Mechanical models for electrical cables

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Abstract

A theoretical and experimental study of mechanical properties of electrical cables with multi order helical structure has been performed. Relations between applied deformations and local strains in the first order helical structure have been developed. The model is then generalized with a hierarchical approach where the strains at any order helical structure are expressed as functions of strains in the upper order helix under the assumption that all components are sticking to each other.

The force balance between the strains and the friction forces is considered. When the cable is exposed to small bending curvature, the slippage of the component is prevented by the frictional force. At this stage, the components of the cable behave as solid beams. Slippage occurs between the components when the tensile force in the components overcomes the frictional force. This state occurs at sufficiently large bending curvatures and results in a variable bending stiffness varying with the magnitude of the applied bending curvature.

The response of the cable to pure bending is measured and the data is evaluated using the theoretical model described above. Magnitudes of unknown properties of the cable are estimated by comparing the theoretical and experimental data. To utilize the model in terms of life time estimation, a number of parameters were suggested to relate the mechanical properties of the cable to wear and fatigue. A parametric study has been done to investigate how these parameters are affected by changing cable properties or the loading condition.

Descriptors: Electrical cable, Industrial robot, Multi order helical structure, mechanical model, fatigue

Contents

Chapter 1

Introduction

Although an electrical cable is used to carry electric power or signal from one point to another and is normally dimensioned with respect to its electrical properties, in certain applications, such as in industrial robots, Fig. 1.1, it is subject to external mechanical stresses and forced motion. To supply the electrical motors located close to the joints of an industrial robots, the cable is mounted on the arms through a number of mounting points at which the movement is restricted partly or completely. The operation of the robots, shown schematically in Fig. 1.2, involves large movements. Thus, the cables are exposed to simultaneous bending and torsion at large magnitudes. The deformation of the cable implies that the stress and strain are introduced in the internal components and this leads to fatigue and wear related problems limiting the usage period. Current methodology for design of cables is reliant on the experience from previous designs and on time-consuming cable fatigue testing. The capability of available theoretical models estimating the life time of the cable is poor and needs improvements before models can be used for accurate engineering work.

This thesis concerns modeling of mechanical properties of cables, in particular those influencing the life-time and aims at building up theories and providing models that can be used for a theoretical evaluation of the properties of cables. The final goal of the work is to estimate the life time of the cable theoretically and present a complete description of behavior of the cable as a response to imposed deformations and stresses.

There is a large variety of cables with different electrical and mechanical properties, see e.g. Fig. 1.3. A typical structure of cables used for industrial robots is shown in Fig. 1.4(a). In general, the design consists of conductors (1), tape (2), fillers (3), shield (4) and a jacket (5). The jacket serves several purposes. It protects the vital parts of the cable from external loads and environmental effects, preserves the cross sectional layout of the cable and thereby keeping the internal organization of the components unchanged and

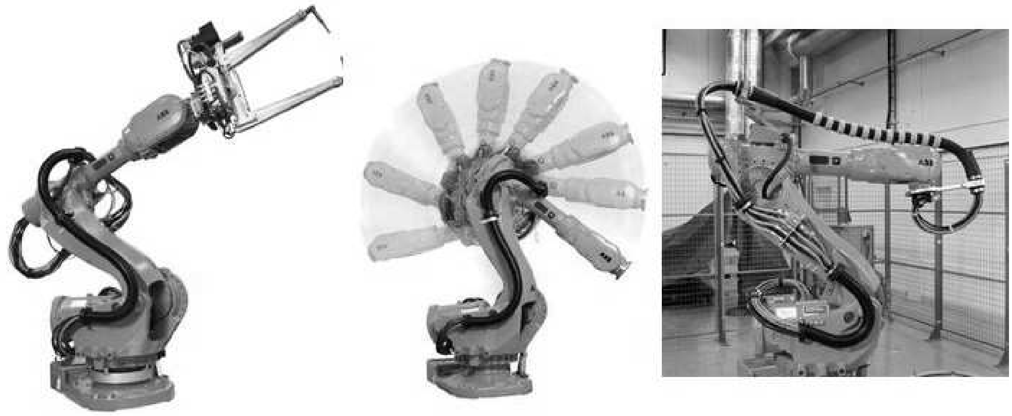


Figure 1.1: Industrial robots.

