LETTERS

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A note on the overlap region in turbulent boundary layers

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Two independent experimental investigations of the behavior of turbulent boundary layers with increasing Reynolds number were recently completed. The experiments were performed in two facilities, the Minimum Turbulence Level (MTL) wind tunnel at Royal Institute of Technology (KTH) and the National Diagnostic Facility (NDF) wind tunnel at Illinois Institute of Technology (IIT). Both experiments utilized oil-film interferometry to obtain an independent measure of the wall-shear stress. A collaborative study by the principals of the two experiments, aimed at understanding the characteristics of the overlap region between the inner and outer parts of the boundary layer, has just been completed. The results are summarized here, utilizing the profiles of the mean velocity, for Reynolds numbers based on the momentum thickness ranging from 2500 to 27 000. Contrary to the conclusions of some earlier publications, careful analysis of the data reveals no significant Reynolds number dependence for the parameters describing the overlap region using the classical logarithmic relation. However, the data analysis demonstrates that the viscous influence extends within the buffer region to $y^+ \approx 200$, compared to the previously assumed limit of $y^+ \approx 50$. Therefore, the lowest Re_θ value where a significant logarithmic overlap region exists is about 6000. This probably explains why a Reynolds number dependence had been found from the data analysis of many previous experiments. The parameters of the logarithmic overlap region are found to be constant and are estimated to be $\kappa=0.38$, $B=4.1$ and $B_1=3.6$ ($\delta=\delta_{95}$). [S1070-6631(00)01901-2]

In the classical theory, the overall description of a turbulent boundary layer is dependent on two separate inner and outer length scales. The outer length scale is commonly taken as the thickness of the boundary layer δ, and the inner length scale as the viscous length $l^*=\nu/u_\tau$, where $u_\tau=\sqrt{\tau_w/\rho}$ is the friction velocity, τ_w is the skin friction and ρ is the density of the air. Dimensional analysis of the dynamic equations with boundary conditions leads to a scaling of the mean velocity profile in the inner and the outer parts of the boundary layer in the form:

$$\overline{U}^+=\frac{\overline{U}}{u_\tau}=f(y^+); \quad \overline{y}^+=\frac{yu_\tau}{\nu}, \tag{1}$$

$$\frac{U_\infty-\overline{U}}{u_\tau}=F(\eta); \quad \eta=\frac{y}{\delta}. \tag{2}$$

At sufficiently large Reynolds numbers, it is assumed that there is a region of overlap, $\nu/u_\tau \ll y \ll \delta$, where the law of the wall (1) and the defect law (2) simultaneously hold. Matching[1] the relations (1) and (2) gives one of the classical results in turbulence theory, i.e., the logarithmic overlap region: in inner variables,

$$\overline{U}^+=\frac{1}{\kappa}\ln(y^+)+B, \tag{3}$$

and in outer variables

$$\frac{U_\infty-\overline{U}}{u_\tau}=-\frac{1}{\kappa}\ln(\eta)+B_1. \tag{4}$$

By combining Eqs. (3) and (4) one obtains the logarithmic skin friction law

$$\frac{U_\infty}{u_\tau}=\frac{1}{\kappa}\ln\left(\frac{\delta u_\tau}{\nu}\right)+B+B_1. \tag{5}$$

Recently, due primarily to inconsistencies with trends of experimental data, several researchers have investigated alternatives to the classical theory.[2–4]

Based on extensive data from two independent experiments, this investigation targets three main issues related to the overlap region between the inner and outer parts of turbulent boundary layers under zero pressure gradient: the functional form of the overlap, the extent of the overlap and any Reynolds number dependence that may exist in the overlap parameters.

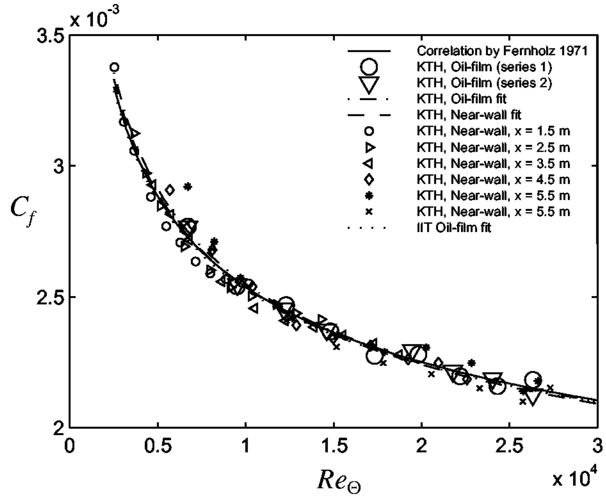

FIG. 1. Skin-friction coefficient using the oil-film and near-wall methods (Ref. 9) shown with best-fit logarithmic friction laws from Eq. (6) and Fernholz correlation (Ref. 10).

