Numerical modelling of the human eye accommodation

by

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Abstract

This thesis addresses the biomechanics of the human eye accommodation. It deals with the development of numerical model of a 29-year-old eye, incorporating the vitreous body as a part of accommodative apparatus.

A more complete understanding of the mechanism of accommodation becomes increasingly important as new types of lens implants and surgical procedures to correct both accommodative loss in aphakia as well as in the aging process are being explored. It is necessary to conduct the experimental and analytical studies to gain a better comprehension on ophthalmologic processes.

The accommodation has been investigated through numerical simulations based on finite element analysis. The extensive literature survey was the platform for establishing relevant modelling procedures. The calculations were carried out using the commercial general-purpose finite element software ABAQUS. All materials were modelled as being linearly elastic and the interiors on the lens and vitreous were assumed to be incompressible.

Present research seeks to investigate the validity of some fundamental assumptions about the construction and functioning of human accommodation system. The model is rather general and involves the synthesis of disparate sets of geometric and mechanical data from a variety of published sources. Different configurations of the structural model can easily be simulated by appropriate adjustments of parameters.

The results of this study are broadly in agreement with published observations. The model behaviour is consistent with classical Helmholtz theory. It is shown that such a modelling exercise captures at least some physiological aspects of human accommodation. The proposed procedures and developed inverse methodology can be a useful tool to derive previously not documented parameters and test the consistency of different sets of experimental measurements. The obtained results can be used to draw some recommendations in pursuit of the clinical imperatives of ophthalmologists.

Descriptors: accommodation, numerical analysis, finite element modelling, biomechanics, eye

Preface

The research presented herein constitutes a thesis for the Licenciate degree at the Department of Mechanics, Royal Institute of Technology (KTH). The work was carried out between 2003 and 2005 under the supervision of Prof. Anders Eriksson. I am very grateful for your endless support, valuable guidance and always positive attitude towards this project, as well as in general. Thank you for giving me the opportunity to work as your student and develop my knowledge of finite element method.

I am much indebted to Prof. Svetlana Bauer both in relation to this particular work and to my professional development in general. Svetlana was actually the one who initiated me to work with eye accommodation and has not stopped to share her vast knowledge in mechanics and ocular biomechanics whenever I have been lost. You were always there for discussions and I experienced that solutions seemed to exist to any problem.

My heartfelt gratitude also to Prof. Andrey Smirnov. His willingness to share his knowledge in engineering and English language is much appreciated.

Many thanks to the people at KTH Mechanics for providing a very nice atmosphere at the department, especially to my former and present colleagues on the 8th floor. It has been great fun and pleasure to work with you.

I also direct special thanks to my friends in Sweden, Holland and Russia: you have all contributed to this thesis more than you think. Sometimes I have been neglecting you, but you are often in my thoughts and I miss you all.

Finally, I want to thank my mother and my sister for their love and never ending patience that made this possible. You will always be with me in my heart. Last, but certainly not least, I thank Alicia for her support and constant encouragement.

Stockholm, August 2005
Darja Ljubimova

Dissertation

This thesis considers the development of an axisymmetric, nonlinear finite element model of an accommodative process of a human eye. It is divided into two parts. The first part consists of introduction to the science of ocular biomechanics: basic concepts, anatomy of an eye, review of relevant experimental and theoretical works. It also includes the summary of the research with the objective to clarify its context. The second part contains two appended scientific papers listed below. The papers are re-set in the present thesis format when necessary.

Paper 1. Ljubimova, D., Eriksson, A. & Bauer, S., Numerical study of effect of vitreous support on eye accommodation. To appear in *Acta of Bioengineering and Biomechanics*, Vol. 7, No 2, November 2005.

Paper 2. Ljubimova, D., Eriksson, A. & Bauer, S., Aspects of eye accommodation evaluated by finite elements. Submitted to *Biomechanics and Modeling in Mechanobiology*

Part of this work was presented at 14^{th} Europian Society of Biomechanics (ESB) Conference, July 4 – 7, 2004 s'-Hertogenbosch, Netherlands, a poster presentation to be, which became oral.

Division of work between authors

Prof. Svetlana Bauer (SB) provided valuable knowledge about the biomechanics of an eye. Modelling and implementations were carried out by the respondent under the supervision of Anders Eriksson (AE). The manuscripts were prepared by Darja Ljubimova with feedback and comments from SB and AE.

Contents

Part 1

Overview and Summary

Chapter 1

Introduction

1.1. Background

We are a highly visual species. Most of our information about the world comes to us through our eyes and most of our cultural and intellectual heritage is stored and transmitted as words and images to which our vision gives access and meaning. Knowing more about our eyes and vision is, therefore, one path to better understanding ourselves. A better understanding of the human eye allows us to intervene more intelligently and purposefully as we attempt to correct or modify disorders of the eye brought on by trauma, disease or ageing.

Despite advanced experimental and clinical methods some visual mechanisms still remain generally unknown. In these cases ophthalmologists use statistics, since existing analytical frameworks are not enough for the prediction of occurred processes and selection of the best medical treatment. However, a statistical approach is unreliable, exposing the patient to the relatively high risk and provoking the essential production costs. Such a science as ocular biomechanics came as a solution, combining the laws of physics and engineering concepts to describe motion of human eye segments, and the forces which act upon them during activity. A systematic mathematical and biomechanical approach may help us to analyse and explain some visual processes and give an impulse for corresponding research in ophthalmology.

More detailed investigation on the functioning of primary eye systems are important both for clinical ophthalmologic practice and for the correction of general concepts of human eye. Conduction of such studies will provide an explanation for normal and pathological performances of intraocular structures, indicate perspective directions for treatment and diagnostics of some ocular diseases, reducing to minimum the risk of surgery of such pathologies.

The combination of various experimental studies, both *in vivo* and *in vitro*, and analytical modelling help us to gain a better comprehension on ophthalmologic processes. Important data, provided by experiments are interpreted within the context of an analytical framework. Theoretical simulations are used to test the consistency of sets of previous experimental measurements and suggest where further experimental investigations are needed. The integration of these techniques becomes widely accepted as investigative tool for systematic analyses.

In the clinical ophthalmologic practice we can segregate two main problems: drainage control and accommodation of an eye. In the present study we performed an attempt to investigate the process of accommodation of a human eye. A more complete understanding of the mechanism of accommodation becomes increasingly important as new types of lens implants and surgical procedures to correct both accommodative loss in aphakia as well as in the aging process are being explored. The detailed study of accommodation would add to our fundamental knowledge about refraction anomaly (e.g. spasm of accommodation), dysfunction caused by surgery on the accommodative apparatus (e.g. cataract surgery) and pathologies of the accommodative system.

1.2. Aims and scope

This thesis aims to contribute to the understanding of the accommodative process of a human eye by investigating and progressively developing a numerical model which simulates its optical and mechanical behaviour. It takes as its background clinical and experimental knowledge on the construction and behaviour of various eye systems and connects this to the computational models, based on finite element (FE) representation. The extensive literature survey has been the platform for establishing relevant modelling procedures. Non-linear qualitative analyses are performed using the commercial general-purpose finite element software ABAQUS, Hibbit *et al.* (2002). Special interest was directed towards the influence of vitreous in the accommodative process and its effects on certain aspects of accommodation. The goal of the present project is to show that addition of vitreous support to the model is consistent both with the mechanical laws and different published data. The improved models of the accommodative apparatus lead to new demands on the physiological experimentation and thereby could provide comprehension of aspects of accommodation, particularly in pursuit of the clinical imperatives of ophthalmologists.

1.3. Outline of thesis

The following chapters seek to provide the background and summary of results for the papers constituting the second part of the thesis.

- Chapter 2 outlines the basic anatomy of an eye. The elements that are essential for understanding of the process of accommodation are discussed in details.

- Chapter 3 is concerned with the mechanism of accommodation: representation from historical point of view and review of present understanding. Various experimental and theoretical studies of accommodation are discussed and new modern ideas presented.
- Chapter 4 reviews the aspects of FE modelling.
- Chapter 5 contains results and discussions. Concluding remarks and suggestions for the future researches are presented.
- An appendix gives a glossary of biological terms to make text accessible to readers for whom subject is new.

Chapter 2

Anatomy of an Eye

This thesis aims at numerical modelling of the accommodation of a human eye. It is a fair guess that the majority of readers will have at least some knowledge of mechanics. To familiarize the reader who has no background in biology with some important features of the science of ocular biomechanics and the process of human eye accommodation, the first chapter of the thesis is therefore dedicated to an introduction of basic anatomy of a human eye. The chapter is far from complete, since only those elements which are considered relevant to the understanding of this thesis are discussed in details, and the reader is referred to the appendix, basic books on physiology or the references cited for further information.

2.1. Human eye

The eye is a complex sensory organ specialized for the gathering of visual information. Human eyes are roughly spherical, filled with a transparent gel-like substance called the vitreous humour, with a focusing lens and an iris which regulates the intensity of the light that enters the eye, see Figure 2.1. Each eye is surrounded by a fibrous protective globe.

In humans the eye works by projecting images onto a light-sensitive retina, where the light is detected and signals are transmitted to the brain via the optic nerve. Light enters the eye from an external medium such as air or water, passes through the cornea, into the aqueous humour and is focused on the lens, which is the convex, springy disk. On the other side of the lens is the vitreous humour, which lets light through without refraction, helps to maintain the shape of the eye and suspends the delicate lens. From behind, the vitreous is bounded by retina, whose photosensitive cells trigger nerve impulses which travel to the brain.

The crystalline lens and the cornea represent the main focusing system of a human eye. The cornea gives a larger contribution (about 2/3) to the total refraction than the lens, but whereas the curvature of the lens can be adjusted

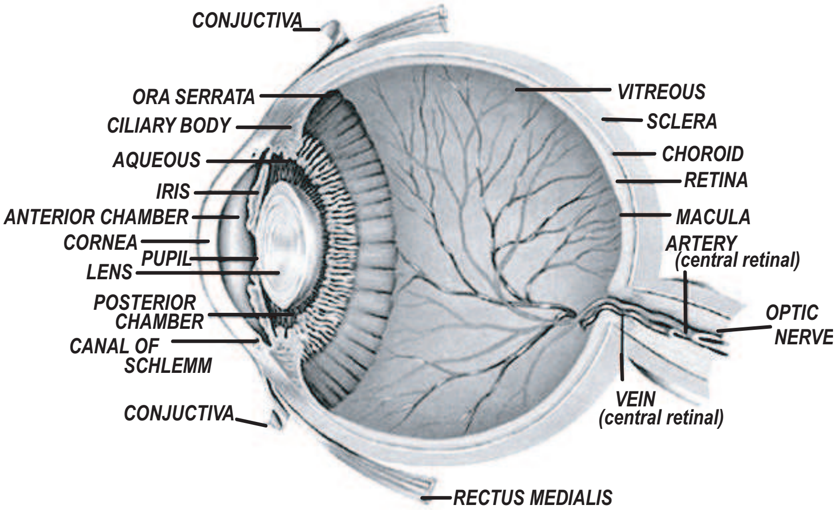


Figure 2.1: Schematic diagram of the human eye. Redrawn from the Discovery Fund for Eye Research.

to "tune" the focus, the curvature of the cornea is fixed. This variable lens contribution is called accommodation and arises both from controlled changes in curvature and thickness along the lens's polar axis, mediated by ciliary muscle contractions. A second, smaller contribution to accommodation occurs through a difference in the index of refraction between the crystalline lens and the surrounding medium, Koretz & Handelman (1982).

The mechanism of visual accommodation, i.e. the ability of the eye to focus on objects at various distances from itself, still remains largely unknown, despite the increased interest in the human eye, the geometry of the crystalline lens and its associated internal structures. Nowadays it is universally accepted that accommodation is primarily achieved by alteration of the shape of the crystalline lens, but the exact nature of the changes has not been identified.

2.2. Components of the accommodative apparatus

All the anatomical elements which are believed to participate directly or indirectly in the accommodative process are described more precisely in the current section. The component elements of the accommodation mechanism are: the crystalline lens, the zonular apparatus, the ciliary body, containing the ciliary muscle, the choroid and the vitreous, see Figure 2.2.

The crystalline lens

The lens is an essential contributor to the performance of the visual system of the human eye. It is a soft flexible elastic body, biconvex with the anterior surface being less curved than the posterior surface, and is composed of three layers, from surface to the center: capsule, cortex, nucleus. The innermost and softest part of the lens — nucleus, is surrounded by the material called cortex. The crystalline lens is encased in a capsular bag, which is a quite elastic, clear, membrane-like structure and maintained by the zonular apparatus. The elasticity of the lens capsule keeps it under constant tension. As a result, the lens naturally tends towards a rounder or more globular configuration, a shape it must assume for the eye to focus at a near distance.

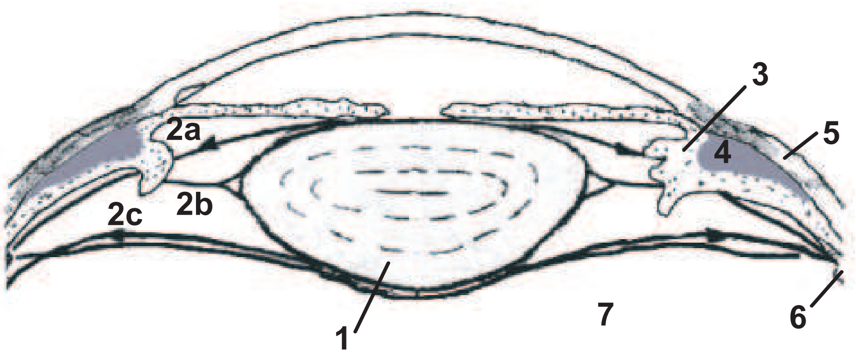


Figure 2.2: Supporting system of the lens.
Redrawn from Svetlova & Koshitz (2001)
1 – lens; 2 – zonular fibres: 2a – anterior, 2b – central, 2c – posterior;
3 – ciliary body, containing the ciliary muscle, 4; 5 – sclera;
6 – choroid; 7 – vitreous.

The lens curvature and axial thickness are controlled by the ciliary muscle. By changing these parameters of the lens, one can focus the eye on objects at different distances from it.

The zonule

The zonule (suspensory ligaments, zonules of Zinn, zonular apparatus) is a supporting system of the lens. The zonule appears to attach and interlace with the lens capsule in rings on anterior, equatorial and posterior surfaces on one side and the elastic choroid structures on other. These three distinct groups of fibres denoted by the places of their attachments to the lens are called anterior (AF), central (CF) and posterior (PF) sets of fibres, respectively.

The ciliary body

The ciliary body is a ring of tissues situated inside the globe lining the anterior of the eye behind the origin of the cornea just between the iris and choroid. It gives rise to iris, secretes the aqueous humour of the anterior chamber and provides attachments to the central zonules of the lens. The anterior part of the ciliary body contains ciliary folds and is called pars plicata. These folds (processes) are essentially vascular structures and do not contain extensions of ciliary muscle. The posterior part of the ciliary body is called pars plana and is limited by the ora serrata, which separates it from the retina.

The anterior and bulk of the ciliary body contains the ciliary muscle that affects zonules in the eye, enabling changes in lens shape for light focusing, Atchison (1995).

The vitreous body

The vitreous body (hyaloid) is located between the crystalline lens and the retina, occupies the posterior compartment of the eye and is about 80% of the volume of the eyeball. It is bounded on all sides: by the lens, ciliary body, the suspensory ligaments and by the retina, representing a single optic complex. The vitreous represents a membrane full of gel-like substance called vitreous humour and looks like a ball flattened out in front. It maintains a certain level of intraocular pressure and ensures normal adjacency of the inside shells of an eye (choroid and retina). It is known that the vitreous membrane is quite elastic, a quality which keeps it under constant tension. If the lens is removed from the eye, the vitreous body takes a spherical configuration, although naturally the crystalline lens is lying on the anterior part of the vitreous hyaloid membrane, pressed into it by zonular fibres and forming a small pit — fossa patellaris.

The choroid

The choroid is the elastic antagonist of the ciliary muscle. It lies between the retina and sclera and is composed of layers of blood vessels that nourish the back of the eye. The choroid connects with the ciliary body toward the front of the eye and is attached to edges of the optic nerve at the back of the eye.

2.3. Construction of the accommodative apparatus

The interaction to be between the elements defined in the previous section is generally believed as follows. The lens is anchored into its place with two sets of connections. Its posterior surface shares, in large part, an interlace with the vitreous anterior membrane. Such a conjugation is very fast and the lens sealed in the cup-like depression of the vitreous body cannot be mobile in relation to hyaloid. The experiments like the one by Bystritsky (1994) show that even if the zonules of Zinn and the supplementary capsule-hyaloid ligament are cut and the optical complex "lens — vitreous" is removed from the eye, its structure is still intact with the lens sealed in fossa patellaris. That interconnection is called Wieger's ligament and can be regarded as an additional annular sheet between the back of the lens and anterior hyaloid vitreous membrane.

Anterior, posterior and equatorial regions of the lens are indirectly connected to the ciliary body through the zonular apparatus. When ciliary muscle is relaxed, the eye is in unaccommodated state. In order to see near object clearly, the muscle contracts, and the collar, which is formed by the ciliary body decreases in diameter and moves slightly forward. The zonule also move inward and anterior, thus changing the degree of stress they apply on the lens capsule. The lens, therefore, exhibiting elastic recovery, becomes more convex and thicker, increasing its optical power. When muscle relaxes, the crystalline lens is returned to its far-accommodated state by the spring force exerted by choroid.

Although the main changes occurring in the human eye are well known and documented, the subtle aspects of the change of lens shape require further explanation, and the next chapter of the thesis is devoted to the discussion of these issues.

The role of the vitreous body in the accommodative process is still a matter of some debate, although it is getting universally accepted that it contributes in at least a part of the process as a support supplying the reaction force, but in what fashion and with how much force is unknown, Coleman & Fish (2001); Koretz & Handelman (1982).

Chapter 3

Mechanism of Human Accommodation

Understanding the eye accommodation requires an exploration of the relationship between its structure and its function – that is, a consideration not only of how the eye and its parts are constructed, but also of what they do and how they work. In general, the details of lens ultrastructure and anatomical connections, as well as the events occurring during accommodation, are agreed on universally and have been described in the previous chapter. The question of the mechanisms by which accommodation is attained, however, is currently not answered unequivocally. The appropriate mechanical model can add a piece to that puzzle. In order to construct an adequate model of the accommodation process, it is necessary to investigate given problem under different angles; check various existing hypotheses on the mechanics of the current biological system, perform an intensive literature survey to determine the exact components of the modelling structures and precisely define its geometrical and mechanical properties. Therefore the following chapter covers not only the present understanding of the mechanism of accommodation, but also contains a small review of evolution of different theories and ideas about this process.

3.1. Accommodation from a historical point of view

The means whereby the human eye brings to focus objects of varying distances from itself has been the subject of intensive study for centuries. Different theories regarding the mechanism of accommodation have been extensively reviewed in several studies, Atchison (1995); Strenk, Strenk & Koretz (2005). To show how ideas about this process evolved, a small overview is presented here.

Preliminary theories

Descartes (1637) explained that the lens must be more curved in order to view near objects. He did not have any experimental data and believed that the fibers suspending the lens were the reason for changing the lens shape. Tomas Young (1801) found that accommodation still occurs after the power of the cornea was neutralized or after the eye was clamped to prevent changes in its length. These observations let him conclude that it is the lens that is responsible for accommodation. The first experimental evidence to support this hypothesis was provided by Cramer (1853), who used Purkinje images to show the increase in anterior lens curvature in response to accommodation. He suggested a theory, stated that ciliary muscle contraction act on the choroid, which in turn compresses the vitreous against the posterior lens. The iris resists subsequent lens pressure and only the pupillary free area of the anterior lens surface increase its curvature. The theory was refuted later in 1861, when the presence of full accommodation was demonstrated in an aniridic patient.