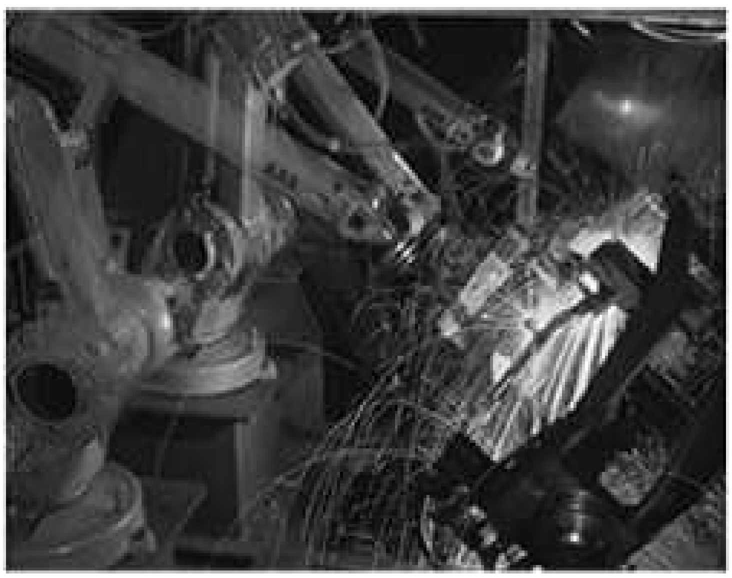


Figure 1.2: Industrial robot in work.

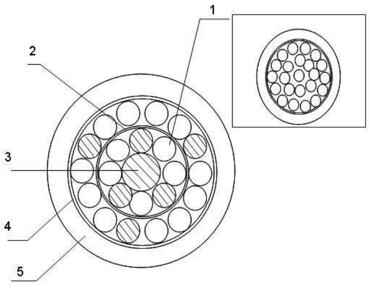
increases the bending radius of the cable and thus reduces internal stresses and strains induced by bending.

A shield is put between the jacket and the tape to protect the surroundings from the electromagnetic field generated by conductors and protect the conductors from electromagnetic disturbance at the same time. The tape is used to hold the components together during the assembly of the cable and it also plays a role in reducing the friction between conductors and a shield. Fillers are introduced in order to make the cable geometrically symmetric and possibly more compressible.

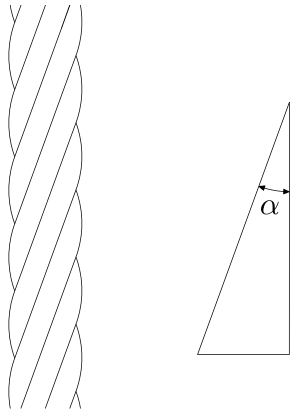
Conductors are the essential pars of a cable and could have a complex internal structure. These carriers of electric power or signals are wrapped helically around the core of the cable. Conductors consist of copper wires bundled to form a helical structure and are wrapped together by an insulation material. That forms a helix-in-helix structure being the characteristic structure of the electrical cable; here, it is called second level helical structure. For the sake of simplicity, the component with the first order helical structure is called conductor and the second order helical structure is called wire in what follows. The configuration of the centerline of a helix is determined by the distance from the center of helix and the lay angle which is illustrated in Fig. 1.4(b).

Figure 1.3: Various cables.

Mechanical properties of components with internal helical structure have been investigated in several scientific papers, most of which oriented towards steel wire ropes designed to carry axial loads. Very little published work



(a) General cross section of a cable.



(b) Illustration of a lay angle.

