nearly parallel flows to the cross stream mean velocity gradient. The generalization of this approach, which is commonly used in two-equation models, implies that the Reynolds stress anisotropy is proportional to the mean strain rate tensor and since the corresponding coefficient (ν_T) is usually determined by scalar quantities (i.e. K and ε), the resulting Reynolds stress anisotropy, $a_{ij} = \overline{u_i u_j}/K - 2\delta_{ij}/3$, becomes unsensitive to rotation. Therefore, this type of models are ill suited for flows in which the effects of rotation and curvature are strong. It is, however, possible to make this type of model respond to rotation by introducing a dependence on rotation in ν_T. This has been achieved by for instance Pettersson et al. (1999).

An alternative intermediate level of modelling between the eddy-viscosity based two-equation models and the DRSM is the explicit algebraic Reynolds stress model (EARSM). In this approach one considers the transport equation of the Reynolds stress anisotropy, $a_{ij} = \overline{u_i u_j}/K - 2\delta_{ij}/3$, and neglects the advection and diffusion of a_{ij} due to the weak equilibrium assumption, see Rodi (1976). Thereafter, the remaining relation, the algebraic Reynolds stress model (ARSM) relation, is solved explicitly by assuming that a_{ij} is dependent on the mean flow gradients. In addition to the relation between a_{ij} and the mean velocity gradients, the turbulence timescale and kinetic energy have to be determined. This is usually done by using a standard two equation platform. A pioneering work on EARSMs was done by Pope (1975) where the ARSM-relation was solved for 2D mean flows. Other formulations have been proposed by Gatski and Speziale (1993), Girimaji (1996) and Wallin and Johansson (2000).

While the DRSM can be expected to capture the flow physics in a wide variety of flows best, its complex nature provides the biggest challenges when it comes to actually solving the system of equations. The EARSM, which on the other hand, has the advantage of being similar to a standard two-equation model in terms of computational cost but still being based on the transport equation of a_{ij} and hence being expected to perform better than standard two equation models in more complicated flows, can in many cases provide an adequate compromise. However, the neglection of the advection and diffusion greatly affects the EARSMs predictive capabilities when the flow is strongly affected by streamline curvature effects, history effects and diffusion. From an engineering point of view, streamline curvature represent perhaps the most important flow feature since essentially all flows under rotation and over curved surfaces are affected by streamline curvature effects, this will supposedly have a negative effect on the predictive capability of EARSMs for a wide range of flows. However, by imposing the weak equilibrium assumption in a streamline oriented curvilinear coordinate system, a term emanating from the advection of a_{ij} can be derived. The inclusion of this term in the ARSM-relation yields a systematic improvement of the weak equilibrium assumption in that it gives an approximation of the neglected advection of a_{ij} in a streamline based system. The formulation of the curvature correction was first presented by Girimaji (1997) and Sjögren (1997). Since then the formulation has been refined by for instance Wallin and Johansson (2002). In the present work the strain rate based curvature correction proposed by Wallin and Johansson has been adopted. For all the test cases in paper 1, the contribution from the curvature correction exactly corresponds to the advection. This is, however, not true in more general flows, but still, the adoption of the curvature correction brings, at least in principle, an EARSM one step closer to a full DRSM.

The purpose of the present work is partly to investigate how far the modelling used in EARSM can be taken in terms of using models nonlinear in the strain and rotation rate and/or the Reynolds stress anisotropy tensor. By using nonlinear modelling more of the flow physics can be incorprated in the turbulence model. For instance, Sjögren and Johansson (2000) adopted models nonlinear in the Reynolds stress anisotropy to achieve realizability. Also, a term tensorially quadratic in the rotation rate tensor was used, and proven necessary to be able to capture the damped oscillations of anisotropic turbulence subject to pure rotation. For a DRSM, nonlinear modelling does not, in principle, complicate the formulation and the implementation is usually straight forward. For an EARSM, on the other hand, nonlinear models, especially those nonlinear in a_{ij}, radically complicates the solving of the ARSM-relation. Therefore, the outcome of the work of extending the WJ-EARSM discussed in paper 1, is very much restricted to what is possible to do rather than what one would like to achieve in terms of, for instance, realizability. Another key ingredient in the work presented, is the investigation of the differences in the predictions between EARSMs and DRSMs. This has been done by using mainly the EARSM proposed by Wallin and Johansson and the corresponding DRSM. The test cases used are rotating and nonrotating pipe and channel flow, see paper 2 and 3 respectively.