Relaxation theory

About this time von Helmholtz (1909) proposed his relaxation theory, in which the zonules play a major role in the accommodation. He described the ciliary muscle contraction as causing a reduction in zonular tension that allows the crystalline lens to increase its curvature, decreasing its equatorial diameter while increasing its thickness. Helmholtz refuted some postulates in early theories, including: (1) the optical system does not alter at all; (2) contraction of pupil size during near vision accounts for accommodation (via increased depth of focus); (3) changes in corneal curvature; (4) movement of the lens alone; (5) change in length of the eye. Later, Gullstrand expanded Helmholtz' mechanism by including the elastic force of choroid as the restoring force to ciliary muscle contraction.

It is necessary to mention a study by Hess (1896), who established that the lens drops forward under the influence of gravity when the eye looks down (here "drops forward" means the forward movement of the center of the mass, not necessarily both lens surfaces). These observations were recently confirmed by studies of Coleman (1970); Glasser & Kaufman (1999).

Fincham (1937) modified the relaxation theory by excluding the role for the iris, since in most cases of aniridia accommodation amplitude was normal. He also observed that upon removal of the capsule the lens assumed its unaccommodated-like shape and concluded that the lens elasticity resides almost entirely in the capsule.

Tschernings' mechanism

Tscherning produced two theories of accommodation, where he challenged Helmholtz' theory, suggesting essentially the completely opposite: the increase of the zonular tension in response to ciliary muscle contraction. In the second of these, Tscherning (1909) considered that during ciliary muscle contraction, tension is exerted upon the choroid, which compresses the vitreous body against the peripheral part of the posterior lens capsule. The anterior surface of the lens is fixed at its periphery by tension of the zonule, thus during the accommodation the lens is flattened at the edges, becoming more bulging in the central part. In order to explain how increasing zonular tension might account for accommodative increase of the lens curvature, Pflugk (1932) modified Tscherning's theory, by adding the vitreous, the iris and anterior chamber pressure. Although the further researches provided almost irrefutable contrary evidence (e.g. Fincham 1937), Tscherning's hypothesis has forced to pay a rapt attention to possible influence of changes occurring in the peripheral part of the lens, on its bulging in the central part and, in particular, on possible participation of the choroid and vitreous body in the act of accommodation.

Catenary theory and the role of vitreous

The catenary (hydraulic suspension) theory by Coleman (1970) propounds that the lens, zonule, and anterior vitreous comprise a diaphragm between the anterior and vitreous chambers of the eye. It is stated that with ciliary muscle contraction, vitreous chamber pressure relative to the anterior chamber pressure increases.

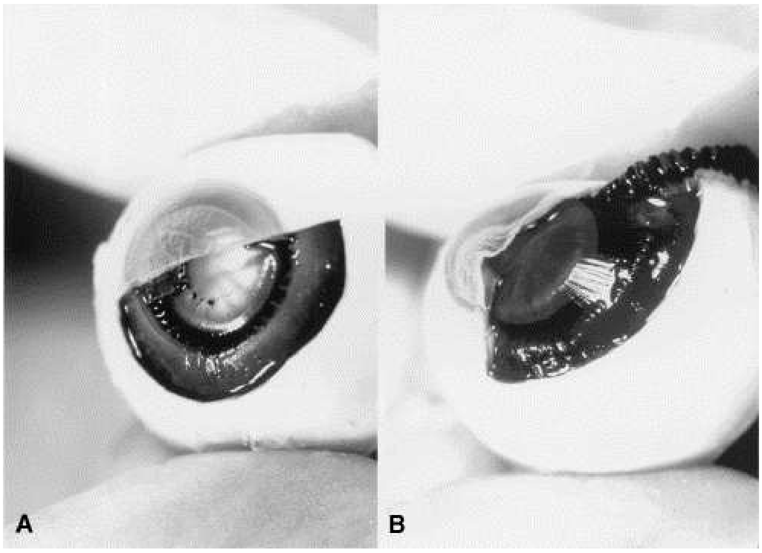


Figure 3.1: (A) A human eye bank eye with the cornea and sclera removed to expose the lens and zonule. (B) The ciliary body is lifted away to demonstrate how the anterior vitreous and zonule remain intact so that compression of the posterior vitreous compartment can cause the lens zonule diaphragm to move forward. Reproduced from Coleman & Fish (2001).

The vitreous pressure applies a force and produces a change in catenary shape for the posterior lens surface leading to the change in anterior lens curvature. The integral relationship of the lens, zonules, and vitreous in the human eye was also demonstrated by Coleman & Fish (2001) as a support for the concept of a diagram between the chambers, Figure 3.1.

Later, Coleman (1986) presented a simple catenary model to prove his hypothesis, where differential pressure measurements between the vitreous and anterior chamber in primates were consistent, as it was claimed, with vitreous compartment support of the lens. At his recent work, Coleman & Fish (2001) simulated aging of the lens using mechanical model, and showed that accommodative loss with age (presbyopia) and other features consistent with his theory.

Interestingly, the catenary model was first proposed in 1970 and has been largely ignored by other researchers. At the present moment with newly achieved observations about construction and geometry of the accommodation apparatus, (e.g. Gorban & Dgiliashvili 1993; Kotliar *et al.* 1999; Svetlova & Koshitz 2001; Dubbelman *et al.* 2005) hydraulic suspension theory gains more and more attention and idea of vitreous being a part of accommodation mechanism is getting universally accepted.

Helmholtz himself did not include the vitreous in his theory, Fincham did accept vitreous support during nonaccommodative state but not in the accommodative state. It is mechanically not possible, since a force cannot exist in one phase of change and not in the reversal of that phase. Koretz & Handelman (1982) believed that the forward movement of the anterior lens surface must be aided by vitreous force, later they suggested that the vitreous itself molds lens shape through the compressible action of muscle-coupled structures on it, Koretz & Handelman (1986). However, Fisher (1983) refuted that influence.

Accommodation by Schachar

Schachar (1992) proposed Tscherning-type mechanism, in which it is supposed that both the lens equator and zonular tension increase upon accommodation. Schachars' theory claimed to be proved in the work, where accommodation was investigated in the primate eye, Schachar *et al.* (1995). But the description of the insertion of the zonule, accepted in that "proof" conflicts with evidence from fresh human tissues, (Glasser & Campbell 1998) and from scanning electron microscopy, (Rohen 1979; Ludwig *et al.* 1999). The experimental study on determination of mechanism of accommodation in rhesus monkeys was undertaken by Glasser & Kaufman (1999), where it was shown that movements of the accommodative structures are contrary to mechanism produced by Schachar.

Later, Schachars' theory was modified by specifying the exclusive role of the equatorial zonules, Schachar *et al.* (1996). This hypothesis assumes that the tension of the central fibers is minimum in the unaccommodated state of an eye. During ciliary body contraction, they pull an equatorial area of the lens capsule, tighten its peripheral part and force the lens to bulge the central area.

Thus, the lens becomes more spindle shaped: thinner near the equator and more curved in the middle. In Schachars' opinion anterior and posterior zonules are auxiliary stabilizing components of the accommodative mechanism, which are tensed during distance vision and relaxed during accommodation. This proposal is supported by a simple physical model, consisting of a gelatine-filled balloon, Schachar *et al.* (1994) and also by analytical and numerical studies of the accommodation process Schachar & Bax (2001); Chien *et al.* (2005). However, Schachar does not explain how an increase in contraction can increase zonular tension and it is not immediately clear why his numerical model gives such a result. Against Schachars' concept state the facts that the ciliary muscle does not have the necessary anatomic rigidity or the attachments to support an equatorial traction to flatten the lens.

3.2. Studies of accommodation

The present thesis deals with mechanical modelling of human eye accommodation. Construction of a numerical model of a biological system involves synthesis of disparate sets of geometrical and mechanical data from a wide variety of sources. In that case it is crucial to know both the limitations and advantages of different experimental techniques used to derive the data. We should be aware of any possible sources of inherent experimental errors which can occur, and use that knowledge to determinate the degree of reliability of the findings.

Experimental studies

Different experimental studies have been carried out on the accommodation process. They fall into two groups: *in vivo* (within a living organism) studies and *in vitro* (in an artificial environment) studies.

In vivo

The first systematic *in vivo* observations using modern instrumentation were obtained using phakometry, (e.g. Duke-Elder & Abrams 1970; Veen & Goss 1988), a technique that relies on measurements of the relative height and location of Purkinje images III and IV. Most of the *in vivo* data published in the literature was obtained using phakometry. Examination of the changes in the lens geometry (e.g. Royston *et al.* 1989; Garner & Yap 1997; Hemenger *et al.* 1995) or accommodative amplitude (Duane 1912; Fincham & Walto 1957; Eskridge 1984) in response to pharmacological stimulation of the ciliary muscle are problematic as drugs affect the iris and the ciliary muscle, complicating the results.

Much of the controversy and confusion regarding the mechanism of accommodation has resulted from the difficulty of fully visualizing the accommodative structures *in vivo* in the intact human eye. Specifically, the crystalline lens is normally partially obscured by the iris, which prevents direct visualization of the ciliary muscle, circumlental space and the lens periphery by screening these regions from optical methods of visualization.

Recently, one of the most popular method to measure the shape of the lens with accommodation has been Scheimpflug photography, a non-invasive optical imaging technique, which provides high-contrast and detailed information about the internal organization of the lens, Brown (1974); Cook *et al.* (1994); Dubbelman & van der Heijde (2001). However, to obtain reliable results using this technique, the Scheimpflug images must be corrected for two types of distortion. The first one can easily be corrected, since it is due to the geometry of the Scheimpflug imaging system, Ray (1995). The second type of distortion is not constant, but particularly depends on the shape of the cornea, the anterior chamber depth and the shape of the lens. It arises because the internal structure of the lens is observed through the preceding refractive surfaces, i.e. the measurements of the anterior lens surface has been influenced by the refraction of the cornea, and the measurement of the posterior surface has additionally been influenced by the anterior lens surface and the refractive elements within the lens. Up till the works by Dubbelman *et al.* (2003, 2005), only the first type of distortion has been corrected. The individual correction for the refraction of cornea and lens was not taken into account by Brown (1973); Koretz *et al.* (1997, 2002), so their results should be treated with caution.

These optical difficulties can be overcome by the use of alternative techniques, such as ultrasound (Glasser *et al.* 1999; Ludwig *et al.* 1999) and magnetic resonance imaging (MRI), (Strenk *et al.* 1999). Unfortunately, ultrasound studies have contrast and field-of-view limitations and MRI method tends to produce data of an unsatisfactory low spatial and temporal resolution.

It appears that mechanical properties of the components of the accommodation apparatus have not been measured *in vivo*, and perhaps this is not a practical possibility. Little is known about the elastic properties of the zonular fibers and it seems not to be possible to estimate experimentally the pattern of forces applied by the ciliary muscle via zonule to the lens capsule or the lens itself.

The rhesus monkey has an accommodative apparatus similar to that of the human and it has been suggested that studies of the mechanism of accommodation in non-human primates may give useful insight into the behaviour of the human eye. Several studies such sort were performed (e.g. Koretz *et al.* 1987; Neider *et al.* 1990; Tamm *et al.* 1992; Croft *et al.* 1999), although *in vivo* investigations in primates are subjected to the same difficulties as *in vivo* in humans.

In vitro

In vitro studies have certain advantages over *in vivo* techniques, providing an opportunity of making more accurate measurements of the way in which changes in the lens shape are related to the changes of the lens equatorial radius during accommodation, Pierscionek (1995); Glasser & Campbell (1998). In principle, they also provide the opportunity to measure the variation of the forces applied to the lens during accommodation as well as mechanical properties of various materials. Measurements on isolated cadaver lenses are generally performed by digitization of cross-sectional photographs of fresh speciemens (Fisher 1977; Glasser & Campbell 1999; Alphen & Graebel 1991; Krag & Andreassen 2003) or frozen sections (Parker 1972). *In vitro* investigations are subjected to the uncertainties about the conditions of the lens, surrounding tissues, their preparation and storage and also the way in which tension through the zonular apparatus is applied to the lens. *In vitro* the lens is in a different mechanical state of stress than *in vivo*: there is a loss of additional forces exerted by both aqueous and vitreous together with the loss of intraocular pressure, normally presented in the eye.

Currently, no method exists that can capture a complete set of correlated geometric and kinematic data for all components of mechanism of accommodation. Eyes of the same age exhibit variations in their mechanical and geometric characteristics, and, it appears that no study has yet provided a consistent set of all important information for individual lenses. The lens must be studied in situ, and ideally in vivo, to preserve the three-dimensional relationship between it and other elements of the anterior segment, as well as the pattern of forces acting upon it; the shape of an isolated human lens depends critically on the support methods used, and its optical properties may also be a function of the preservation method.

Theoretical studies

Sometimes, the relevant experimental archive is too sparse to validate the data by comparison with compatible results from different sources. In other cases, reliable published data on some parameters do not appear to be available. Theoretical analysis involves the incommensurable bodies of experimental measurements and can be used for testing the consistency of sets of previously reported data. Although a structural model is based on geometric and material parameters derived in experimental studies and heavily relies on initial assumptions, modelling is a useful supplement to laboratory examinations. It can also suggest where improved experimental data are needed. When such empirical measurements are accessible, the model modification can easily be accomplished by appropriate adjustments of parameters.

Previous theoretical models

At the beginning, some simple mechanical models were presented in the fields of mechanical engineering, (e.g. Koretz & Handelman 1988; Wyatt 1993; Weale 2000), but new computational techniques, powerful computer-aided design and software created a possibility for more sophisticated analysis, (e.g. Schachar *et al.* 1998; Chien *et al.* 2005), sometimes using finite element calculations, (e.g. Schachar & Bax 2001; Burd *et al.* 2002).

In analyses by O'Neill & Doyle (1968); Koretz & Handelman (1982); Weeber (1999) the lens is modelled as a closed axisymmetrical membrane shell (the capsule) enclosing an incompressible fluid (the matrix). To simulate zonular tension occurring during accommodation, an axisymmetrical radial force or displacement is imposed around the shell equator. These calculations are based on a linearised form of the governing equations, neglecting the important geometric non-linear terms that govern the behaviour of membranes as displacements become large. That approach later was proved to be unacceptable, (e.g. Burd *et al.* 1999) and different nonlinear analyses were performed, Schachar & Bax (2001); Burd *et al.* (2002); Chien *et al.* (2005).

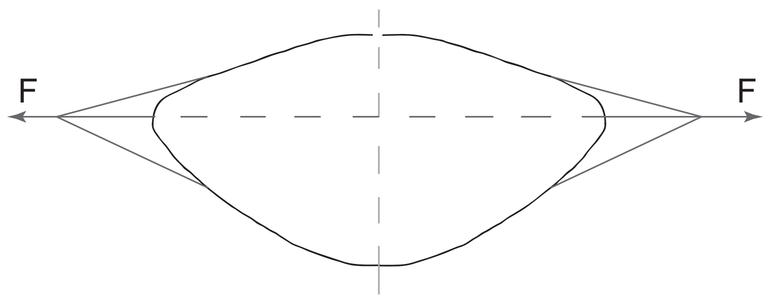


Figure 3.2: Previous theoretical model, (e.g. Weeber 1999; Schachar & Bax 2001; Burd *et al.* 2002).

It should be noted, that all previous studies assumed either that the zonular fibers attach to the ciliary processes at a single point parallel to the equatorial plane or that the displacement boundary conditions are applied around the equator, which means that the resultant of stretching forces is horizontal, see Figure 3.2. All these works also ignored the influence of the vitreous on the mechanical behaviour of the lens. The last mentioned feature, according to the modern findings, appear to be incorrect. This point is to be shown in the following section.

3.3. Present understanding

Our understanding of the mechanism of accommodation is based on the theory by von Helmholtz (1909) with regard to broad issues. Direct support for this theory, obtained from *in vivo* studies of accommodative subjects with intact, normal eyes, has been recently provided by Strenk *et al.* (1999) using high-resolution MRI images. Figure 3.3 includes both Helmholtz' drawing illustrating his theory of accommodation, and a corresponding MR image.

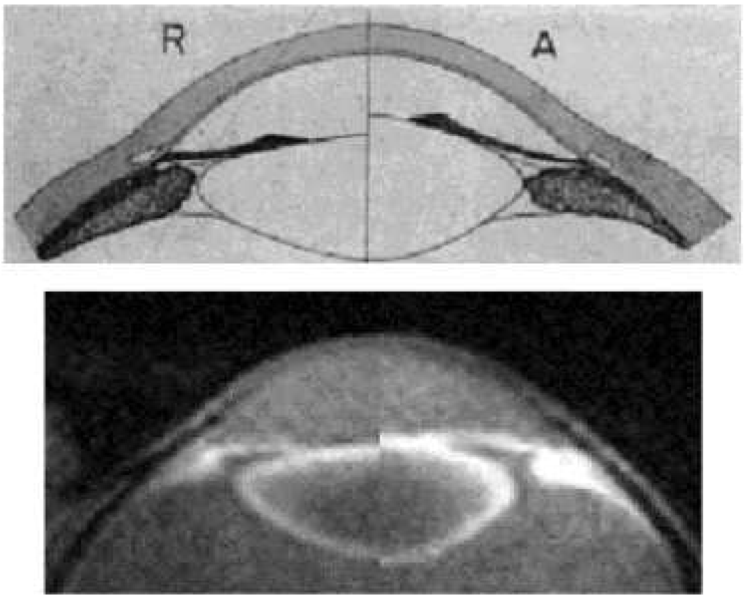


Figure 3.3: a) Helmoholtz' drawing demonstrating his theory. The left half of the image shows relaxed accommodation. (b) A composite of two MRI images. The left half is an image acquired with relaxed accommodation, while the subject, a young adult, views a far target. The right half is an image acquired during accommodation, while the subject views a near target. Reproduced from Strenk *et al.* (2005).

Nevertheless, the classical theory opens to some modifications, since it fails to describe a number of actual facts occurring during accommodation. The most difficult objection of non-modified classic theory to overcome is the precise, rapid, and anatomically reproducible shape of the lens in the accommodative state. The capsule itself does not have the elastic properties to round up the lens mass reproducibly and rapidly. It is getting universally accepted now that the vitreous must play an important role in the lens rounding up.

Helmholtz did not mention the vitreous in his theory, and it has been generally assumed that the process of accommodation is provided by the lens and zonular structures alone. Till present the exact location of bundle termination of the suspensory apparatus in or posterior to the ciliary body has been a matter of discussion, (Farnsworth & Burke 1977; Farnsworth & Shyne 1979; Rohen 1979). Now, according to the studies by Gorban & Dgiliashvili (1993); Glasser *et al.* (1999); Svetlova & Koshitz (2001), it appears that most likely the powerful anterior zonular fibres (AF), going from the lens capsule, do not attach directly to the ciliary body. Instead, they run along the inner surface of the ciliary muscle, go through the ciliary folds (processes) and interlace with the choroid near ora serrata. Thinner posterior fibres (PF) cover the vitreous anterior hyaloid membrane like a web (e.g. Rohen 1979; Ludwig, Wegsheider, Hoops & Kampik 1999), stretching from Wieger's ligament, which is fixing the lens to the vitreous, to the choroid structures near the ora serrata area. Central zonular fibres (CF) appear less numerous and thinner than anterior and posterior. With this refined description of the accommodative apparatus, Helmholtz theory should state the following:

In the unaccommodated state the lens is flattened by the passive tension of the zonular fibres which are pulled by the elastic choroid structures. During accommodation the ciliary muscle contracts, whereby it slides forward and towards the axis of the eye. It pulls the choroid structures, which causes the tension in the zonular fibres to reduce. With release of the resting tension of the zonular fibers, the elastic capsule reshapes the lens; it becomes more sharply curved and thickens axially. The equatorial diameter thereby decreases, increasing the refractive power of the eye.