Figure 1.4: Geometry of a cable.

dealing with mechanical aspects of electrical cables was found. The most important mechanical differences between steel wire ropes and electrical cables are the following ones:

1. Electrical cables are constructed from several materials, both metallic and non-metallic.

2. Electrical cables can be concentric or non-concentric. However, geometrically concentric cables are generally not concentric in terms of stiffness, density and friction as filler threads are mixed with conductors. Steel wire ropes are always concentric with respect to all mechanical properties.

Despite the differences mentioned above, the structural similarities suggest that wire rope theory could be used for electrical cables to some extent, possibly with some modification. The main basic theory for analyzing helical structures was made by Costello [1]. The wire rope is regarded as a collection of many individual helical wires and the nonlinear equations of equilibrium for thin rods are applied. The equations of equilibrium have been linearized by Velinsky et al. [2] taking radial contraction into account. It was also generalized for multi strand wire rope.

The relation between applied deformation and resultant force has been derived assuming that no friction works within the cable. Two extreme cases of cables for bending was clearly stated by Utting and Jones [3]. Upper bound of the bending stiffness is defined with Bernoulli-Euler hypothesis, where the cross sectional planes of the cable remain plane after bending. It is assumed that the cable behaves as a solid beam, i.e. no slippage occurs between components, which yields maximum bending stiffness.

The other extreme situation is the case where no friction force works. Each component is free to slip with respect to its neighbors, which implies minimum bending stiffness. In practice, bending stiffness will be between these two extreme values.

Leclair [4] developed a theoretical model for single-layer helical strands that provides an upper bound of the relative motion between wires in bending by considering the geometry of the deformation. The equilibrium equations were solved taking the friction between the core and the wire into consideration.

Lanteigne [5] obtained the axial strains in a first level helical structure, consisting of several concentric layers of wires, through geometrical considerations. A general stiffness matrix was obtained through investigation of the strains, and a rudimentary treatment of internal friction and slippage in the structure was included. In that paper, the stick-slip transition was also proposed.

In a multi layer strand, all layers are initially in the no-slip state. As the imposed bending curvature increases, the outer layer reaches a point where it is in a full-slip state, with the wires bent independently with respect to their own bending axis. Papailiou [6] developed another model for bending which takes into account the slippage. In his model, a variable bending stiffness has been introduced which considers the inter layer friction and it also considers the additional pressure caused in the individual wires from outer layers.

Chapter 2

Proposed approach

In this thesis, the existing models are extended to take multi layer and multi order helical structure of the cable into account. Based on the model developed by Lanteigne, the relation between the applied deformation to the cable and the strains in the conductors in sticking state is formulated. Then the same formulae are applied to the conductor deformation and the strains in wires. Repeating this procedure hierarchically, strains in the helical structure of any order can be obtained as a function of the global deformations.

As the second step, the model developed by Papailiou is extended to consider the transition from sticking state to slipping state for a multi order helical structure. Maximum friction force, which is produced between neighboring layers, is specified from the force balance on the small segment of conductors. The force caused by bending is also calculated, which is obtained from hierarchical model mentioned above. The slippage of the conductor occurs when the frictional force is overcome by the force caused by bending.

Once the behavior of all conductors are specified, the same calculation is applied to wires. The result of the calculation on the conductor works as an imposed condition on wires, i.e. elongating and twisting strains of the conductor, to which the wire belongs, affect the pressure between the wires and the curvature applied to the wire can be calculated from the applied curvature on the conductor. The comparison of the frictional force and the force caused by the wire deformation shows if the slippage takes place and the tensile force on the wire cross section is calculated.

This model is applicable for any order helical structure although only the second helical structure is studied in detail. When the strains in the highest order helical structure caused by cable deformation are calculated, the bending stiffness of the cable is calculated as an entire response of the cable. The bending stiffness can be a good object to compare theoretical and experimental results.

The response of the cable to pure bending is measured in simple test as shown in Fig. 2.1 and the data is evaluated using the theoretical model described above. The pressure caused by the jacket and the insulation is not measured explicitly, but instead, theoretical model is used to estimate its magnitude. It is worth mentioning that the internal pressure in the cable is parameter depending on both the configuration of the cable and the manufacturing process.