The present work has to a large extent been carried out within the HiAer project. HiAer, which stands for "High level modelling of high ligft aerodynamics", is a collaboration between DLR, ONERA, KTH, HUT, TUB, Alenia, Airbus-D, QinetiQ and FOI and hence involves six European countries. FOI is the coordinator. The goal of the HiAer project is to develop state of the art tools for high lift computations. The scientific work is divided into three workpackages dealing with transition-modelling and prediction, turbulence modelling and implementation and testing. Each workpackage contains a number of subtasks in which one or more partners are involved. KTH has contributed to the turbulence modelling work, and the work presented here describes our contribution so far to the subtasks "Extensions of the classical EARSM" and "Test of new EARSM ideas through comparative studies with a new DRSM".

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Bibliography

Alvelius, K, and Johansson, A.V. 1999, "Direct numerical simulation of rotating channel flow at various Reynolds numbers and Rotation numbers", In doctoral thesis of K. Alvelius, Dept. of Mechanics, KTH, SE–100 44 Stockholm, TRITA-MEK Technical Report 1999:09, ISSN 0348-467X, ISRN KTH/MEK/TR–99/09–SE.

Boussinesq, J. 1877, "Thorie de l'coulement Tourbillant", *Mem. Prsents par Divers Savant Acad. Sci. Inst. Fr.*, **23**, pp. 46–50.

Daly, B.J. and Harlow, F.H. 1970, "Transport equations in turbulence", *Phys. Fluids*, **13**, pp. 2634–2649.

Gatski, T.B., and Speziale C.G., 1993 "On explicit algebraic stress models for complex turbulent flows", *J. Fluid Mech.*, **254**, 59-78

Girimaji, S.S., 1997, "Fully-explicit and self-consistent algebraic Reynolds stress model", *Theor. and Comp. Fluid Dyn.*, **8**, 387-402

Girimaji, S.S., 1997, "A Galilean invariant explicit algebraic Reynolds stress model for turbulent curved flows", *Physics of Fluids*, **9**, 1067-1077

Launder, B.E., Reece, G.J. and Rodi, W., 1975, "Progress in the development of a Reynolds-stress turbulence closure.", *J. Fluid Mech.*, **68**, 537–566.

Pettersson-Reif, B.A., Durbin, P.A., Ooi, A., 1999, "Modeling rotational effects in eddy-viscosity closures", *International Journal of Heat and Fluid Flow*, **20**, 563-573

Pope, S.B. 1975, "A more general effective-viscosity hypothesis", *J. Fluid Mech.*, **72**, pp. 331–340.

Rodi, W., 1976, "A new algebraic relation for calculating the Reynolds stresses", *Z. Angew. Math. Mech*, Vol. 56, pp 219-221.

Sjögren, T. and Johansson, A.V., 2000, "Development annd calibration of algebraic non-linear models for terms in the Reynolds stress transport equations", *Phys. Fluids*, **12**, pp. 1554–1572.

Sjögren, T., 1997, "Development and Validation of Turbulence Models Through Experiment and Computation, *Doctoral Thesis, Royal Inst. of Tech.*, TRITA-MEK 1997:5.

Speziale, C.G., Sarkar, S. andGatski, T.B., 1991, "Modelling the pressure-strain correlation of turbulence : an invariant dynamical systems approach", *J. Fluid Mech.*, **227**, pp. 245–272.

Wallin, S. and Johansson, A.V., 2000, "An explicit algebraic Reynolds stress model for incompressible and compressible turbulent flows", *J. Fluid Mech.*, **403**, pp. 89–132.

Wallin, S. and Johansson, A.V., 2002, "Modelling streamline curvature effects in explicit algebraic Reynolds stress turbulence models", *Int. J. Heat and Fluid Flow*, **23**, pp. 721–730.

Wilcox, D.A., 1988, "Reassessment of the scale Determining Equation for Advanced Turbulence Models", *AIAA Journal*, **26**, No.11, 1299–1310.