This is summarized in Figure 3.4:

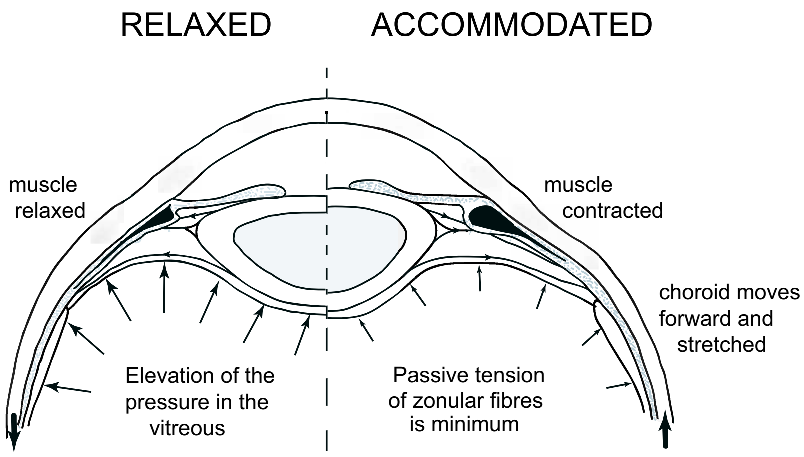


Figure 3.4: Mechanism of accommodation according to the modified Helmholtz theory. Based on Svetlova & Koshitz (2001).

Under the influence of elastic forces from the lens capsule and choroid, the anterior, posterior and central fibres are under a stressed state of dynamic equilibrium in all phases of accommodation. Zonular stresses are at a maximum in an unaccommodated condition, with the focus of the eye at its far point. In the unaccommodated form the lens is not only radially stretched, but also "squeezed" between the stressed anterior fibres and an additional stressed surface of the vitreous body together with the covering the anterior hyaloid membrane posterior set of fibres. Such a response of a vitreous membrane is accomplished by relaxation of the ciliary muscle which causes both movement of the lens towards vitreous, and posterior zonules backward by effort of the elastic choroid, providing a pressure elevation in vitreous.

The ciliary muscle itself does not change directly the tension of the anterior and posterior fibres, but produces the necessary displacement through the displacement of ora serrata, increasing or decreasing the elastic tension of the choroid structures, see Figure 3.5 (equatorial fibres are not shown). Such organization of the ciliary muscle allows to diminish the amount of volume and effort of the work carried by the muscle. This fact is corresponding to the findings of Beers & van der Heijde (1996); Adler (1959).

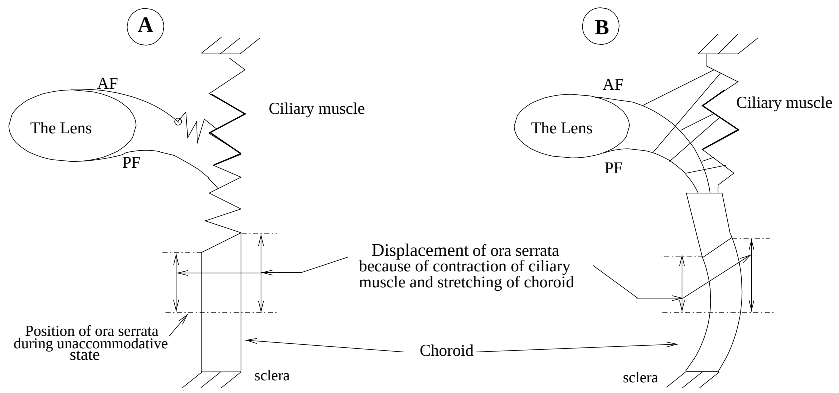


Figure 3.5: Diagram of doubtful (A) and reliable (B) notions of contribution of ciliary muscle in the process of accommodation. Redrawn from Svetlova & Koshitz (2001)

Till recent time modern western ideas about the accommodative apparatus have been based on study by Rohen (1979). It should be noted, that during his experiment Rohen did one fatal error: he was investigating the supportive system of the lens after the separation of the posterior fibres from the anterior hyaloid membrane and cutting the capsule-hyaloid ligament. Thus, this researcher followed the ideas of Helmoholtz, excluding the possible participation of the choroid and the closed volume of vitreous body in the act of accommodation, Svetlova & Koshitz (2001).

Since anterior and posterior fibres originate near ora serrata, the resultant of the tensile forces is directed at some angle to the equatorial plane, and it is necessary to consider additional forces for the equilibrium conditions. These balancing forces cannot be supplied by zonules alone, and the vitreous is, therefore, necessary as a support for the lens during accommodation, Figure 3.6.

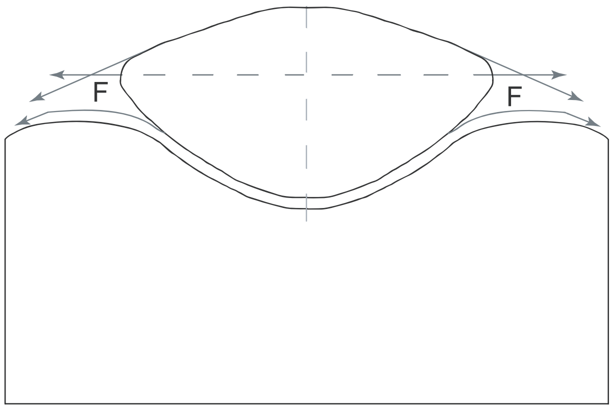


Figure 3.6: New theoretical model.

Conclusions

1) By author's belief, the exploration of the validity of various hypotheses considering the mechanism of accommodation has proven, that Helmholtz theory of accommodation still stands with regard to the broad issues. With addition of some up-to-date modifications and more accurate definitions concerning the supportive system of the lens, the classical theory explains specific experimental facts and does not contradict the basic laws of mechanics during all phases of the accommodation process.

2) New observations lead us to the consideration of a new analytical model, which should consist of the lens, zonular system and vitreous body, see Figure 3.6. The proposal of the present study is that an improved description of human eye accommodation can be obtained by such a model, which incorporates posteriorly sloped forces applied by the ciliary muscle, and the support of the lens, provided by the vitreous.

Chapter 4

Finite Element Model of Accommodation

4.1. General requirements for the numerical study

A finite element analysis is a valid tool in biomechanics when used correctly. A numerical study should obey several important demands before its predictions can be considered to have any clinical value. To achieve success in biomechanical research, the following conditions should be satisfied, Viceconti *et al.* (2005):

1. *Model selection:*

- representation of the physical problem at hand to a sufficient degree of accuracy;
- establishment of the detailed geometric model and suitable reference configuration;
- assignment of material properties for all components of the modelled structure;
- specification of the boundary conditions and operative forces;

2. *FE formulation:*

- elements consideration and solution methods;

3. *Model verification:*

- estimation of the convergence tolerance, e.g. numerical accuracy;
- proper identification of model parameters, (e.g. independence from the time or the repetition in the experiment used to derive them; association with measurable physical/physiological quantities);

4. *Model validation:*

- ability to fit the experimental results;
- sensitivity analysis of the results, consideration of the uncertainties in the model input parameters;

In the following sections of the present chapter we describe an attempt to construct an adequate finite element model of accommodation process with the fulfilment of the mentioned requirements. The model is developed by bringing the anatomy and geometry of the accommodative apparatus together with mechanical properties of the lens, zonular system and vitreous body as far as they are known. It should be noted that lenses from persons of the same age exhibit variations in their mechanical and geometric characteristics, and as it appears, the complete correlated geometric and kinematic data for individual lenses are not available. Because of these features we need to assemble the data from different sources and our modelling procedures lead to an idealized model of a human eye at the particular age. This thesis is devoted to study a accommodation of a 29-years-old human eye. It is generally accepted that eyes of that age are young and non-presbyopic, and this assumption is adopted in the current research.

4.2. Component modelling

One of the main postulates of the Helmholtz theory is that without any zonular tension the lens is in a fully accommodated state. Experiments show that when the lens is isolated and the zonules are cut, the system "lens — capsule" is at equilibrium. With this knowledge it seems reasonable to assume an accommodated state as a reference configuration for the model, with the lens, the capsule and the zonule being in a stress-free state. In that case during the numerical analysis we perform a reverse process – disaccommodation by applying force to the ends of zonular fibres. Such an approach is the most convenient and simplest from the computational point of view. However, it should be noted that in a non-linear analysis the lens behaviour heavily relies on the stresses in the capsule at the beginning of simulation. Smith (1883) reported that in an isolated young lens the capsule serves to mould the interior into a more rounded shape and without capsule the lens substance is more like the form of the unaccommodated state. These geometric changes would mean that significant pre-stresses may exist in the capsule and the modelling would require additional data and procedure. This is beyond the scope of the present research.

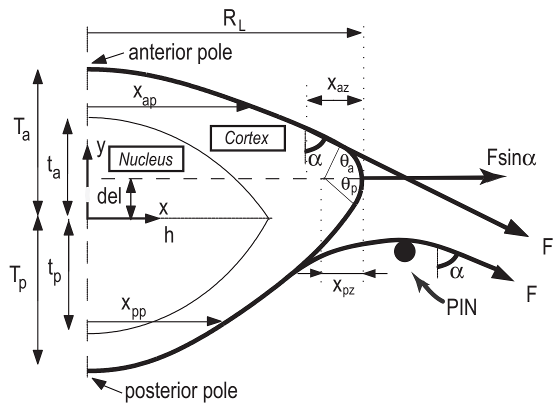
General procedures on modelling the accommodation mechanism are described more or less explicitly in the following sections of the current chapter.

Geometry

The process of disaccommodation is simulated using some basic procedures initially established by Burd, Judge & Cross (2002). We find their study a reasonable and unique attempt of interpretation of experimental data in developing the human lens model, although it must be noted that Burd *et al.* excluded the potential role of the vitreous in accommodation and limited themselves to the modelling of the lens and zonular apparatus alone.

Lens

Using Scheimpflug photography Brown (1973) measured *in vivo* changes in the lens of the human eye. He recorded geometric information for the lens under different accommodative conditions. Since we follow the presumption that 29-years-old lenses are young and fully capable of accommodation, the reported data by Brown for the highest accommodative demands of 10 Diopters are assumed to correspond to a fully accommodated state, representing our initial configuration. It is generally accepted that curves approximating the crystalline lens profile can be fitted with sufficient accuracy by fifth order polynomials, (e.g. Fincham 1937; Schachar *et al.* 1993).



Dimention	Value	Source
R_L(mm)	4.3136	Strenk et al.(1999)
T_a(mm)	2.04	Brown(1973)
T_p(mm)	2.09	Brown(1973)
x_{ap}(mm)	2.8	Brown(1973)
x_{pp}(mm)	2.4	Brown(1973)
θ_a(degree)	63°	Fincham(1937)
θ_p(degree)	37°	Fincham(1937)
t_a(mm)	1.38	Brown(1973)
t_p(mm)	1.58	Brown(1973)
α (degree)	70°	Brown(1973)
h(mm)	2.85	Fisher(1971)
del(mm)	0.5119	Burd et al.(2002)
x_{az}(mm)	0.38	Farnsworth et al.(1979)
x_{pz}(mm)	0.22	Gorban et al.(1993)
F(N)	unknown	derived during our simulations

Figure 4.1: Geometric parameters of the lens and zonule initial state. Configuration of the lens redrawn from Burd *et al.* (2002).

Brown's measurements (e.g. central and peripheral curvatures together with the lens axial thickness) are used to derive these polynomials and define the shape of the anterior and posterior surfaces of the lens, (Burd *et al.* 2002). The polynomial functions are described in the form

$$y = ax^5 + bx^4 + cx^3 + dx^2 + f$$

where x [mm] is radius, and the coefficients of the polynomials for different surfaces are

Anterior		*Posterior*	
$a = -$	0.00153004454939	$a =$	0.00375558685672
$b =$	0.01191111565048	$b = -$	0.03036516318799
$c = -$	0.02032562095557	$c =$	0.06955483582257
$d = -$	0.07692307692308	$d =$	0.09433962264151
$f =$	2.04	$f = -$	2.09

Different *in vitro* studies show that radii are identical at the equator where anterior and posterior surfaces meet, so a circular cap is used to model the equatorial region of the lens. At the points where polynomial surfaces join the circular end cup the slope should be continuous, which is done by specifying angular positions of these points: $\theta_a = 63°$ and $\theta_p = 37°$.

The slope of the anterior and posterior surfaces on the lens axis is set to zero. The radius of the end cap, r_e is assumed to be related to the lens equatorial radius R_L by the ratio $r_e/R_L = 0.1208$. The lens equatorial radius R_L is taken from the data reported by Strenk *et al.* (1999), who have provided *in vivo* measurements of R_L for eyes subjected to accommodation demands of 8 Diopters.

Based on the appearance of the zones in a lens on slit-lamp examination, the lens should be divided into two regions: the inner nucleus and the outer cortex. Anterior and posterior boundaries are defined as circular arcs, based on the study by Brown (1973). Geometric parameters used in description of the initial state of the model are summarized in Figure 4.1.

Lens capsule

The lens capsule is a uniform substance which varies in thickness with radial position. Using capsule thickness data provided by Fisher & Pettet (1972) for the lenses of different age fifth order polynomials for the variation of capsule thickness with position are obtained, see Figure 4.2. To derive the data for the 29-year-old lens, linear interpolation of the experimental results for 22- and 37-year-old lens are performed, Burd *et al.* (2002). The polynomial function is described in the form

$$t(\mu m) = a_c\nu^5 + b_c\nu^4 + c_c\nu^3 + d_c\nu^2 + e_c\nu + f_c$$

where coefficients of the polynomial are

$$a_c = 4.7464 \qquad d_c = -31.6306$$
$$b_c = 16.8928 \qquad e_c = -5.4702$$
$$c_c = -2.7198 \qquad f_c = 21.0881$$

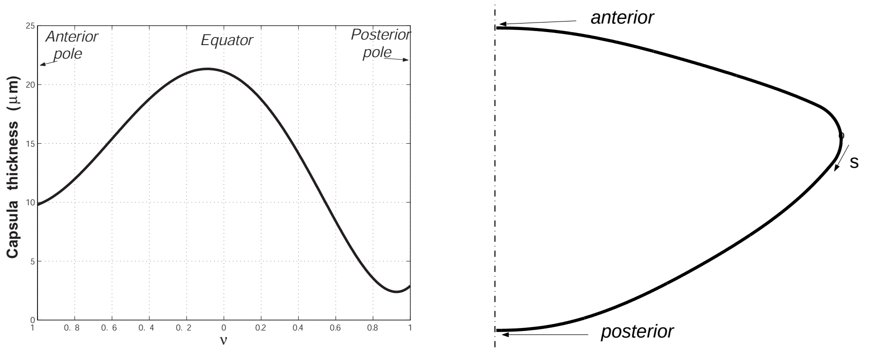


Figure 4.2: Assumed variation of capsule thickness with position. Redrawn from Burd *et al.* (2002). For the anterior surface $\nu = -s/s_a$, where s is a distance measured along the lens outline from the lens equator and s_a is the value of s at the anterior pole. For the posterior surface $\nu = s/s_p$, s_p is a value of s at the posterior pole.

The procedures of modelling the initial lens profile is previously described by Burd *et al.* (2002) and listed here for clarification.

Zonule

By scanning electron microscopy, (Farnsworth & Shyne 1979; Canals *et al.* 1996; Ludwig *et al.* 1999, e.g.), it was observed that the zonule could be represented in the model as three separated sets, denoted by the places of their attachments to the lens capsule: anterior (AF), posterior (PF) and central (CF). Each set of the zonule is composed of a number of individual fibrils, with the anterior group containing more fibrils than posterior and the central considerably fewer. Although the geometry of the zonular fibres is highly complex, the main three groups of the zonule are idealized in the geometric model shown in Figure 4.1. In the preliminary Paper 1 only the anterior and central zonular sets were modelled to simplify the numerical simulations. It was, however, later shown that the omission of posterior zonule and, respectively, Wieger's capsule-hyaloid ligament had a noticeable effect on the behaviour of the model. In Paper 2 all groups of zonular fibres were included in the analyses.

A slope angle of the anterior fibres is measured by Fisher (1971) and we adopt the reported value of $\alpha = 70°$ in our study. Anterior and posterior fibres are interlaced with the crystalline lens capsule on the radii 0.38 mm, (Farnsworth & Shyne 1979), and 0.22 mm, (Gorban & Dgiliashvili 1993), from the lens equator respectively. It is adopted that central fibres connect to the ciliary body and we assume reference values for the ciliary body equator as reported by Strenk *et al.* (1999) for 8 Diopters accommodation demands.

It is shown in the *in vivo* studies that the posterior fibres are quite branchy, covering the anterior hyaloid membrane. A simple way to emulate such a configuration was to create a rigid pin relatively close to the vitreous membrane with posterior fibres thrown over it. That approximation would be helpful to avoid contact problems and simplify the model, while at the same time, being close to reality. Such an approach was assumed in Paper 2, where all anatomical elements mentioned in the previous chapters were modelled. The muscle force is applied to the ends of posterior fibres with the same angle to the lens axis as to anterior fibres, see Figure 4.1.

Vitreous

Traditionally the role of vitreous has been neglected in accommodation and archives of data related to mechanical and geometric properties of the vitreous is still rather sparse. The vitreous body represents a membrane full of vitreous humour and looks like a ball flattened out in front. The small pit, so-called fossa patellaris, is shaped by the crystalline lens which is pressed into the vitreous anterior hyaloid membrane by the zonular fibres. The lens is locked in to the vitreous by Wieger's capsule-hyaloid ligament, which together with the posterior fibres form an additional annular sheet between it and the back of the lens, (Starkov 1967). At the reference model state it is adopted that all stresses in the lens, lens capsule and zonule are zero. It seems reasonable to assume that at the beginning of simulation the vitreous body is also stress-free, and that there is no contact constrains in the model (e.g. between the lens and the vitreous), although the possibility of a self-equilibrating low stresses cannot be ruled out. To assure the latter, a gap of 1 mm is modelled between these structures. It is known, that if the lens is removed from the eye and the zonule with capsule-hyaloid ligament carefully cut, the vitreous takes spherical configuration due to elasticity of its membrane. Therefore, it should be represented as a sphere at the initial state, although in Paper 1 the globular vitreous medium was replaced by the rectangular one for the simplicity. The initial radius 10 mm of vitreous is manually scaled from MR images by Strenk *et al.* (1999).

Mechanical properties

The model consists of several distinct materials and, for the purpose of the study, we use an assumption that each of them is isotropic and purely elastic. Although some of these materials can be described as plastic (lens: Fincham 1937) or viscoelastic (lens, zonule, choroid, vitreous: Ejiri *et al.* 1969; Lee *et al.* 1992; Beers & van der Heijde 1994) or otherwise non-linear (lens capsule: Krag *et al.* 1997; Krag & Andreassen 2001), isotropy and linearity are adopted on the basis that published archives of experimental data is too scarce to develop more complex material descriptions.