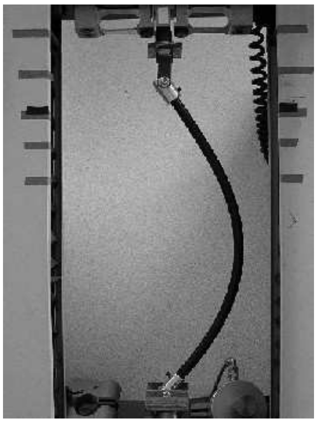


Figure 2.1: Test rig for bending.

Based on the detailed knowledge of the mechanical properties of the cable, a number of parameters can be defined to relate these properties to the lifetime of the cable. The magnitude of the maximum stress in wires and where it occurs are suggested as one of the key properties. It is strongly related to the fatigue which can cause cracks and also tells where the breakage might occur. Another property of interest is the number of the slipping wires because the slippage of the wire lead to wear damage inside the cable. The developed model is used to study how these key properties are affected by changing an external loading condition or the configuration of the cable geometry.

Chapter 3

Results and Discussion

The model developed in this study is applied to the actual cable and the result is shown in this section. Fig. 3.1 shows axial strains occurring in a wire in sticking state, where the cable is exposed to a combination of elongation, twisting and bending. The axial strains are presented for different combinations of lay angle, where α_1 and α_2 are the lay angles of conductor and wire, respectively.

From Fig. 3.1, it can be found that the strain consists of a low frequency and a high frequency component. For the case $\alpha_1 = 1$ degree and $\alpha_2 = 5$ degree, the two frequencies are clearly separated and could easily be distinguished, where as in the case $\alpha_1 = 5$ degree and $\alpha_2 = 1$ degree, the two frequencies are close which is reflected in Fig. 3.

It can also be noticed that the amplitude of the high frequency component is reduced by using a smaller lay angle for the first order helix. This is because this component is governed by curvatures on the first order helix. Using a larger lay angle for the second order helix gives a higher frequency and it also affects the amplitude, which is prominent when the lay angle of the first order helix is relatively large.

It can be expected from the figure that the maximum stress, which occurs in the wire, has a positive correlation with the lay angles of both first and second order helix while the influence of the second order helix is relatively smaller than that of the first order helix.

Fig. 3.2 shows variation of bending stiffness of the electrical cables used in industrial robots as a function of curvature. These results are calculated using assumed values for the pressure as the pressure from both jacket and insulation material are unknown. It is worth noting that the pressure is an important parameter depending on the manufacturing process.

Both cables are showing constant, high stiffness within a small range of curvature. That can be related to the fact that all of the conductors are in a sticking state. When the curvature reaches a limiting value, a number of the

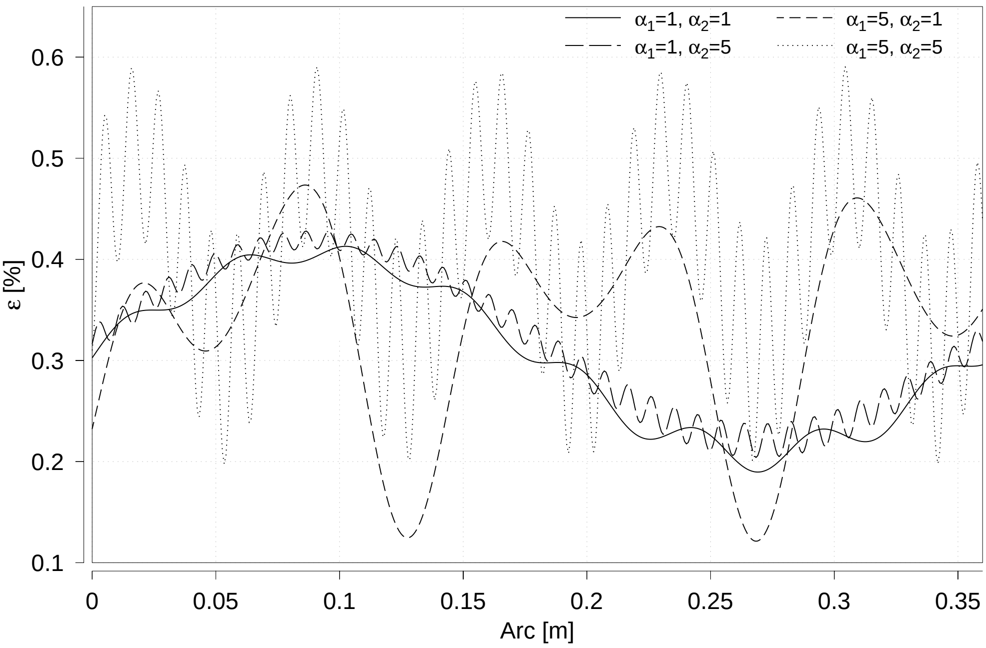


Figure 3.1: Wire strain profile for different lay angles for first and second level helix.

conductors enters the slipping state and consequently the value of the bending stiffness decreases. The theoretical values are compared with experimental data to validate the model. The assumed values for the pressure are adjusted so that the theoretical cable response meet the experimental result as shown in Fig. 3.3.

Using detailed models presented in this work, the order of magnitude for a number of parameters describing mechanical properties of cable with strong influence on the life-time of the cable can be estimated.

In Fig. 3.4, the maximum stress occurring in the wire cross section is shown. One important feature is that the maximum stress does not necessarily occur in the outermost wires in the outermost conductors. The reason is that the inner wire is radially compressed not only by the pressure from insulation material but also by the outer conductors. Consequently, the effect of the friction can be larger in an inner layer which results in a larger stress.

The result presented in Fig. 3.4 consists of five distinguishable parts. From Fig. 3.5, it can be found that the number of slipping wires is first constant and equal to zero. After that the curvature has reached a threshold the wires start to slip and the number of slipping wires increases until nearly all of the wires in the outer layer of each conductor come into the slippage

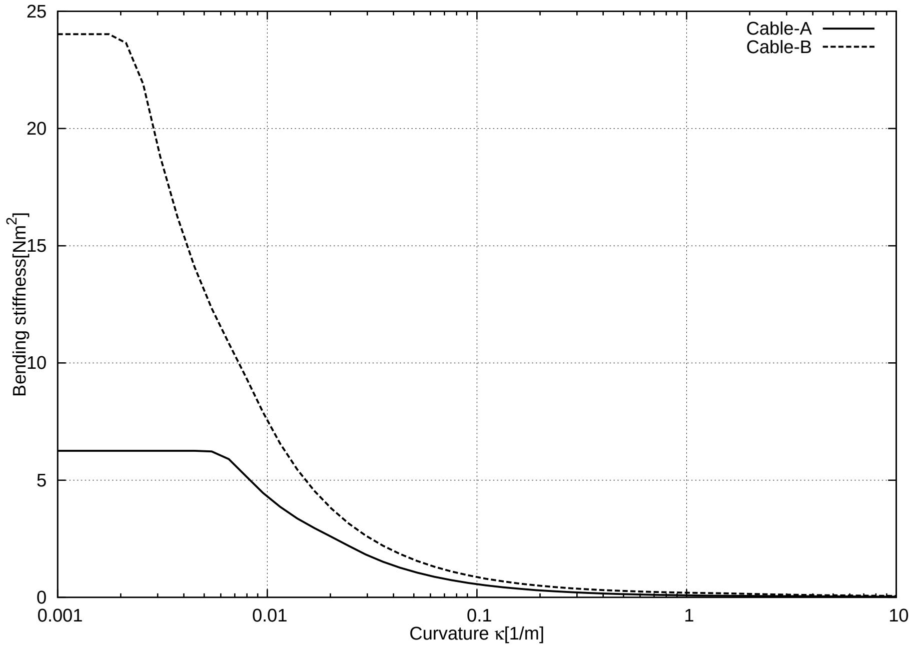


Figure 3.2: Theoretical response of the cables.