Lens and its capsule

Fisher (1969) performed a biaxial testing procedure, assuming a linear elastic behaviour of the lens capsule. He reported a Poisson's ratio of 0.47 for the capsule and that value is incorporated in the present numerical model. The uniaxial experiments to measure the Young modulus of the capsule at strain of 10% was described by Krag *et al.* (1996). With the emphasis on the marked material non-linearity, authors derived an elastic modulus of the lens capsule equal to $1.27\,\mathrm{Nmm}^{-2}$ and we adopt that value in our modelling exercise.

The substance of the lens consists of cortex and nucleus, and their stiffnesses were deduced by Fisher (1971, 1973), who proposed linear elasticity of these materials with Young's modulii of $3.417 \times 10^{-3}\mathrm{Nmm}^{-2}$ and $0.5474 \times 10^{-3}\mathrm{Nmm}^{-2}$, respectively. We follow the assumption that the lens matrix is practically incompressible (which was also adopted by Schachar & Bax (2001); Burd *et al.* (2002)), and this feature of the model is carefully accounted when finite element formulation is chosen.

Zonule

It is difficult to specify the combined stiffness of any zonular fiber set since it depends on the number of fibrils, their sizes and the elastic modulus of the material. Alphen & Graebel (1991) proposed a value of $1.5\,\mathrm{Nmm}^{-2}$ for the Young modulus of zonule during their *in vitro* experiment and this value is adopted in our study. In the finite element model three separate thin sheets of material are used to model each group of zonular fibres. Due to accepted assumptions their material stiffnesses correspond to the product of elastic modulus and the representative thicknesses of these annular sheets. To evaluate the latter parameters, an inverse experiment is carried out as described below, since experimental data on their cross-sectional area seem not to be available.

It was recently reported by Svetlova *et al.* (2003), that the role of anterior and posterior fibres appear to be the most functionally essential, while the role of central fibres is auxiliary. In the simulations with the addition of a posterior zonular group we considered two sets of analyses including and excluding the central annular sheet, representing the central zonule. This was done to test the hypothesis of Svetlova *et al.* and to examine the effect of an active central zonular set on the numerical model.

Vitreous

In the current work, the vitreous body is approximated as a membrane full of fluid, since interior gel-like substance almost entirely consists of water (99%). From a mechanical point of view the response of such a construction is quite difficult, depending not only on the external loads but also on the pressure exerted by the fluid, which, in turn, is affected by the deformation of the structure. Recently, the data on the rheological properties of human vitreous as a function of location within the eye were reported by Lee *et al.* (1992). In this study the vitreous humour is represented as a practically incompressible isotropic material. In that case, measurements of Lee *et al.* cannot be used in the model description directly, because their data do not include the pressure produced by the contained fluid. To derive the Young modulus for vitreous humour an inverse methodology is used, which is described below. As a first approach we use a hypothesis that the vitreous membrane has the same stiffness as the lens capsule. Later this assumption is explored and the influence of that parameter on the model behaviour is discussed.

4.3. Experimental methodology

In order to construct a general model of accommodation reliable experimental data are of a primary necessity. Unfortunately, appropriate published measurements on some parameters required by the model seem not to be available. To deduce unknown quantities such as force, zonular stiffnesses and Young's modulus of the vitreous we use an alternative approach, so-called inverse methodology. This is a well known engineering technique used to establish unknown parameters, by performing "what if" calculations in these parameters from which their values are deduced by comparing the resulting behaviour to known factors.

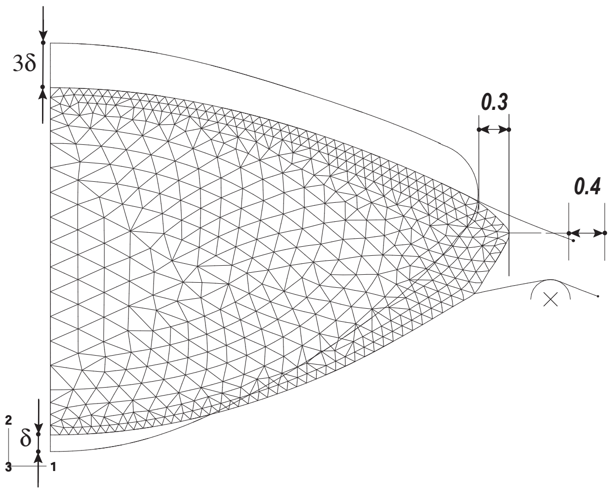


Figure 4.3: *Inverse experiment.* At the end of simulations the following changes should occur: a) forward movement of posterior pole, δ, is three times less than backward movement of anterior pole, Drexler *et al.* (1997); b) lens equator moves on 0.3 mm with corresponding change in ciliary body radius on 0.4 mm, Strenk *et al.* (1999).

Finite element calculations are done by applying the outward tension to the ends of zonular fibres. It is assumed that the resulting geometry should matched measured data in chosen respects and values of unknown parameters are manually adjusted at the end of each calculation to give the known deformed configuration. The papers listed at the second part of current thesis describe an accommodation models, which are different at some aspects. In Paper 2 more advanced model is considered and the observations chosen to be a target for the inverse experiment of this model are cited here for comprehensibility of the method, see Figure 4.3.

4.4. Modelling procedures

A numerical analysis has been carried out using the commercial general-purpose finite element package ABAQUS, (Hibbit *et al.* 2002), which is widely used in mechanical and civil engineering. The eye was regarded as being a body of revolution and an axisymmetric non-linear analysis was adopted, which allowed the three-dimensional performance of the eye structures to be modelled using two-dimensional meshes of its cross-sections. Axisymmetric boundary conditions were applied to the nodes located along the symmetry axis.

Two models of accommodation were considered in the study, described respectively in Paper 1 and Paper 2. The first model was a preliminary attempt to establish the basic modelling procedure for the finite element analysis of the human eye accommodation. It used first-order triangular elements to model the interior of the lens and vitreous. Two-noded axi-membrane elements were adopted to model lens and vitreous capsules. These were based on the approach that the capsules act like structural membranes, possessing axial stiffnesses but with zero bending stiffnesses. The axial stiffness for the crystalline lens capsule certainly varies with position because of the assumed variation in capsule thickness. Posterior fibres and, correspondingly, Wieger's ligament and Berger's space were neglected. These introduced assumptions were adopted to simplify the complex contact simulations and the results of such simulations should be used with care, since the mentioned features have significant effects on the model behaviour.

In the second model a posterior fibres set was added. Unlike the first model, second-order axisymmetric elements were used to emulate the eye structures involved in the accommodation process. One of the common feature in both models was the assumption about near incompressibility of the cortex, nucleus and vitreous humour. It is known, that the use of normal elements in the case of material incompressibility can be a reason for unsatisfactory model performance, (Sloan & Randolph 1982). This issue was carefully addressed when developing the second model by using six-noded hybrid elements available in ABAQUS, specially formulated to avoid problems (such as volumetric locking) associated with modelling incompressible materials.

The calculations were based on the use of a Poisson's ratio of 0.4999 for the cortex, nucleus and vitreous humour. This value is adopted on the basis that it corresponds closely to the value of 0.5 that would specify a perfectly incompressible material. A control was performed for both models to ensure that the enclosed lens volume was conserved. It was shown that the adopted Poisson's ratio was sufficient to guarantee the maintenance of the constant lens volume during calculations. Several meshes with different densities were created for the crystalline lens to investigate the accuracy of obtained solutions. The lens mesh represented by 1007 continuum and 200 membrane (100 for the first model) elements was regarded as being acceptable, since further mesh refinement showed insignificant influence on the results. In the second model both lens and vitreous capsules were modelled using two-noded axi-shell elements. Although these elements are incompatible with six-noded triangles used to model cortex and vitreous humour such a choice was accepted on the basis that both capsules are much stiffer than the interiors. This approach helps to decrease the complexity of contact formulation and thereby significantly speeds up the analyses. It should be noted that since we had, in preparations, ascertained a sufficiently fine mesh, no major differences were observed when capsules of lens and vitreous were modelled using either shell or membrane elements.

Following Burd *et al.* (2002) we assumed that the lens capsule is fully bonded with the cortex. Anterior, posterior and central sets of zonular fibres were modelled as three separate annular sheets using two-noded axi-shell elements. The contact between zonules and lens capsule was modelled as frictionless. The ligament of Wieger is represented in the model as an additional annular sheet, created by adhesion of the posterior surface of the lens and the vitreous anterior hyaloid membrane. Extra analyses were carried out taking into consideration this anatomical element. The ligament has a certain stiffness and as a first step we assume that it is identical to the stiffness of the posterior fibres, which was in turn deduced via inverse experiments. Later, the sensitivity of this assumption was tested.

Further we present aspects and the results of modelling exercise described in Paper 2, where all components of the accommodative apparatus which were introduced in Section 2.2 are included in the modelling.

Preliminary steps

The reference configuration of the main model is shown in Figure 4.4(a). At the starting point of simulation no contact exists between the lens capsule and the vitreous. Several preliminary steps were required to achieve such a configuration of the model, that process of disaccommodation can be performed.

1. Creation of the pit on the anterior surface of hyaloid (fossa patellaris), emulation of stresses in the vitreous;
2. Establishment of the contact between the lens and the vitreous avoiding the appearance of rigid body motion;
3. Performing the process of disaccommodation by applying ciliary muscle forces to the ends of zonular fibres.

The shape of fossa patellaris should be the same as the shape of the posterior surface of the lens, since the adopted lens configuration was based on *in vivo* measurements by Brown (1973). The lens was replaced by the rigid surface with the same shape as the lens posterior surface to facilitate the contact simulation. Displacement boundary conditions were applied to the bottom of the vitreous. During this procedure the anterior part of the vitreous was pressed into the rigid surface, creating the required pit, Figure 4.4(b). As a matter of fact, the depth of this pit is unknown. Preliminary simulations showed that the variation of the depth of fossa patellaris in analyses of the sort described here did not affect the lens optical power. Thus, the calculated optical power is not sensitive to this parameter. For the purposes of the study the depth of pit was prescribed to be 3 mm.

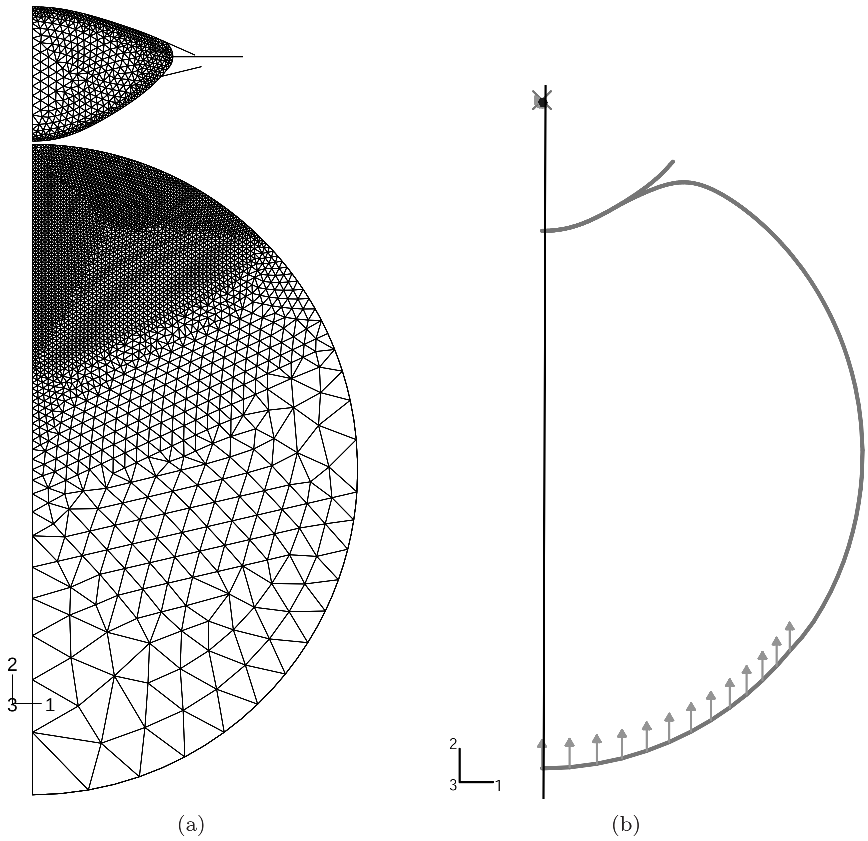


Figure 4.4: Procedures: a) reference configuration for the model with all zonular groups; b) first preliminary step.

At the second step, the rigid surface was replaced with the crystalline lens and contact between the lens and vitreous was carefully established with a friction coefficient equal to $\nu = 0.01$ as for lubricated surfaces. After these preparations numerical calculations were carried out by applying outward tension to the lens via the zonules to simulate the process of disaccommodation.

Postprocessing

The finite element calculations were conducted by applying a ciliary muscle force in small increments to the ends of zonular fibres. Previously undefined parameters were derived and the resulting geometry of the model was computed us ing non-linear algorithms available in ABAQUS. Geometrical solutions for the eye structures were available at the end of each calculation increment; these solutions corresponded to particular values of muscle force as it was increased during the analysis. According to Dubbelman *et al.* (2003, 2005) during *in vivo* experiments compared to the anterior lens surface only a smaller part of the posterior lens surface can be seen, which, due to the pupil construction, becomes even smaller with accommodation. On the basis of this observation different diameters of anterior and posterior optical zones were chosen for consideration. The portions of the obtained anterior lens surface within a circular aperture of radius 2 mm and one for the posterior lens surface were regarded as all-important. For every increment, deformed positions of all points within considered zones were fitted to second order polynomials of the form $z = a + bx + cx^2$, with z is the axial deformed coordinate and x is the deformed radius.

Since the curves are all symmetric around the lens pole, the radii of curvature could be calculated using formula $r = (1+(2cx)^2)^{3/2}/2c$, (Koretz, Handelman & Brown 1984). This equation was solved for the anterior and posterior lens surfaces using $x = 1$ mm. On the basis of simple paraxial theory, the optical power of the lens was determined using data within this important vision zone. Refractive indices of $n_{av} = 1.336$ for aqueous and vitreous humour and $n_l = 1.42$ for the crystalline lens were adopted, as proposed by Bennett & Rabbetts (1989) for the Gullstrand-Emsley schematic eye. Substituting these values and the computed polar lens thickness t together with anterior r_a and posterior r_p radii of curvature to the conventional thick lens, formula the lens optical power can be calculated,

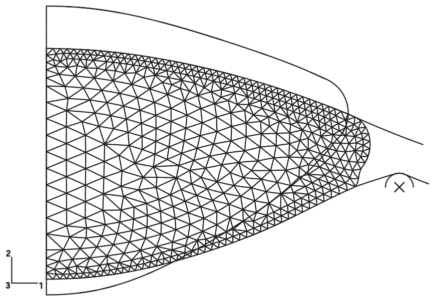
$$\text{Power}_{\text{lens}} = \frac{n_1 - n_{av}}{r_a} + \frac{n_1 - n_{av}}{r_p} - \frac{t(n_1 - n_{av})^2}{r_a r_p n_1}$$

4.5. Results

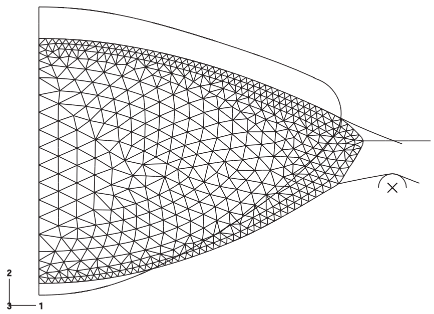
A short selection of obtained results is given in the present section. Only general outcomes are outlined here to show the achievements of our modelling exercises. More detailed quantitative results are presented in the attached papers.

Support for Helmholtz theory

Deformed meshes together with the reference lens surfaces for the models with and without zonular central set are shown in Figure 4.5. During disaccommodation, outward tension is applied to the lens, which pulls it into a relatively flattened state. The central anterior and posterior surfaces of the lens flatten and the axial thickness decreases, thus decreasing the lens power; these results are in agreement with the Helmholtz hypothesis of accommodation.



(a)



(b)

Figure 4.5: Deformed lens meshes together with reference surfaces. No Wieger's ligament. a) Without central zonular fibres. b) All three sets of fibres.

Optical power of the isolated lens and complete eye

We have shown that model with only anterior and posterior sets of fibres behaved better than with inclusion of the central zonular group, giving results consistent with published experimental data. For this case, the estimated accommodation amplitude was varied in the range from 12 to 5.4 Diopters, depending on the thickness of Wieger's ligament. It should be noted that data from all numerical models described in the thesis so far relates to the optical power of the isolated lens. To determine the accommodation amplitude of the complete eye we use the schematic eye of Bennett & Rabbetts (1989), where it is assumed that a change of lens power of 13.2 Diopters is required to change the optical power of the eye by 10 Diopters. Thus, the obtained change in lens power should be divided by 1.32 to estimate the accommodation amplitude of the optical system of the complete eye. The computed data corrected in this way to give estimated accommodation amplitude for the complete eye are plotted in Figure 4.6, together with amplitude curves derived by Duane (1912) from clinic patients.

In connection to the present issue several points should be emphasized:

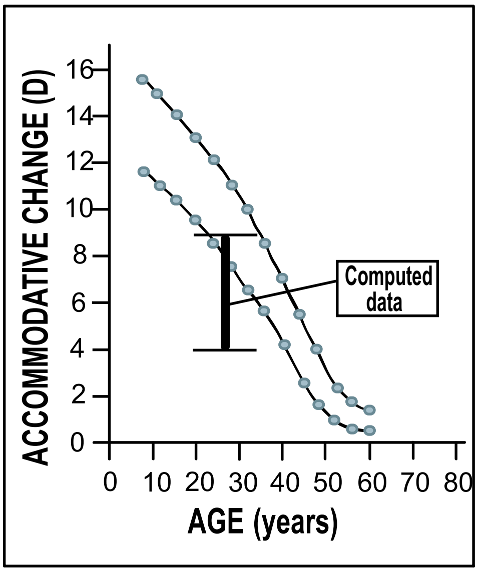


Figure 4.6: Amplitude of accommodation compared with Duane (1912)

1. Our model predicts the objective accommodative amplitude, while Duane's data are concerned with the subjective accommodative amplitude. There can be a considerable difference between the two, the subjective amplitude always being higher that the objective amplitude.

2. In our model the amplitude of lens equator displacement was derived from *in vivo* data by Strenk *et al.* (1999) based on an accommodation demand of 8 Diopters. It seems possible that data do not represent the full range of available movement for young, non-presbyopic eyes, which should be capable of an accommodation range in excess of 8 Diopters.

Thus, there is likely to be a tendency for under-prediction of accommodation amplitude in the present analyses.

Pressure inside the lens

Svetlova & Koshitz (2001) raised the question about the lens pressure, stating the maximal pressure inside the lens is at a near sight, when the lens is more roundish. This point of view is disputed by the majority of ophthalmologists who claim the opposite, e.g. Nesterov *et al.* (1999). The computed variations of the pressure inside the lens were estimated in our analyses for the main model by taking the average value for the elements, representing the crystalline lens.

All models show the similar behaviour in pressure change patterno. For the model without central fibres the pressure distribution within the lens is shown in Figure 4.7.

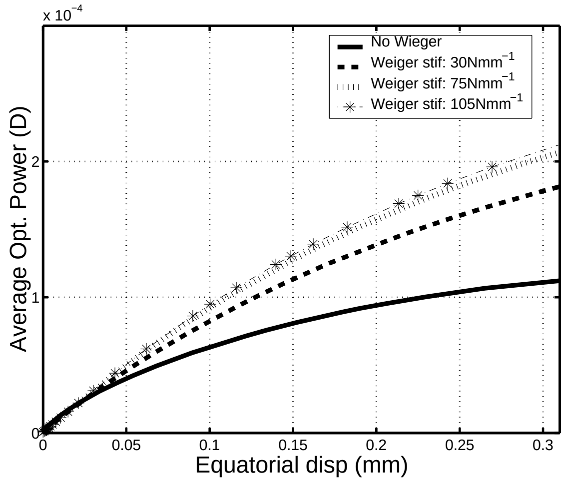


Figure 4.7: Lens pressure variations for different thickness of Wieger's ligament

According to our calculations, the pressure inside the lens capsule at near fixation lower than when looking into the distance, which contradicts suggestion by Svetlova & Koshitz (2001) .