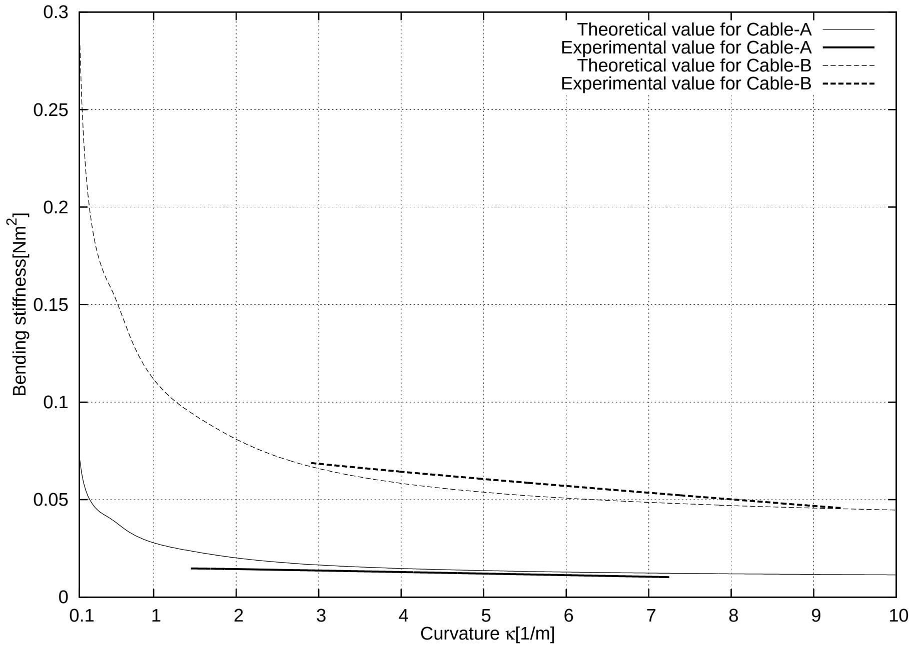


Figure 3.3: Bending stiffness versus curvature.

state. The wires at the second layer of each conductor start to slip when the bending curvature of the cable has become large enough. After that all wires at the second layer have reached the slippage state, the number of slipping wires becomes constant at least up to a curvature of $10m^{-1}$.

Due to the internal pressure created by the outer layers of conductors, the critical value of curvature for slippage is larger in the inner layers. The conditions for slipping of layers 3, 4 and 5 are never satisfied within the realistic range of the curvature. For this particular case, the outermost wires in the conductors shifts to slippage mode within the range $0.2 < \kappa < 0.6$, and the wires in the penultimate layers start to slip within the range $1.3 < \kappa < 2$.

The average value of tensile stress in the wires is shown in Fig. 3.6. It is clear that the slope decreases as the curvature increases due to the effect of wire slippage.

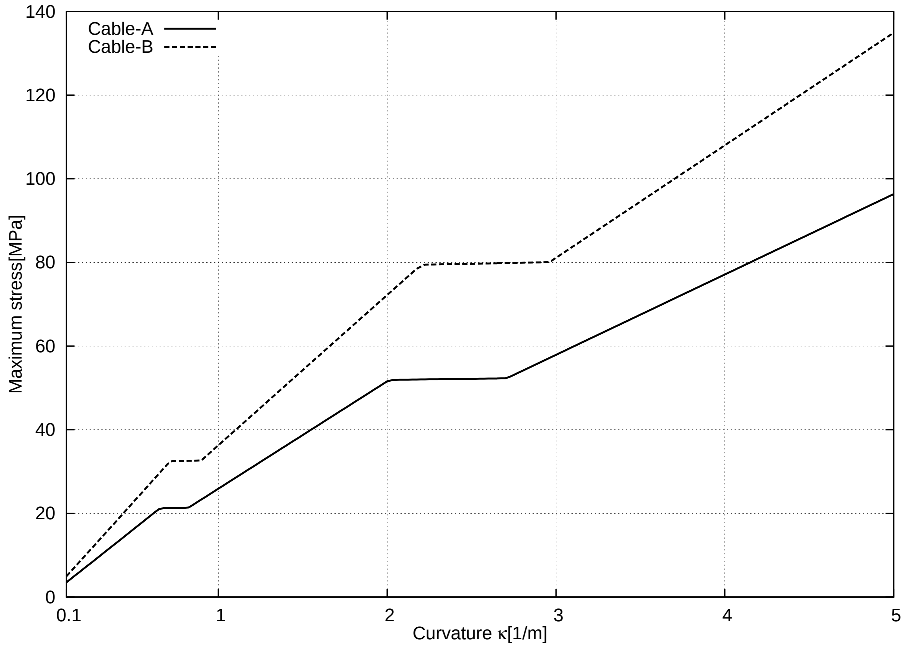


Figure 3.4: The maximum stress in different cables.

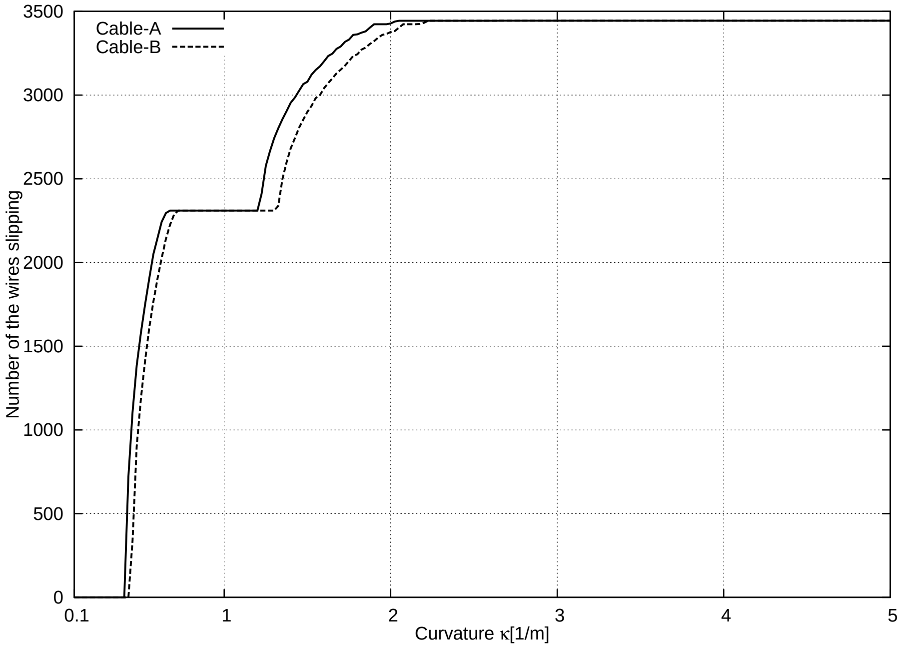


Figure 3.5: The number of the slipping wires for different cables.