Chapter 5

Conclusions and Outlook

The present chapter brings conclusions together to form a summarized and generalized version, upon which suggestions are based as to further topics for fruitful research.

5.1. Concluding remarks

This thesis uses as an experimental basis the results of previous studies concerning the construction of the lens and its supportive apparatus and their material and mechanical properties. We have attempted to investigate the validity of some fundamental assumptions about the structures involved in the accommodation process that seem to be taken for granted by most investigators. This includes such issues as the direction of the resultant of the stretching forces and presence of vitreous.

As was pointed out before, *in vitro* measurements are subjected to the nascent uncertainties about tissue preparation and storage and also the way in which tension is applied during the mechanical stretching experiment. Unfortunately, these studies are necessary when it comes to the estimation of mechanical properties of different components, since there is still no technique to measure these qualities *in vivo*. Nevertheless, we should avoid using *in vitro* observations for evaluation of the geometrical properties. Thus, in the present work most received results (and all geometry characteristics) are discussed in comparison with different *in vivo* studies.

Comparison of results with experimental observations

The results of lens curvature measurements *in vivo* together with our derived values are summarized in Table 5.1 for the unaccommodated eye (in our simulation it is the deformed state). We present the results for the model without central set of fibres, since it shows the best behaviour. All results are within the range of reported data.

Some experimental measurements presented on Table 5.1 are obtained from the eyes which are relatively younger than the age accepted in our study — 29 years. It is universally accepted that younger lenses should be capable of larger accommodation, which imply larger changes in curvatures.

Table 5.1: Our derived values and published values of the radii of curvature of the human crystalline lens measured *in vivo* (unaccommodated eye).

Source	Mean age	R_{ant} (mm)	R_{post} (mm)	Technique
Our derived data	29	$8.7 \div 9.7$	$4.3 \div 6.4$	FEM
Royston et al. (1989)	22	$7.33 \div 11.18$	$4.74 \div 6.97$	phakometry
Goss et al. (1997)[a]	26	9.85	5.95	phakometry
Brown (1973)[b]	29	13.4[9.8 at 3D]	9.4[7.3 at 3D]	slit-lamp
Hemenger(1995)	19-31	11.21 ± 1.08	6.45 ± 0.56	phakometry
Koretz et al. (2001)	29	10.575	7.687	Scheimpflug
Dubbelman (2001)	29	11.247	5.852	Scheimpflug
Garner, Yap (2001)	19-24	11.54 ± 1.27	6.62 ± 0.94	phakometry
Dubbelman (2005)	29	11.8	6.1	Scheimpflug

[a] Average of all measured data for different ages.
[b] Posterior lens surface not corrected.

It is known that radii for both surfaces become larger towards periphery, so, fitting only the central zone of the lens which is relatively smaller than accepted by other researchers could produce substantially smaller radii. For example, Dubbelman & van der Heijde (2001); Dubbelman *et al.* (2005) adopted the vision zone of 3 mm, Koretz *et al.* (2001) fitted the lens data within 4 mm central zones. For the comparison, we accepted the diameter of vision zone to be equal to 2 mm.

It was said before that we cannot use directly the value of vitreous humour found in the literature, since it does not include the pressure produced by the contained fluid. In our modelling exercise the derived value for the modulus of elasticity is, as can be expected, larger than the value reported in experimental studies. This observation is confirmed by measurements performed by Lee *et al.* (1992), where the internal modulus of elasticity is two orders of magnitude less than those derived from our inverse methodology.

Critique of experimental basis

In our analyses the internal profile of the lens was constructed using geometry data from a study by Brown (1973), since it is the only study which in detail describes paraxial and peripheral radii of curvature for the lenses under different accommodative conditions. At present it has been proved, that these results (particularly about posterior surface) should be treated with caution, because Brown did not correct the optical distortion caused by refraction by the cornea and lens itself, (Dubbelman *et al.* 2003, 2005).

Radii reported by Brown (1973) for 0 Diopters are significantly larger than those obtained in the present study. At the same time his measurements at an accommodative demand of 3 Diopters give almost the same values as by our model, see Table 5.1. A following explanation can be adopted for the last statement: Brown's (1973) geometric information for accommodative demands of 10 Diopters are used to describe the reference state of the model. At the same time as an initial lens radius Strenk *et al.* (1999) data for an accommodative demand of 8D is adopted. At the end of the simulations it is assumed that the deformed lens radius increased by 0.31 mm; which was also measured Strenk *et al.* (1999) for 0D. Taking these facts into the account we cannot claim, that our simulations correspond to the amplitude of an accommodative demand of 10D; most likely the amplitude is around $9-7$D. In that case, Brown's (1973) measurements at 3D (corresponding to the amplitude of an accommodative demand of 7D) agree very good with our data.

To summarize, the consistency with experimental measurements Table 5.2 is created, where data are presented about changes of radii of curvatures for the anterior and posterior surfaces for different accommodative demands. Our results are in a relative agreement with published archives.

Table 5.2: Our derived values and published values of the changes of radii of curvature of the human crystalline lens during accommodation/disaccommodation.

Source	Mean age	ΔR_{ant} (mm)	ΔR_{post} (mm)	Amplitude Demands
Our derived data	29	$2.4 \div 3.4$	$0.3 \div 2.4$	$9-7$ D
Brown (1973)[b]	29	6.9	4.1	10D
	29	3.3	2.7	7D
Garner, Yap (2001)	19-24	appr. 4.3	appr. 1.3	8D
Dubbelman (2005)	29	3.5	1	7D

[b] Posterior lens surface not corrected.

By the comparison with experimental data we can claim that the present model of accommodation with adopted assumptions captures at least several physiological aspects of accommodation. We do believe that the present study shows that the vitreous support is an important feature and its addition is consistent with mechanical laws for such structures.

A new model can be a useful tool for giving relevant clinical suggestions to a developing area in ophthalmology, such as surgical correction to presbyopia. A benefit of the finite element formulation is the possibility to relatively easily change properties and different configurations of the model. This study is a step towards the understanding on how the mechanisms of human accommodation work. Using an already established model of the young eye with reasonable behaviour we can expand it to eyes of different ages. Such modelling exercise would involve modelling of age-related changes appearing in related eye structures, and for this purpose more relevant ophthalmological data are necessary for wider range of subjects.

5.2. Future research

In considering the possibility of future work a series of suggestions are listed below:

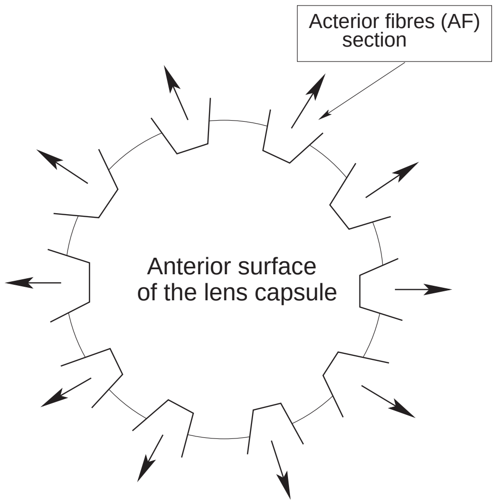


Figure 5.1: Scheme of distribution of the anterior fibres.

Theoretical

According to research by Radivoj (1991) the anterior fibres are distributed over the place of fastening not uniformly but by portions, i.e. sectionally. A scheme of attachment of the anterior fibres to the anterior lens capsule is shown in Figure 5.1. As stated by Svetlova & Koshitz (2001), such a partitioning of the anterior group of fibres makes it possible for the ciliary muscle to stretch the segments of the anterior lens capsule differentially, thus eliminating congenital or acquired astigmatism of the optical systems of a human eye.

A new model that would take into account this feature is suggested for consideration.

Experimental

The data for the actual geometry and material properties of the eye structures are very sparse, some of them are old and some are not available at all. It is a first necessity to gain better geometric and mechanical high quality data to calibrate more complex model and, later, models of eyes of different ages.

Chapter 6

Review of Papers

Two papers are included in the current thesis. Both studies regard numerical non-linear simulations featuring an axisymmetric finite elements model of accommodation of the human eye. The analyses are carried out using the commercial finite element code ABAQUS, Hibbit *et al.* (2002).

Paper 1

This paper addresses the problem of establishing theoretical procedures based on finite element analysis for modelling human eye structures that are involved in the accommodation process. It is a forerunner to Paper 2 presenting a preliminary attempt to construct the model of accommodation using a modified description of the accommodative apparatus which consists of the lens, zonule, ciliary body and vitreous. The model employs vitreous support for the back of the lens but neglects such anatomical elements as the posterior zonular set of fibres and the capsule-hyaloid ligament between the posterior surface of the lens and the anterior part of vitreous body. Those adopted assumptions affect the model performance. In particular, upon this modelling exercise only a certain phenomenon of accommodation is studied — the so-called "squeezing" effect. Nevertheless, the model behaves according to the expectations giving results which are consistent with the Helmholtz theory of accommodation. Realistic values of total resulting optical power are obtained. Some limited sensitivity analysis is performed and the effect of various input parameters on the models' behaviour is evaluated. Developed procedures and algorithms are verified by comparing the model performance to existing archive of experimental published data. The overall results show that the proposed method of modelling is sufficiently accurate to be used in further studies, employing more complex models.

Paper 2

Paper 2 is an extension of the model proposed in Paper 1. It implements specially formulated elements for the incompressible analyses, and includes previously omitted anatomical structures, e.g. the posterior fibres and Wieger's ligament. The model's mechanical and optical performance is studied. A proposed inverse methodology is used to find some parameters which could not be reliably found in the literature. It is shown that the model captures at least some physiological aspects of accommodation. The results are in agreement with the range of experimental data. The model behaves according to the classical theory expectations. The current modelling exercise can be used to obtain clinically relevant recommendations to presbyopia surgery by allowing easy adjustments of modelling parameters to simulate the eyes of different ages.

Appendix

The ***amplitude of accommodation*** is the difference, expressed in diopters (D), between the far point of accommodation (the rest state of an eye) and the near point of accommodation (fully accommodated state of an eye) measured clinically from the spectacle plane. It is equivalent to the inverse of the distance in which healthy, so-called emmetropic eye can focus clearly.

The ***aniridia*** is the congenital absence of the iris.

The ***aphakia*** is the absence of the natural crystalline lens of the eye.

The ***anterior chamber*** is the area bounded in front by the cornea and in back by the lens, and filled with aqueous.

The ***aqueous (aqueous humour)*** is a watery solution in the anterior and posterior chambers. It is a clear mass which connects the cornea with the lens of the eye and helps maintain the convex shape of the cornea, which is necessary to the convergence of light at the lens. In health the aqueous humour does not mix with the firm, gel-like vitreous humour. Aqueous is produced by the ciliary body.

The ***artery (central retinal)*** is a branch of the ophthalmic artery that enters the retina from the middle of the optic nerve and supplying blood to the eye.

The ***canal of Schlemm*** is the passageway for the aqueous fluid to leave the eye. Blockage of this canal can result in glaucoma, an increase in intraocular pressure.

The ***choroid***, which carries blood vessels, is the inner coat between the sclera and the retina. It is darkly colored, so that reflection within the eye is limited.

The ***ciliary body*** is an unseen part of the iris, and these together with the ora serrata form the uveal tract. Ciliary body is responsible for producing the aqueous humour that circulates in the chambers of the eye. It contains the multi-unit smooth ciliary muscle.

The ***conjunctiva*** is a clear membrane, which is covering the white of the eye (sclera) and lines the inside of the eyelids.

The ***cornea*** is a clear, transparent portion of the outer coat of the eyeball through which light passes to the lens. It is a powerful refracting surface, providing 2/3 of the eye's focusing power. This surface is extremely sensitive - there are more nerve endings in the cornea than anywhere else in the body.

The ***Extraocular Muscles (EOMs)*** are six tiny muscles that surround the eye and control its movements. The primary function of the four rectus muscles is to control the eye's movements from left to right and up and down. The two oblique muscles move the eye rotate the eyes inward and outward.

An ***intraocular lens (IOL)*** is an implanted lens in the eye, usually replacing the existing crystalline lens because it has been clouded over by a cataract. They usually consist of a plastic lens with plastic side struts called haptics to hold the lens in place within the capsular bag.

The ***iris*** is a coloured ring of muscle fibres. It functions like the aperture on a camera, enlarging in dim light and contracting in bright light. The aperture itself is known as the pupil. The iris is flat and divides the front of the eye (anterior chamber) from the back of the eye (posterior chamber). The color, texture, and patterns of each person's iris are as unique as a fingerprint.

Lag of accommodation is the dioptric difference between the accommodative response and the stimulus to accommodation. It can be thought of as a measure of accommodative accuracy.

The ***lens*** is a transparent structure that lies between the iris and vitreous body. It helps to focus light on the retina, by changing its shape.

The ***macula*** is a oval yellow spot located roughly in the center of the retina, temporal to the optic nerve. It is a small and highly sensitive area that provides our most central, acute vision.

The ***optic nerve*** conducts visual impulses to the brain from the retina. It connects to the back of the eye near the macula. When examining the back of the eye, a portion of the optic nerve called the optic disc can be seen. The retina's sensory receptor cells of retina are absent from the optic disc. Because of this, everyone has a blind spot there. This is not normally noticeable because the vision of both eyes overlaps.

The ***ora serrata*** is serrated extremity of the optic part of the retina, located a little behind the ciliary body and marking the limits of the percipient portion of the membrane. Together with the ciliary body it forms the uveal tract, an unseen part of the iris.

The ***posterior chamber*** is the area behind the iris, but in front of the lens, that is filled with aqueous humour.

The ***presbyopia*** is a condition that occurs with growing age and results in the inability of the human eye to focus on objects up close. It is not a disease as such, but a condition that affects everyone at a certain age, that cannot be cured, but can be corrected with glasses or contact lenses.

The ***pupil*** is the opening, or aperture, of the iris. The size of the pupil determines the amount of light that enters the eye. The pupil size is controlled by the dilator and sphincter muscles of the iris. It appears black because most of the light entering it is absorbed by the tissues inside the eye.

The ***rectus medialis*** is one of the six muscles of the eye, see EOMs.

The ***retina*** is the innermost coat covering the back two-thirds of the eyeball, formed of light-sensitive nerve endings that carry the visual impulse to the optic nerve, producing the sensation of vision. The retina may be compared to the film of a camera. It is actually an extension of the brain, formed embryonically from brain tissue and connected to the brain proper by the optic nerve.

The ***sclera*** is the dense fibrous opaque white outer coat enclosing the eyeball, except the part covered by the cornea. It gives the eye its shape and helps to protect the delicate inner parts. The optic nerve is attached to the sclera at the very back of the eye.

The ***suspensory ligament (zonules of Zinn, zonular apparatus)*** supports the lens of the eye and hold it in place, stretching and loosening (due to relaxation and contraction of the ciliary muscle) to change the shape of the lens to focus at far and at near objects.

The ***vein (central retinal)*** is formed by union of the veins draining the retina and which passes into the middle of the optic nerve and empties into the superior ophthalmic vein. It is the vessel that carries blood away from the eye.

The ***vitreous (vitreous humour)*** is a transparent, colorless mass of soft, viscous material filling the eyeball behind the lens. It is enclosed by a delicate quite elastic hyaloid membrane. Vitreous is composed mainly of water and comprises about 2/3 of the eye's volume, giving it form and shape. The viscous properties of the vitreous allow the eye to return to its normal shape if compressed.

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Part 2

Papers

Paper 1

1

Numerical study of effect of vitreous support on eye accommodation

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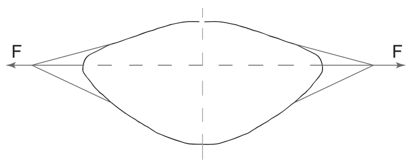
The aim of the current work was to extend previously created finite element models of accommodation such as the one by Burd, Judge & Cross (2002), by addition of vitreous. The zonule consisted of anterior and central sets and vitreous was modelled as a linear elastic incompressible body. An inverse method was used to find some important, previously not documented, aspects. The model was found to behave according to the expectations, with results consistent with classical Helmholtz theory.

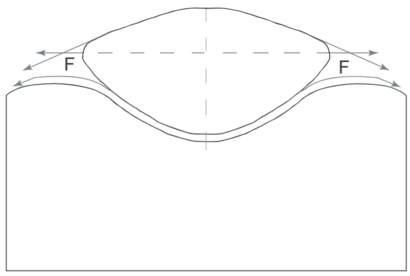
Introduction

Accommodation of an eye is the process of adjusting its focus distance to allow near objects to be focused on the retina. According to the widely accepted classical von Helmholtz theory and recent research of the accommodative apparatus of an eye, (e.g. Svetlova & Koshitz 2001), in the unaccommodated state the lens is flattened by the passive tension of the zonular fibres which are pulled by the elastic choroid structures. During accommodation the ciliary muscle contracts, thereby slides forward and towards the axis of the eye. It pulls the choroid structures, which causes the tension in the zonular fibres to reduce. With release of the resting tension of the zonular fibers, the elastic capsule reshapes the lens; it becomes more sharply curved and thickens axially. The equatorial diameter thereby decreases, increasing the refractive power of the eye.

It appears that in the improved description of the accommodative apparatus (e.g. Svetlova & Koshitz 2001), anterior and posterior zonular fibres originate near ora serrata. From these observations one can conclude that the resultant of the tensile forces applied to the ends of these zonular fibres is not horizontal at the equator but slopped posteriorly. It would contradict mechanical laws if such a construction did not have any support that is supplied by vitreous. It should be noticed that previous theoretical studies of accommodation, (e.g. Burd *et al.* 2002; Schachar & Bax 2001) assumed that all zonular fibres attach to the ciliary processes at a single point, which means that the resultant of stretching forces lies in an equatorial plane, Figure 1(a). These works did not include in the model the vitreous support, which should not be neglected.



(a)



(b)

Figure 1: Theoretical models: a) previous theoretical models (Schachar & Bax 2001; Burd *et al.* 2002); b)a new model.

We came to the conclusion that an improved theoretical model should consist of lens, zonular system and vitreous, Figure 1(b). The hypothesis of the present work is that an improved description of eye accommodation can be obtained by such a model.

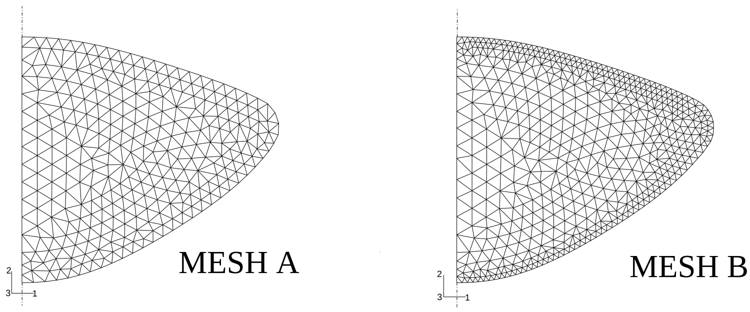
In this study we propose a finite element model of accommodation in which the effects of vitreous support are included. Some basic procedures established by Burd *et al.* (2002) were used to model the accommodative lens and perform post-processing, as described below. Since this is the first step towards establishing an appropriate numerical model of the accommodative eye with the vitreous support, some reasoned assumptions were made to simplify the process and few of them have a noticeable influence on the behaviour of the model. These features were explored and are discussed in the paper. Our findings are of a preliminary nature and further work is necessary to develop a theoretical model that is capable in representing in detail the mechanism of accommodation of an eye.