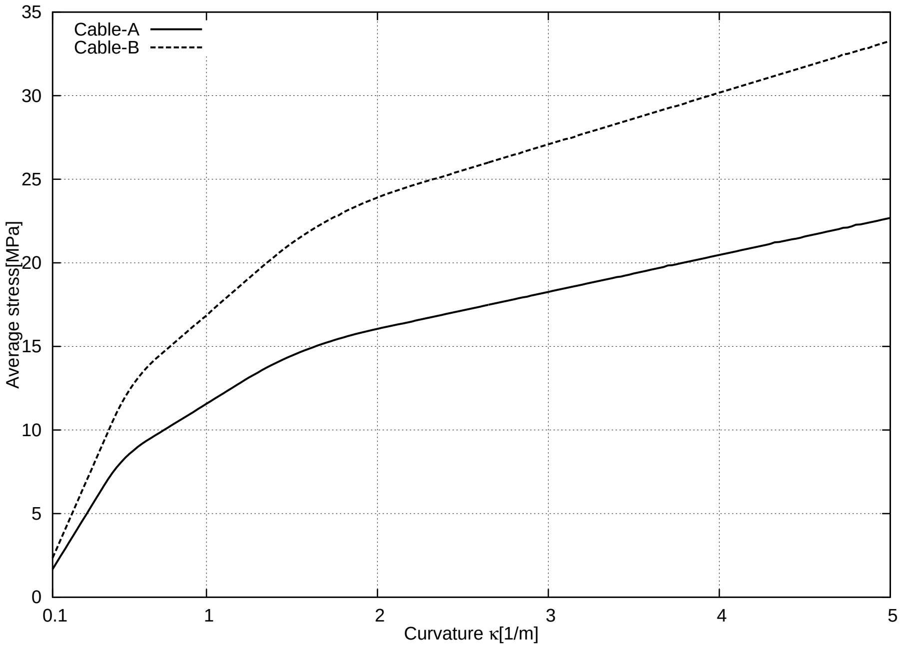


Figure 3.6: The average stress in different cables.

Chapter 4

Concluding remarks

A new model for analyzing complex structure of cables were developed. Strains in helical structure of any order are expressed with a hierarchical approach, where the strains in the helix of arbitrary order is calculated as functions of the strains in the helix of next upper order. Together with the geometrical approach, the effect of the friction between components were taken into account. This model enables to introduce a variable bending stiffness for the cable taking sticking and slipping of wires into account and also calculate some key properties which are strongly related to the lifetime of the cable. This model can be used to find a good configuration of the cable geometry depending on the possible loading condition and the requirement of the cable.

A new methodology for evaluating cables was suggested which involves pure bending or pure twisting tests. It was found in the bending test that radial compressibility of the cable significantly affects the behavior. Absence of a compressible core filler implied that internal stresses were increased with bending even more than the case with compressible core filler. This methodology enables to investigate the quality of the cable with simple tests.

Chapter 5

Summary of papers

5.1 Paper A

Method to relate the global deformations to the local strains have been developed for multi level helical structures of any order. The applied deformation is expressed as combination of tension, torsion and bending. In the proposed method, the relation between applied deformation and induced strain in the first level helix is formulated and it is applied to calculate strains in the conductor. Once the deformed state of each conductor is found, the strains working in conductors are interpreted to the strains in wires and they define the conditions and the same process is repeated to calculate the response of wires. In a step-by-step manner, the method is generalized so that the mechanical response of helical structure of any order is computed.

The model is used in a parametric study of a second order helical structure subjected to small amounts of tension combined with severe torsion and bending. It has been shown that increasing lay angles generated larger strains in wires. Increasing the radius of the second level helix also implied larger strains whereas the change of the radius of the first order helix did not affect the strains.

5.2 Paper B

Simple tests have been done for three specimens of cables. A method to evaluate the internal mechanical response is developed and it is applied to the experimental results. Theoretical maximum and minimum stiffness of the cable is calculated based on the hierarchical model developed in paper-A. The theoretical values are related to the measured value to calculate the maximum stress in the cable. It is shown that the incompressible cable without core filler suffers from larger internal stress compared to the cable which has compressible core filler.

5.3 Paper C

The model developed in paper-A is extended to take frictional effect into account, which is of particular interest in bending. In order to calculate quantitatively the force and stress situation, the force balance working in the small segment of conductor is considered, where the pressure from the jacket is included. This model is first applied to the outermost conductor and the inner layer conductor is considered next.

Once the calculation has been done for all layers, the same approach is repeated for wires in the similar way as in paper-A. Then the response of the cable is obtained as a sum of responses from each wire. The pressure from the jacket and insulation material was not explicitly measured but was estimated by comparing theoretical bending stiffness and experimental values.

Maximum stress, average stress and the number of slipping wires in the cable, which are calculated in this model, are suggested as key parameters in terms of life time expectation. The parametric study is done to investigate how the cable geometry and loading condition affects these properties.

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