Modelling process

An attempt was performed to study an accommodation of a 29-year-old eye. A numerical analysis has been carried out using the commercial general-purpose finite element package ABAQUS (ABAQUS Inc., USA) which is widely used in mechanical and civil engineering. The eye was regarded as being a body of revolution and an axisymmetric analysis was adopted. Axisymmetric boundary conditions were applied to the nodes located along the symmetry axis. For purpose of our study we adopted an assumption that all materials were isotropic and purely elastic, Fisher (1969). Unfortunately, few published data are available to develop more complex material descriptions.

The present model involves complex contact simulation and first-order elements are more appropriate in that case, (e.g. Cook, Malkus, Plesha & Witt 2002). Such elements were chosen to avoid problems that usually appear in contact simulations when second order elements are used. Since triangular elements are geometrically versatile, it was convenient to use them meshing the interior of the lens, as it has a complex shape. Two-noded axi-membrane elements were used to model capsules of lens and vitreous to prevent the out-of-plane bending behaviour. We introduced the assumptions that interiors of the lens and vitreous are practically incompressible. It is known that it is better to use specially formulated elements (so-called hybrid-elements) to avoid problems associated with modelling incompressible materials. But since each hybrid triangular axisymmetric element introduces a constraint equation in an incompressible problem, a mesh containing only these elements will be overconstrained. That was the reason we used normal axisymmetric elements for the lens and vitreous with Poisson's ratio 0.49 representing near-incompressibility, Cook *et al.* (2002).

Lens. We assumed that the crystalline lens profile has the initial geometry, previously described by Burd *et al.* (2002) and is composed of three layers, from surface to the center: capsule, cortex, nucleus. The variation in thickness of capsule was provided by Fisher & Pettet (1972).



(a) (b)

Figure 2: Lens meshes. a) mesh with 593 elements; b) mesh with 1107 elements.

Burd *et al.* (2002) developed a fifth order polynomial using their data and these values were adopted in the present work to determine the axial stiffness of the membrane elements. Axisymmetric continuum three-noded elements were used to represent nucleus and cortex. Elastic moduli for capsule, cortex and nucleus were, respectively, $1.27\,\mathrm{Nmm}^{-2}$(see Krag, Olsen & Andreassen 1997), $3.417 \cdot 10^{-3}\,\mathrm{Nmm}^{-2}$ and $0.5474 \cdot 10^{-3}\,\mathrm{Nmm}^{-2}$, (see Fisher 1971).

Two meshes were created for the crystalline lens to investigate the influence of mesh density on the results, Figure 2. Here, we assumed that when the eye was in a fully accommodated state (our reference configuration) all the stresses in the lens, capsules, vitreous and zonule were zero. An emphasis of the present work was given to the addition of the vitreous support to the model of accommodation. The assumptions proposed by Burd *et al.* (2002) about the reference state of the lens were accepted. Therefore, questions about the relevance of these assumptions are still open. These features are beyond the scope of the paper and left for the future research.

Zonule. The zonule is a supporting system for the lens. By scanning electron microscopy (e.g. Farnsworth & Shyne 1979; Ludwig, Wegsheider, Hoops & Kampik 1999), the zonule appeared to attach to the lens capsule in three distinct sets, denoted by the places of their attachments to the lens: anterior, posterior and central. Each set of the zonule is composed of a number of individual fibrils, where anterior fibres (AF) are more numerous than posterior fibres (PF) and the number of central fibres is quite low. Fisher (1971) measured an angle α of inclination of the anterior zonular fibres to the lens axis, Farnsworth & Shyne (1979) reported the value for X — the distance between the anterior zonule attachment and the lens radius. We also use the assumption that central fibres (CF) connect to the ciliary processes. Strenk, Semmlow, Strenk, Munoz, Gronlund-Jacob & DeMarco (1999) obtained the reference value for ciliary body equator $R = 6.474\,\mathrm{mm}$. These values were adopted in our study, Figure 3.

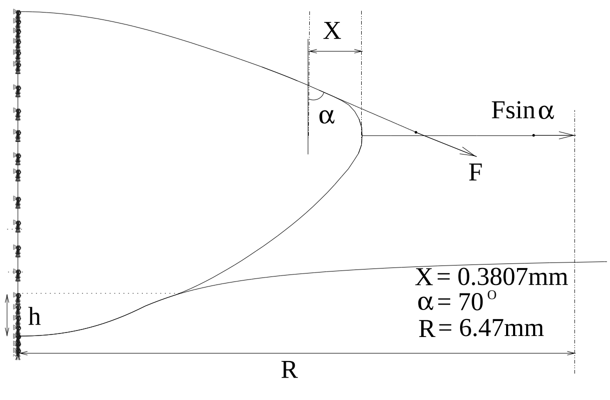


Figure 3: Geometry of fully-accommodated eye.

Addition of the vitreous support in the model complicates the simulations significantly, due to the appearance of complex contact conditions (anterior fibres/lens capsule, posterior fibres/lens capsule, posterior fibres/vitreous). For the present modelling it was assumed that PF together with Wieger's ligamentum and Berger's space could be neglected to simplify numerical analysis. It should be noticed that this assumption has a significant effect on the behaviour of the model. That feature is discussed later in the paper. Such an approach is considered to be satisfactory, since we make an emphasis on studying the certain aspect of accommodation, namely the "squeezing effect", which is achieved by stressed AF and an additional stressed surface of the vitreous body in unaccommodated state.

It is difficult to specify the stiffness of zonule fibres since it depends on fibres number, size and elastic modulus. Van Alpen & Graebel (1991) proposed a value of $1.5\,\mathrm{Nmm}^{-2}$ for the Young's modulus of a zonule during their *in vitro* experiment of equatorially stretching lens. To determine thickness which could not be found in literature, we performed an inverse experiment described below. We followed the assumption that contact between the lens capsule and AF was frictionless.

Vitreous. Traditionally the role of vitreous was neglected in accommodation. From a biomechanical point of view, the vitreous body, which occupies the posterior compartment of the eye and is about 80% of the volume of the eyeball, represents a membrane full of vitreous humor. It maintains a certain level of intraocular pressure and ensures normal adjacency of the inside shells of an eye (choroid and retina). It is known that the vitreous membrane is quite elastic, a quality which keeps it under constant tension. As a result, the vitreous body naturally tends towards spherical configuration although the lens lying on the anterior part of the vitreous forms a small pit — fossa patellaris. To simplify the numerical analysis we replace the globular vitreous media by the rectangular one. It can be justified by the fact that effect of the far side of vitreous on the accommodation mechanism may be neglected. In the modelling, initial geometry of the vitreous was manually scaled from MR images, Strenk *et al.* (1999). We assumed that it is a nearly incompressible material with length and height 10 mm and 17 mm, respectively. No data were found about mechanical properties of the vitreous and as the first approach we assumed that the vitreous humor and membrane are elastic materials. We modelled the membrane using two-noded axi-membrane elements. It was assumed that vitreous membrane has almost the same stiffness as the lens capsule with Young's modulus $2\,\mathrm{Nmm}^{-2}$ and thickness 0.02 mm.

Procedure. As a starting point in the simulation no contact existed between lens capsule and vitreous. A first step was done to achieve the needed configuration for simulation: to create fossa patellaris and to establish contact between lens and vitreous avoiding the appearance of rigid body motions. The finite element model at its initial state is shown in Figure 4(a), and, after the preliminary step in Figure 4(b).

In order to emulate stresses in vitreous which occur when we put the lens on the anterior hyaloid membrane, a first step was taken. Since the lens configuration was based on *in vivo* measurements (Brown 1973), the shape of fossa

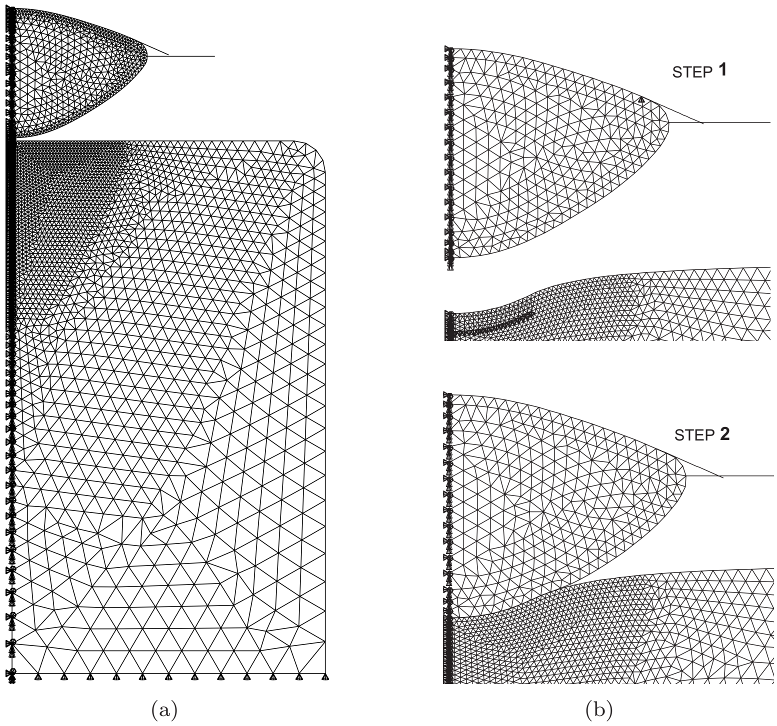


Figure 4: Procedures: a) reference configuration, mesh with 1107 elements; b) preliminary steps (h=1).

patellaris should be the same as the shape of the posterior surface of the lens. To achieve this state we applied displacement boundary conditions. The depth of the pit, h, was unknown and we varied it from 1mm to 3mm, Figure 3. The second step established the contact between vitreous body and lens capsule with a friction coefficient equal to $\mu = 0.01$ as for lubricated surfaces.

Two simulations using different meshes were done by applying forces to the ends of the zonular fibres. To deduce unknown parameters (force, anterior and central zonule stiffness, Young's module of vitreous humor) we performed inverse experiment. This is a well known engineering methodology used to establish unknown parameters, by performing "what if" calculations in these parameters from which their values are deduced by comparing the resulting behaviour to known factors. Finite element calculations were done by applying the outward tension to the ends of zonular fibres. It was assumed that the final deformed geometry should match the data reported by Brown (1973) and Strenk *et al.* (1999) for non-accommodated state. Strenk *et al.* (1999) suggest that when the ciliary body moves 0.4 mm, the corresponding change in lens equator radius would be 0.3 mm. Brown (1973) proposed that during

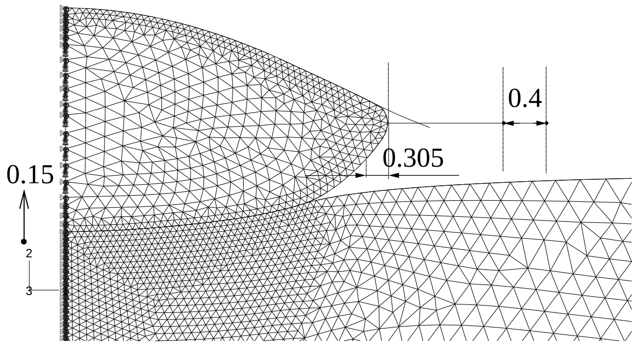


Figure 5: Inverse experiment.

disaccommodation, the posterior pole moves 0.15 mm (see Figure 5). Unknown parameters were manually adjusted at the end of each calculations to give the known deformed configuration.

Results

Calculations were performed and unknown parameters were calculated for different depths of fossa patellaris ($h = 1, 2, 3\,\text{mm}$). We report data for configuration with $h = 2\,\text{mm}$, other results are given in the "Sensitivity studies". Solutions for lens geometry were received and each portion of the obtained lens surface was fitted with circular fits to calculate the anterior (r_a) and posterior (r_p) radii of curvature. A circular aperture of radius 0.6 mm was used, regarded as the optical zone and the most important for vision. A polar lens thickness (t) was calculated from the finite element simulation results. The optical power was determined by using the thick lens formula:

$$\text{Power}_{\text{lens}} = \frac{n_1 - n_{av}}{r_a} + \frac{n_1 - n_{av}}{r_p} - \frac{t(n_1 - n_{av})^2}{r_a r_p n_1}$$

We assumed refractive indices of $n_{av} = 1.336$ for aqueous and vitreous humour and $n_1 = 1.42$ for the crystalline lens, i.e., the values proposed for the Gullstrand-Emsley schematic eye. It should be noted that the procedure described here follows closely that described by Burd *et al.* (2002).

During the simulation we manually deduced unknown parameters (force, Young's modulus of vitreous humor, zonular stiffness) by ensuring that the final geometry matched the data for the unaccommodated state of lens, as discussed above. For considered values h, the final value of zonular force was 0.09 N and the thicknesses of anterior and central fibres were 70μm and 29μm, respectively.

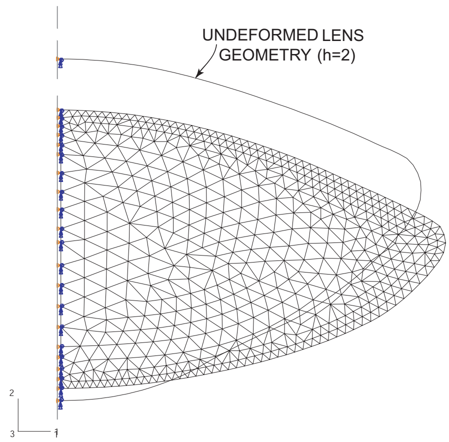


Figure 6: Deformed mesh with 1107 elements for the lens

The deformed mesh for mesh B (see Figure 2(b)), together with the reference lens surfaces is shown in Figure 6. The simulations show that the central thickness of the crystalline lens increases with accommodation and paraxial anterior and posterior curvatures become steeper.

During disaccommodation, outward tension is applied to the lens and pulls it into a relatively flattened state, thus decreasing the lens power; these results are in agreement with the Helmholtz hypothesis of accommodation, Figure 7. During stretching from accommodated to unaccommodated state, computed results indicated an 11% reduction in central anterior curvature and a 51% reduction in central posterior curvature. The corresponding decreases in optical power of the anterior and posterior surfaces were 2D and 10D, respectively, and the decrease in optical power of the whole lens 11.3D. The decrease in geometric thickness during disaccommodation was 0.79 mm and occurred entirely in the nuclear region — 84%. This agrees well with measurements by Brown (1973). The anterior pole moved backwards 0.6 mm, when the eye changed its vision from near to far point, which is approximately three-quarters (76%) of the total change of lens thickness.

The computed variations of optical power with equatorial strain for two meshes with different densities are shown in Figure 8. The difference in mesh densities has a small effect on the variation of the optical power. As the difference is limited the finer mesh B with 1107 elements is assumed to be acceptable.

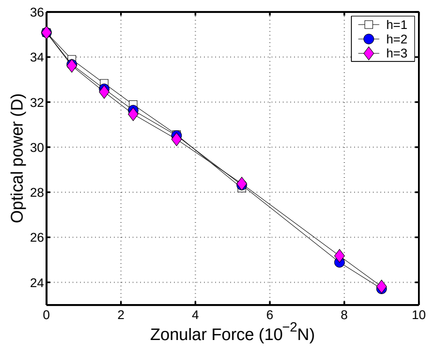


Figure 7: Relationship between zonular force and optical power

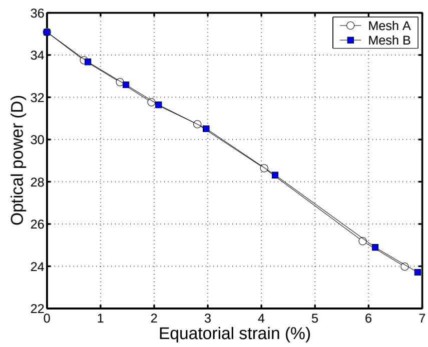


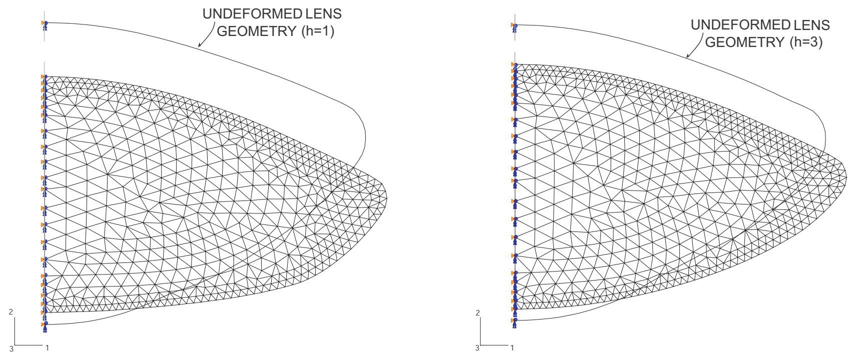
Figure 8: Variation of optical power with equatorial strain

Sensitivity studies

Since theoretical modelling critically depends on input parameters and assumptions, it is susceptible to errors. Any aspect of the model can be responsible for yielding poor quality results. Thus, the effect of variability of different input data on performance of the model is an important question. Although the primary interest of this study was the qualitative numerical analysis of accommodation process with vitreous support, some limited variations have been studied, to decide the sensitivity of obtained results.

Depth of fossa patellaris. Since the depth of fossa patellaris was unknown in our numerical simulations, three computational analyses with inversely estimated parameter values have been carried out for eye models with different depths of vitreous pit ($h = 1, 2, 3\,\mathrm{mm}$). It is clear from Figure 9 that variation of h has noticeable effect on the behaviour of the model. Here we assumed that computed values for zonular force $0.09\,\mathrm{N}$, anterior and central stiffness $105\,\mathrm{Nm}^{-1}$ and $43.5\,\mathrm{Nm}^{-1}$ of the zonular fibers were the same for all three models.

During the process of disaccommodation, the posterior central curvature is flatter and changes in thickness are greater for smaller values of h. This feature of the model would be expected, on the basis that during calculations we fixed the displacement of the posterior pole and lens equator and achieved the final configuration by varying the elastic modulus of the vitreous humour. Thus, for smaller h, the vitreous substance was assumed to be stiffer with higher stresses during the second step of simulation. As a consequence, the deformation of the posterior surface was higher during the contact. For $h = 1$, when the values of force exceeded $0.05\,\mathrm{N}$, the posterior curvature varied so significantly with radius of circular aperture in the optical zone that we exclude these points from further consideration. Posterior curvatures for $h = 2, 3$ showed smooth behaviour during the analysis and these calculations were considered to be



(a) (b)

Figure 9: Deformed mesh with different depths of pit: a) $h = 1$; b) $h = 3$

acceptable. The effect of difference in depths of pit on radius of posterior curvatures of the lens is shown in Figure 10.

It is less clear from Figure 9 how the anterior paraxial curvature changes. The computed results, Figure 11, indicated greater increase in anterior radius of curvature with higher h, as the lens was stretched from accommodated to unaccommodated state (i.e. $6\%, 11\%$ and 14% for $h = 1, 2, 3$, respectively).

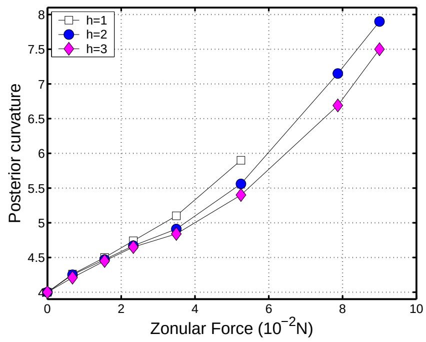


Figure 10: Effect of difference in depths of pit on radius of posterior curvature of the lens

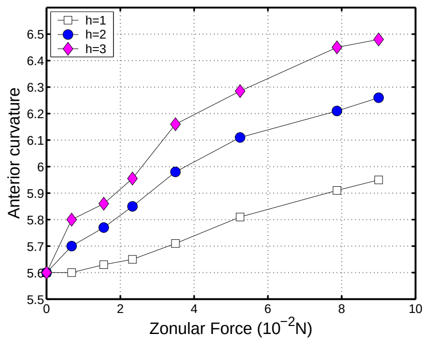


Figure 11: Effect of difference in depths of pit on radius of anterior curvature of the lens

Incompressibility of the lens. Our calculations were based on the assumption that lens and vitreous matrix are incompressible. That means that Poisson's ratio should be equal to 0.5, which provokes numerical difficulties during finite element simulations. Another calculation was carried out with Poisson's ratio inside the lens set to 0.499. We calculated the deformed volume, which should be constant for an incompressible material. For a computation with Poisson's ratio 0.49 there is a loss in lens volume about 2% through the shortening process. For a Poisson's ratio 0.499 volume is conserved. The difference in accommodation amplitude was about 5% between the cases, which is not considered significant.

Discussion

Classically it has been believed that the anterior lens surface is the main contributor in the accommodative process. Recently, partial coherence interferometry showed a posterior movement of the posterior surface, anterior movement of anterior surface and small forward translation of the center of mass of the lens, (Drexler, Baumgartner, Findl, Hitzenberger & Fercher 1997). It was reported that the forward movement of the anterior pole of the lens is approximately three times larger than the backward movement of the posterior pole during fixation from the far point to near point. According to our results the forward axial translation of the anterior surface is 76% of the increase of lens thickness, which is in agreement with other measurements (e.g. Drexler *et al.* 1997), although polar movements are about three times as great as those observed by Drexler *et al.* It should be mentioned that we fixed the posterior pole displacements according to Brown (1973) on 0.15 mm, and Drexler *et al.* reported a value only one third of this.

It was shown (e.g. Cook, Koretz & Kaufman 1991; Brown 1973) with Scheimpffug slitlamp photography that the posterior curvature might increase, although the magnitude is unknown due to refraction by the lens and may be zero. We used the values given by Brown (1973) to recreate the initial geometry of the lens in our study, but it should be noted that because of optical distortions involved in this method (Dubbelman & der Heijde 2001), some inherent experimental errors can exist. Garner & Yap (1997) present measurements, showing that about one third of the change in lens power with accommodation of 8D is caused by changes in posterior lens curvature. In our model the change in optical power of the posterior surface was significantly greater than the anterior one. The posterior surface made not only substantial, but main contribution to the overall increase in lens power with accommodation. It is expected for such a model because of the assumptions made about the back of the lens. In reality posterior surface of the lens is adhere to the vitreous membrane creating Wieger's ligamentum, which include a capsule-hyaloidal interspace so-called Berger's space. PF take origin from the line of attachment of the lens to the vitreous. The assemblage of these anatomical elements can be regarded as an additional annular sheet which is situated in between the back of the lens and vitreous membrane. It has certain stiffness and is confronting sudden changes of posterior curvature of the lens. In our model this sheet is omitted to simplify the numerical analysis. During the contact simulation the lens is pressed into the vitreous by powerful AF, thus producing major changes in posterior curvatures. Future models should include all anatomical components mentioned above to model the accomodative process even more precisely.

The computed variations of optical power with equatorial strain for two meshes with different densities were shown in Figure 8. The picture shows, that with increasing equatorial strain, optical power is reduced as predicted by classical theory, giving the average power reduction rate of approximately 1.6D per percent strain. Storey & Rabie (1985) report in their *in vivo* measurements a reduction of 1.2D per percent equatorial strain, Glasser & Campbell (1999) show *in vitro* experiments that in young eyes the optical power of the lens is reduced at a rate between 0.7D and 1.25D per percent equatorial strain. These physiological measurements were closely matched by our results.

Since the movement of the posterior pole had the same fixed value for all calculations, change in thickness occurred because of axial translation of the anterior pole. For values of $h = 1, 2, 3$ movements of the anterior surface were 0.733, 0.608, 0.555 mm, respectively. The resulting variation of optical power with zonular force for mentioned sets of analysis was shown in Figure 7. It seems that different depths of pit had a small effect on the resulting optical power. That difference was regarded as being small enough to suggest that variation of fossa patellaris in our numerical exercise does not play a significant role and that the model is not sensitive to this parameter.

Computed results for the zonule stiffness satisfied the suggestion by van Alphen & Graebel (1991) that lens capsule and zonule are both three magnitudes stiffer than lens substance.

Conclusions

Nonlinear finite element analysis was used to construct a numerical model of accommodation with vitreous support. It is not claimed that the model captures all physiological aspects of the accommodation, since some anatomical details are omitted. Better to say, it captures certain aspect of the mechanism, being the influence of vitreous, so-called "squeezing" effect. The behaviour of the model seems to be in reasonable agreement with different published data. Accommodation of a young (not presbyopic) eye occurred similarly to a Helmholtz mechanism.

Further work is needed to develop a numerical model, sufficiently close to the real process of accommodation of a 29-year-old eye. Better knowledge of geometric and mechanical high quality test data are required to calibrate more complex models. Some of these can be obtained or verified by an inverse methodology described in the paper.

Acknowledgements

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Paper 2

2

Aspects of eye accommodation evaluated by finite elements

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Axisymmetric nonlinear finite element models of accommodation with vitreous have been studied with respect to their effectiveness in mechanical and optical performance. All materials were assumed to be linearly elastic, vitreous and lens matrices were incompressible. Results of numerical analyses support the Helmholtz theory of accommodation.

Introduction

The mechanism of visual accommodation still remains largely unknown, despite the increased interest in human eye, the geometry of the crystalline lens and its associated internal structures. It is universally accepted that accommodation is achieved by alteration of the shape of the crystalline lens, but the exact nature of the changes has not been identified. It is agreed that during accommodation the crystalline lens undergoes an increase in the central thickness together with a decrease in the radius of curvature of the anterior and posterior lens surfaces; these lead to a decrease in the equatorial diameter (e.g. Brown 1973; Glasser & Campbell 1999). Classical von Helmholtz theory states that during the accommodation process the ciliary muscle contracts, releasing tension on the zonules suspending the lens of the eye; this allows the elasticity of the crystalline lens capsule to "round up" (Fincham 1937), and increase its central thickness and optical power, Figure 1.

Helmholtz did not mention the vitreous in his theory, and it has been generally assumed that the accommodation is provided by the lens and zonular structures alone. As a confirmation of this particular assumption, *in vitro* mechanical experiment by Glasser & Campbell (1998) with a stretching zonular force directed radially outwards at the equator of the lens showed, that in such a setup the lens accommodation was rather similar to *in vivo* accommodation. However, these results are not reliable to draw final conclusions from, since *in vitro* measurements are subjected to the uncertainties about conditions of

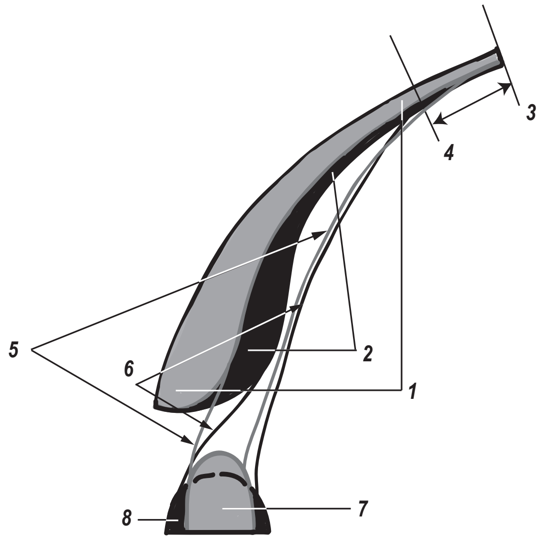


Figure 1: Mechanism of accommodation according to the classical theory.
1, 2 – meridional cut of ciliary body containing the ciliary muscle (relaxed and contracted, respectively); 3, 4 – positions of posterior border of the ciliary body - ora serrata, anterior and posterior fibres (5, 6) and crystalline lens (7, 8) during relaxation and contraction of the muscle.

the lens, surrounding tissues and the way in which tension through the zonular apparatus is applied to the lens. Some earlier studies (e.g. Coleman 1970; Koretz & Handelman 1982) tried to indicate the contribution of the vitreous in the process of accommodation. Now with improved knowledge about the accommodative apparatus (Svetlova & Koshitz 2001), we come to the conclusion that the role of vitreous can hardly be omitted. According to the studies by Gorban & Dgiliashvili (1993); Svetlova & Koshitz (2001), it most likely appears that powerful anterior zonular fibres (AF), going from the lens capsule, do not attach directly to the ciliary body. Instead, they run along the inner surface of the ciliary muscle, go through ciliary folds (processes) and interlace with the choroid near ora serrata. Thinner posterior fibres (PF) cover the vitreous anterior hyaloid membrane like a web (e.g. Rohen 1979; Ludwig, Wegsheider, Hoops & Kampik 1999), stretching from Wieger's ligament, which is fixing the lens to the vitreous, to the choroid structures near the ora serrata area. Central zonular fibres (CF) appear less numerous and thinner than AF and PF (they are not shown on the Figures 1 – 2). The ciliary muscle itself does not change directly the tension of AF and PF, but produces only the necessary displacement through the displacement of ora serrata, increasing or decreasing elastic tension of the choroid structures, see Figure 2.

Since anterior and posterior fibres originate near ora serrata, the resultant of the tensile forces is directed at some angle to the equatorial plane, and it is necessary to consider additional forces for the equilibrium conditions. These balancing forces cannot be supplied by zonules alone, and the vitreous is thus necessary as a support for the lens during accommodation.

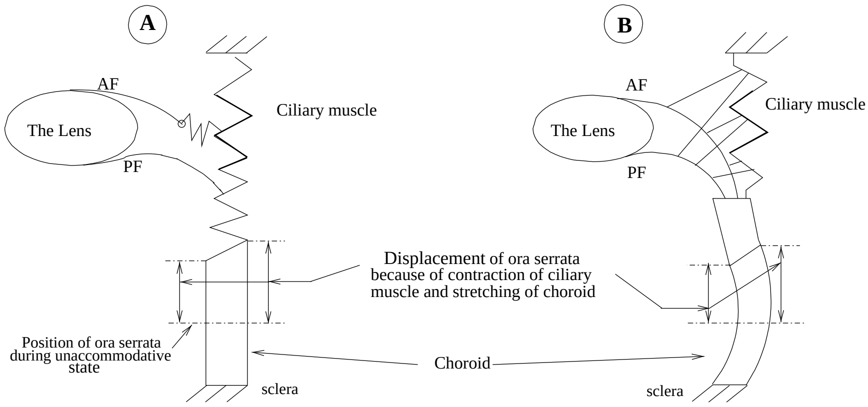


Figure 2: Diagram of doubtful (A) and reliable (B) notions of contribution of ciliary muscle in the process of accommodation. Redrawn from Svetlova & Koshitz (2001).

In this paper we attempt to extend the generally accepted theoretical model by bringing the anatomy and geometry of the accommodative apparatus together with mechanical properties of the lens, zonular system and vitreous body so far as they are known. In contrast to previous analyses, which assumed that the resultant of stretching forces lies in an equatorial plane, this study incorporated posteriorly sloped force and the vitreous, which provide a support for the whole construction, Figure 3. A nonlinear finite element model of accommodation was proposed and its effectiveness in mechanical and optical performance was investigated. Since current archives of published data contain too few sources, reasoned assumptions and inverse investigations were made to specify the model, when needed parameters were unavailable. Thus, the primary interest in this study was directed towards the qualitative numerical analysis of accommodation with vitreous support. The sensitivity study of the performance of the model is an important question and it is left for future research, with exception of some limited issues. A better knowledge of geometric and material properties of components of an eye are required to obtain a better quantitative understanding of accommodation.

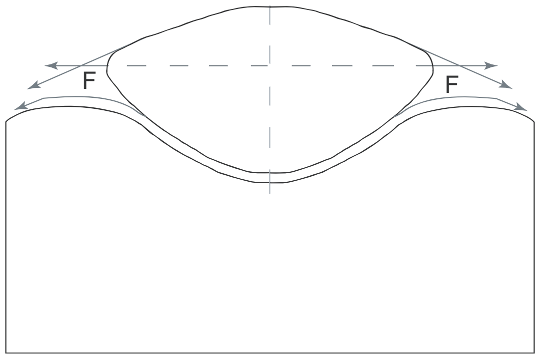


Figure 3: New theoretical model of accommodation.

Materials and methods

A new model represented the lens, together with a zonules and vitreous (see Figure 3). One of the main postulates of the Helmholtz theory is that without any zonular tension the lens is in the fully accommodated state. With this knowledge it was reasonable to choose an accommodated eye as a starting stress-free configuration for the numerical analysis, as it was previously done by Burd, Judge & Cross (2002). In that work a finite element model of the crystalline lens was constructed based on experimental data from a variety of sources. We find the study by Burd *et al.* a reasonable and unique attempt of interpretation of experimental data in developing the human lens model and some procedures established by the authors are used in present paper for modelling a 29-year-old eye.

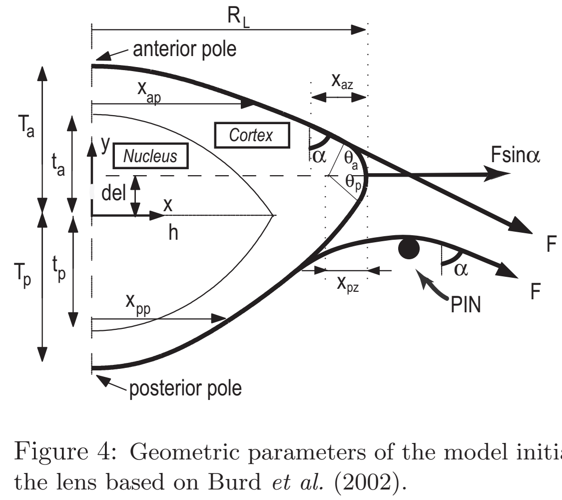
Preprocessing

We adopted non-linear axisymmetric analysis, assuming that the eye was a body of revolution. Axisymmetry perpendicular to the optic axis was applied. The commercial finite element package ABAQUS Standard, Hibbit *et al.* (2002), was used to carry out the numerical simulations. All materials were assumed to be isotropic and elastic, Fisher (1969).

Geometry and material properties

The initial geometry of the fully accommodated crystalline lens and its material properties were taken from the study by Burd *et al.* (2002). The interior of the lens consisted of nucleus and cortex and the variation in capsule thickness was included in the analyses. The capsule was assumed to be bonded with the cortex. The vitreous body was represented by a membrane full of vitreous humour, confined by the inside shells of an eye (choroid and retina), thus maintaining a certain level of intraocular pressure. Since a humour is a gel-like substance and almost entirely consists of water (99%), it was modelled as a practically incompressible material. If the lens is removed from the eye, the vitreous takes a spherical configuration, although naturally the crystalline lens is lying on the anterior hyaloid membrane, pressed into it by zonular fibres and forming a small pit. At the starting point of simulation there was no contact between the lens and vitreous membrane in our model, so the vitreous can be approximated as a sphere. The initial radius 10 mm of vitreous was manually scaled from MR images by Strenk *et al.* (1999). The Young modulus for vitreous humour was derived by inverse method during simulations, as further described below. For the purpose of our modelling it was assumed that the vitreous membrane has the same stiffness as the lens capsule and is totally tied to the humour.

The incompressibility of the interior of the lens and vitreous was modelled by using axisymmetric second-order hybrid elements with Poisson's ratio 0.4999. These elements are specially formulated to avoid problems associated with modelling of contact simulations together with almost incompressible materials. Since the capsule was much stiffer than the interior of the lens, we followed the assumption that it is sufficient to use two-noded axi-shell elements to model it. Using the same line of reasoning we modelled the vitreous membrane with two-noded shell elements. This approach helped to decrease the complexity of contact formulation and to significantly speed up the analyses. It should be noted that since we had, in preparations, ascertained a sufficiently fine mesh, no major differences were observed when capsules of lens and vitreous were modelled using either shell or membrane elements.



Dimention	Value	Source
R_L(mm)	4.3136	Strenk et al.(1999)
T_a(mm)	2.04	Brown(1973)
T_p(mm)	2.09	Brown(1973)
x_{ap}(mm)	2.8	Brown(1973)
x_{pp}(mm)	2.4	Brown(1973)
θ_a(degree)	63°	Fincham(1937)
θ_p(degree)	37°	Fincham(1937)
t_a(mm)	1.38	Brown(1973)
t_p(mm)	1.58	Brown(1973)
α (degree)	70°	Brown(1973)
h(mm)	2.85	Fisher(1971)
del(mm)	0.5119	Burd et al.(2002)
x_{az}(mm)	0.38	Farnsworth et al.(1979)
x_{pz} (mm)	0.22	Gorban et al.(1993)
F(N)	unknown	derived during our simulations

Figure 4: Geometric parameters of the model initial configuration. Configuration of the lens based on Burd *et al.* (2002).

It was observed that the zonule could be divided into three sets, according to their attachments to the lens capsule, (Canals, Costa-Vila, Potau, Merindano & Ruano 1996). In our model they were represented by two-noded axi-shell elements forming continuous sheets. A slope angle of the anterior fibres was measured by Fisher (1971) and we adopted the reported value of 70° for our analyses. Anterior (AF) and posterior (PF) fibres were attached to the crystalline lens capsule on the distances 0.38mm, (Farnsworth & Shyne 1979), and 0.22mm, (Gorban & Dgiliashvili 1993), from the equator respectively. We assumed that central fibres (CF) connected to the ciliary body. The reference value for ciliary body equator was 6.474 mm reported by Strenk *et al.* (1999).

It was mentioned above that posterior fibres *in vivo* are quite branchy, covering the anterior hyaloid membrane. The simplest way to emulate such a configuration is to create a rigid pin relatively close to the vitreous membrane with posterior thrown over it. That approximation would be helpful to avoid contact problems and to simplify the model. At the same time, the model would be close to reality. Such an approach was assumed in the present study, and the force was applied to the ends of posterior fibres with the same angle to the lens axis as to anterior zonule, see Figure 4.

The Young modulus of a zonule was set to 1.5Nmm^{-2} as proposed by Alphen & Graebel (1991). To specify the stiffness of the zonular fibres we needed to evaluate representative thicknesses. Noting that only the product of material stiffness is of importance, this was manually done through the inverse experiment described below. We presumed, that the contact between zonules and lens capsule was frictionless.

It was recently reported by Svetlova *et al.* (2003), that the role of anterior and posterior fibres appears to be the most functionally essential, while the role of the equatorial zonule is auxiliary. We considered two sets of analyses including and excluding the central annular sheet, to test the mentioned hypothesis and examine the effect of active central fibres on the numerical model.

Procedures

The reference configuration of the model is shown in Figure 5(a). The vitreous and lens were represented by 6338 continuum, 208 shell elements and 1007 continuum, 200 shell elements, respectively. No convergence test was performed for the model verification, since Burd *et al.* (2002) and preliminary tests verified that in similar axisymmetric non-linear analyses even coarser lens meshes were converged with respect to the mesh density.

At first we needed to recreate a fossa patellaris, which was done by applying displacement boundary conditions to the bottom of vitreous with radius 7.8 mm. During this procedure the anterior hyaloid membrane was pressed into the rigid surface, which had the same shape as the posterior surface of the lens. The lens was in this shape modelled as a rigid body, in order to lighten the contact simulation, Figure 5(b). The depth of the pit was prescribed to

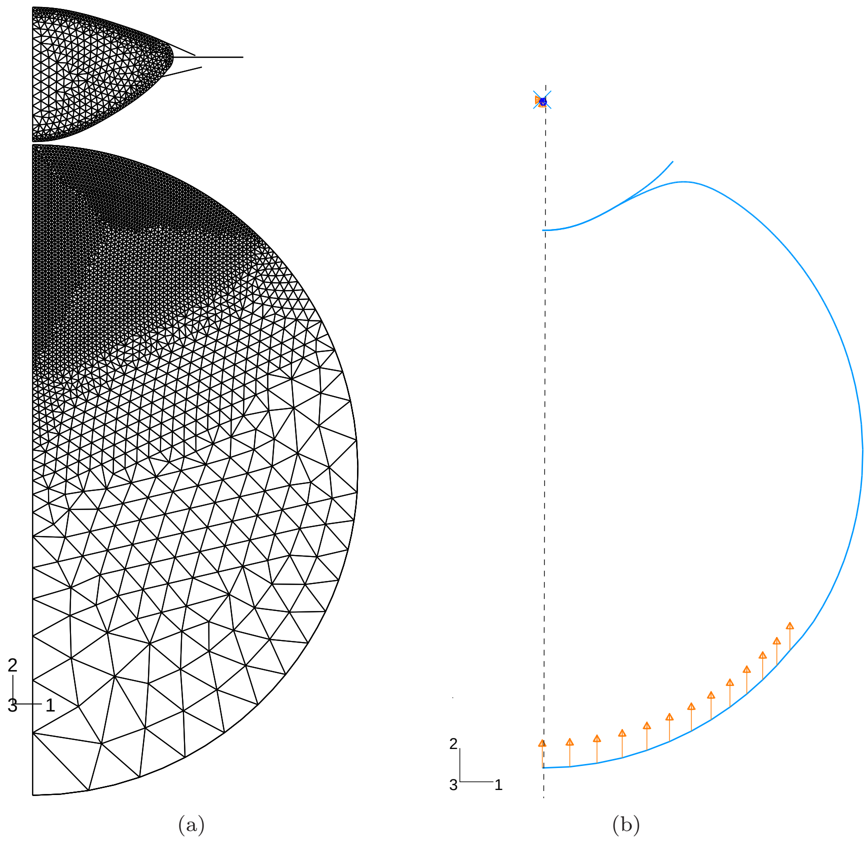


Figure 5: Procedures: a) reference configuration; b) first step.

become 3 mm. Later, the rigid body was replaced with the crystalline lens and contact between the lens and vitreous was carefully established with a friction coefficient equal to $\nu = 0.01$ as for lubricated surfaces.

It is known that Wieger's ligament is an additional annular sheet, created by adhesion of the posterior surface of the lens and the vitreous membrane. The posterior zonule are plaited into it. We performed extra analyses taking into consideration this anatomical element. The ligament has a certain stiffness and as a first step we assumed that it is identical to the stiffness of the posterior fibres.

Experiment

In order to construct a general model it is necessary to have relevant experimental measurements. Unfortunately, reliable published data on some parameters required by the model seem not to be available. An alternative approach, a so-called inverse methodology, was used to derive or estimate unknown data, such as force, zonular stiffness and Young's modulus of the vitreous. Numerical simulations were performed and the values of unknown parameters were deduced by ensuring that resulting configurations matched measured data in chosen respects. We have chosen the following observations as a target for the inverse experiments:

- during disaccommodation the backward movement of the anterior lens pole should be approximately three times larger than the forward movement of the posterior lens pole as reported by Drexler *et al.* (1997).
- when the lens equator moved 0.3 mm, the corresponding change in ciliary body radius is 0.4 mm, as measured by Strenk *et al.* (1999).

It should be mentioned that although it is confirmed that during the process of accommodation the anterior pole displacement is larger than the posterior, it is not exactly clear to what extent, since various studies report different values for these movements. The influence of this particular feature on the model behaviour was tested in our simulations.

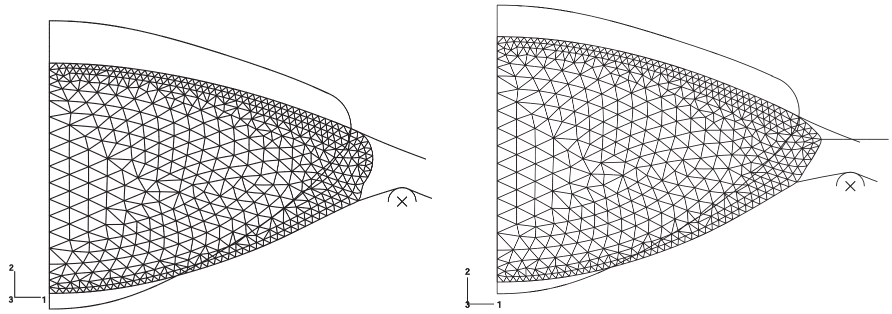
Postprocessing

The finite element calculations were conducted and previously undefined parameters were derived. The ciliary muscle force was applied by increments to the ends of zonular fibres. For every increment the calculated deformed positions of all points on the anterior surface within an initial zone of 4 mm, and a similar 2 mm posterior zone of the lens surface were fitted to second order polynomials of the form $z = a + bx + cx^2$, with z is the axial deformed coordinate and x is the deformed radius. During *in vivo* experiments only a smaller part of posterior surface than of the anterior lens surface can be seen, as reported by Dubbelman, van der Heijde & Weeber (2005). On the basis of this observation different diameters of anterior and posterior optical zones were chosen for consideration.

Since the curves are all symmetric around the lens pole, the coefficient for x is always very closed to zero, thus the radii of curvature were calculated using formula $r = (1 + (2cx)^2)^{3/2}/2c$, (Koretz, Handelman & Brown 1984). This equation was solved for the anterior and posterior lens surfaces using $x = 1$ mm. Refractive indices of $n_{av} = 1.336$ for aqueous and vitreous humour and $n_l = 1.42$ for the crystalline lens were assumed, (Bennett & Rabbetts 1989). Substituting these values and the calculated polar deformed lens thickness, t, to the thick lens formula the optical power of the lens was determined as proposed by Burd *et al.* (2002).

Results

The final configuration of the lens with and without central fibres is shown in Figure 6. As expected the force required for a needed equatorial displacement was less in analysis with all three sets of fibres, 0.06 N, than without central zonular sheet, 0.1 N. Although during both simulations the central anterior and posterior surfaces of the lens flatten and the axial thickness decreases, these changes were greater in the model without the central set of fibres. Therefore the lens accommodation amplitude was bigger in that case. It should be noted that force, predicted by present analyses have to be almost an order of magnitude greater that the physiological force capacity of the ciliary muscle, Fisher (1977); this result is consistent with other numerical studies, such as Burd *et al.* (2002); Schachar & Bax (2001).



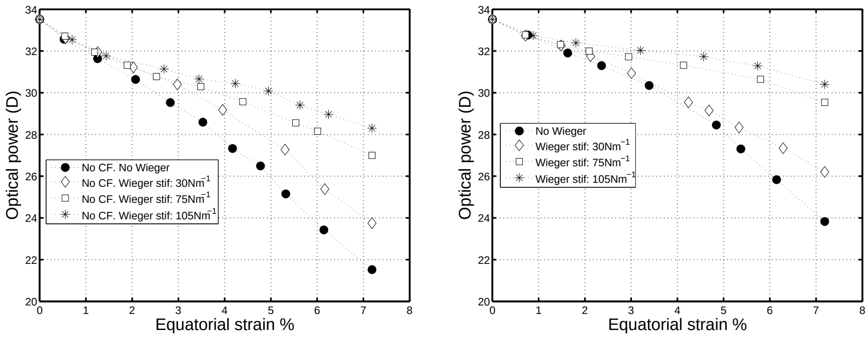
(a) (b)

Figure 6: Deformed lens meshes together with reference surfaces. No Wieger's ligamentum. a) Without CF. b) All three sets of fibres.

As described above, the unknown parameters were manually adjusted to achieve needed deformed geometry. For the model without CF, we report $30\,\mathrm{Nm}^{-1}$ and $22.5\,\mathrm{Nm}^{-1}$ for anterior and posterior zonular stiffnesses. When we added CF, derived zonular thicknesses were in the ratio 3 : 1 : 2 for anterior, central and posterior sets, with the values for stiffnesses of 45, 15 and $30\,\mathrm{Nm}^{-1}$, respectively.

Three additional analyses were conducted for both cases, where Wieger's ligament was modelled between the posterior surface of the lens and the vitreous. First, we assumed that it had the same stiffness as posterior fibres, later the value was increased to $75\,\mathrm{Nm}^{-1}$ and $105\,\mathrm{Nm}^{-1}$. The computed variations of optical power are shown in Figure 7.

During disaccommodation the decrease in central thickness and changes in radii of curvature became smaller with increasing stiffness of Wieger's sheet. The changes in thickness were from 0.8 mm to 0.65 mm for the model without CF and from 0.62 mm to 0.47 mm for analysis with three sets of fibres. The decrease in lens size along the polar axis is caused almost entirely by a change in the thickness of the nucleus in this direction (approximately $75\% - 80\%$ for all



(a) (b)

Figure 7: Optical power of crystalline lens vs equatorial strain. Evaluated from model a) without CF, b) with all three sets of fibres.

cases), which is consistent with measurements by Brown (1973); Dubbelman, van der Heijde, Weeber & Vrensen (2003).

Anterior and posterior central radii of curvature are presented in Table 1, as functions of the model thickness of Wieger's ligament, in μm. The curvature is one divided by the radius and it is proportional to the power of the surface. Figure 8 shows the changes in curvature for different models. It is obvious that the changes in posterior curvature become smaller with increasing thickness of Wieger's membrane.

	NO CF				WITH CF
	`No Wieger`	`Tk=20`	`Tk=50`	`Tk=70`	`No Wieger`
r_{ant}	9.7	8.9	8.7	8.7	8.2
Δ_{ant}	3.4	2.6	2.4	2.4	1.9
r_{post}	6.4	5.8	4.7	4.3	6.1
Δ_{post}	2.4	1.8	0.7	0.3	2.1

Table 1: Anterior, posterior radii of curvature and their alterations at the end of simulations, mm. Changes of the central radii during disaccommodation, mm. `Tk` – thickness of additional sheet, representing the Wieger's ligament, μm.

The computed results for the model without central fibres indicate the reduction in anterior and posterior curvatures on $0.06\,\mathrm{mm}^{-1}$ and $0.09\,\mathrm{mm}^{-1}$, when there was no additional sheet between the back of the lens and vitreous; $0.05\,\mathrm{mm}^{-1}$ and $0.07\,\mathrm{mm}^{-1}$, when the thickness of Wieger's membrane was equal to 20μm; $0.05\,\mathrm{mm}^{-1}$ and $0.04\,\mathrm{mm}^{-1}$ and $0.05\,\mathrm{mm}^{-1}$ and $0.02\,\mathrm{mm}^{-1}$, when it was 50μm and 70μm, respectively.

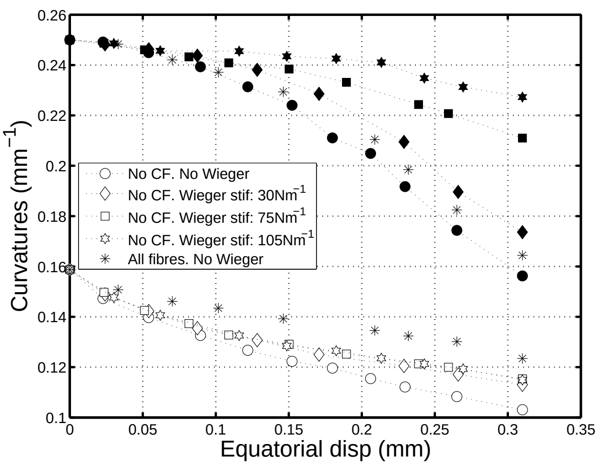


Figure 8: Changes in curvature for anterior and posterior surfaces. Curvatures of the posterior surface are shown with shaded areas. They are bigger than anterior, since posterior profile is more convex.

The next computation was performed to investigate the sensitivity of the model to the distribution of zonular stiffnesses. In the analysis with three active groups of fibres the derived stiffnesses were in ratio 3 : 1 : 2 for the anterior, equatorial and posterior zonular sets, respectively. Present calculations assumed identical stiffnesses for three portions of fibres. The magnitudes of the stiffnesses were selected using the same approach as before: through the inverse experiment.

A rigid pin was used to throw the posterior fibres over, thus enabling us to lighten a contact simulation. The pin was modelled as a part of the vitreous, relatively close to its anterior hyaloid membrane. We conducted two further numerical runs to study the influence of pin position on the behaviour of the model. The resulting variation of optical power with outward displacement of the lens equator evaluated from models with three sets of fibres with equal stiffnesses and stiffness in ratio 3 : 1 : 2 is shown in Figure 9. It is clear that the differences in pin position has insignificant effect on the amplitude of accommodation.

It was observed that movement of the crystalline lens in the axial direction without altering radii of curvature in the optical zone occured as a result of variations of the elastic modulus of the vitreous humour and the position of pin. As a result, in our modelling exercise with all adopted assumptions and procedures, we always can find a combination of these parameters that during the simulation the movement forward of the posterior lens pole was in a specified ratio to the backward movement of the anterior lens pole (to satisfy the requirements of the inverse experiment).

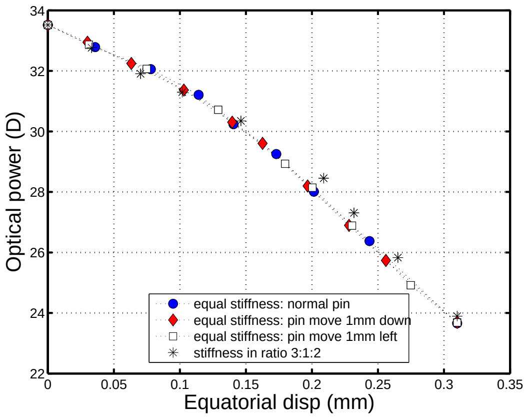


Figure 9: Relationship between optical power and equatorial displacement.

The deformed lens volume was calculated to verify incompressibility. The result was that the volume was conserved within 0.1% for all studied cases.

Discussion

Clinical studies confirm that during accommodation the changes in position and curvature are larger in the anterior surface than in the posterior surface. It should be noted that *in vitro* the lens is in a different mechanical state of stress than *in vivo*. When removing the eye, there is a loss of additional forces exerted by both aqueous and vitreous together with the loss of intraocular pressure, normally present in the eye. These observations as well as nascent uncertainties through the tissue preparation and assumptions about mechanical stretching experiment bring us to the conclusion that *in vitro* observations should not be used as the basis for any final conclusions. Thus, in present work all the results are discussed in comparison with different *in vivo* studies.

Zonular fibres attach and interlace with the lens capsule quite close to the equator region, which, in turn, has a significant shift in the anterior direction. The anterior lens capsule is approximately three to five times thicker than the posterior lens capsule and its stiffness more than hundred times greater than a lens interior, Krag & Andreassen (2003). Combining the mentioned facts and applying zonular force in the equatorial direction, major changes in the anterior part of the lens could be predicted in the *in vitro* experimental studies, (e.g. Glasser & Campbell 1998), as well as explained for the different theoretical models, (e.g. Schachar & Bax 2001; Burd *et al.* 2002). It is not obvious and consequently much harder to explain why changes predominantly occur in the anterior part of the lens, when the lens is located in the eye. As a step towards this goal and better understanding of the process of accommodation we have undertaken the present theoretical study.

Movements of the central poles were specially adopted to follow observations by Drexler *et al.* (1997), i.e. to be in ratio 3 : 1 for the anterior and posterior poles, respectively. Nevertheless, the values of total changes in thickness in our models were about two, one and a half times those reported by Drexler *et al.* (1997); Dubbelman *et al.* (2003); but favourably consistent with results shown by Brown (1973).

In our analyses the internal profile of the lens was constructed using geometry data from study by Brown (1973), since it is the only study which in detail describes paraxial and peripheral radii of curvature for the lenses under different accommodative conditions. At the present moment it has been proved, that these results (particularly about posterior surface) should be treated with caution, because Brown did not correct the optical distortion caused by refraction by the cornea and lens itself, (Dubbelman *et al.* 2003, 2005). So, some mistakes could occur in our results; however, they agree qualitatively with reported measurements.

We performed numerical analyses of accommodation in two cases: with and without central zonular fibres. The model with all three zonular sets and chosen stiffness relations behaved worse than the model with only anterior and posterior fibres. Changes in the posterior radius of curvature were larger than those in the anterior, which contradict experimental results, making the posterior surface the main contributor to the accommodative lens power change. Thus, we exclude further detailed analyses of that model, accepting the fact that with our adopted assumptions and established procedures, the model without central zonular fibres gives a behaviour closer to reality.

Recent studies report that about one third of the change in the lens power with accommodation is caused by the posterior lens surface, (e.g. Garner & Yap 1997; Dubbelman *et al.* 2005). It is within our results, although, Wieger's ligament played an important role: without this sheet, the posterior surface contributed 63% to the accommodative power changes, with increasing stiffness the values become 60%, 44% and 30%.

The vitreous can be approximated as a membrane full of fluid. It is known, that the response of such a structure depends not only on the external loads but also on the pressure exerted by the fluid, which, in turn, is affected by the deformation of the structure. In our modelling exercise the derived value for the modulus of elasticity is, as can be expected, larger than the value reported in experimental studies, since it shall include the pressure, produced by the contained fluid. This observation is confirmed by measurements performed by Lee *et al.* (1992), where the internal modulus of elasticity is two orders of magnitude lower than derived from our inverse methodology.

Figures 7 show the relationship between equatorial strain and optical power of the lens for different assumed thicknesses of Wieger's ligament. It is clear from the picture, that optical power is reduced with increasing outward tension;

this is consistent with the classical Helmholtz hypothesis of accommodation. *In vivo* measurements by Storey & Rabie (1985) predict a reduction of 1.2D per percent strain, which is within the range of our results.

In the numerical model with three sets of fibres the stiffness of the zonule was in the ratio 3 : 1 : 2 between the anterior, central and posterior fibres, respectively. In the next analysis the stiffnesses of all sets were set equal. The effect of different zonule configurations on the performance of the lens is shown in Figure 9. It has some small effect on the computed response, however not significant in that particular case.

Conclusions

The present study represents an attempt to investigate the process of accommodation through numerical simulations. In contrast to previous numerical analyses that by default assumed that the zonular sets attach to the ciliary processes at a single point and stretching forces lie in equatorial plane, this study incorporates a posteriorly sloped zonule. This, and additional forces supplied by vitreous support is a necessity for the equilibrium conditions. A basic and concrete empirical knowledge of geometrical and mechanical properties of the eye and its internal structure is necessary for the theoretical modelling, since the true value of theoretical work is realized only when theoretical results and the empirical measurements can be shown to agree. Although further work is needed to develop more complex models, the present study showed that addition of all anatomical elements (such as vitreous, Wieger's ligament) captures some aspects of accommodation. The results of this study are broadly in agreement with different published data, and give indications on the values of some new parameters, which are needed in numerical simulation but are not given reliable an consistent results in literature.

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