The experiments were carried out in the MTL wind tunnel[5] at the department of mechanics, KTH, and the NDF wind tunnel at IIT. At KTH, a 7 m long flat plate was mounted in the test section of the MTL wind tunnel. Measurements of the turbulent boundary layer were performed at five different streamwise stations, $x=1.5, 2.5, 3.5, 4.5$, and 5.5 m for ten different speeds. At IIT, a 9 m long and 0.457 m diameter cylinder was mounted in the test section of the NDF tunnel.[6,7] Measurements were taken at x = 1.84, 3.65, 7.33 m using five free-stream velocities. In both experiments, the measurement of the velocity profiles was done using hot-wire techniques, the skin friction was measured using oil-film interferometry, and the Reynolds numbers based on the momentum thickness ranged from 2500 to 27 000.

The skin friction was measured independently of the velocity measurements using oil-film interferometry in a setup similar to Fernholz *et al.*,[8] (see Fig. 1). The reproducibility of c_f obtained with this technique was $\pm 1\%$. A fit to c_f by a variant of the logarithmic skin friction law (5), namely,

$$c_f = 2\left[\frac{1}{\kappa}\ln(\mathrm{Re}_\theta)+C\right]^{-2}, \tag{6}$$

was made and the friction velocity used in scaling the data was calculated as $u_\tau = U_\infty (c_f/2)^{1/2}$. The value of the von Kármán constant determined in this way was, $\kappa=0.384$ and additive constant $C=4.08$. However, it is not possible to determine the additive constants B and B_1 by this method. In Fig. 1, the results from the oil-film measurements, together with the values of the skin friction determined from the mean velocity by the near-wall technique[9], are shown together with the calculated best fits using Eq. (6). The determined logarithmic skin friction laws agree very well with each other and also with the correlation developed by Fernholz.[10]

In order to investigate the scaling in the overlap, a normalized slope of the mean velocity profile,

$$\Xi = \left(y^+ \frac{d\overline{U}^+}{dy^+} \right)^{-1}, \tag{7}$$

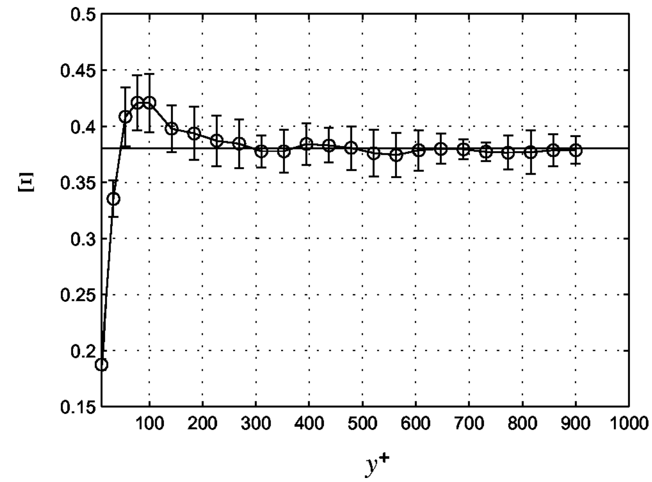

FIG. 2. Normalized slope of mean profile, Ξ, as function of y^+; only the part of the profiles in which $\eta<0.15$ was used and the horizontal line corresponds to $\kappa=0.38$.

was utilized. In a logarithmic region of the profiles Ξ is constant and equal to κ. The value of Ξ was calculated by taking an average of the individual profiles at a constant wall distance in inner scaling while omitting the part of the profiles where $\eta > M_o$. Similarly, the profiles were again averaged at constant outer-scaled distances from the wall for $y^+ > M_i$. The parameters M_i and M_o are the inner and outer limits of the overlap region. In Figs. 2 and 3, the Ξ values averaged over all Reynolds numbers for the KTH data are shown together with error bars representing a 95% confidence interval. A region where a nearly constant Ξ very accurately represents the data is evident in both figures. This clearly supports the existence of a logarithmic overlap region within the appropriate range of the parameters M_i and M_o. The choice of the appropriate limits was subsequently selected based on the y values where the error bar deviates significantly from the horizontal line in the figures. This was based on an iteration of the limits until a consistent result was obtained. The resulting values for the inner and outer limits are $M_i \approx 200$ and $M_o \approx 0.15$, respectively. Taking κ as the average value within the determined limits gives a κ of about 0.38.

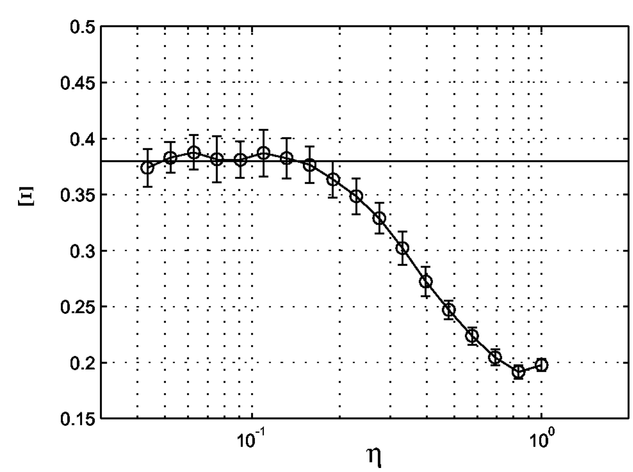

FIG. 3. Normalized slope of mean profile, Ξ, as function of η; only the part of the profiles in which $y^+ > 200$ was used and the horizontal line corresponds to $\kappa=0.38$.

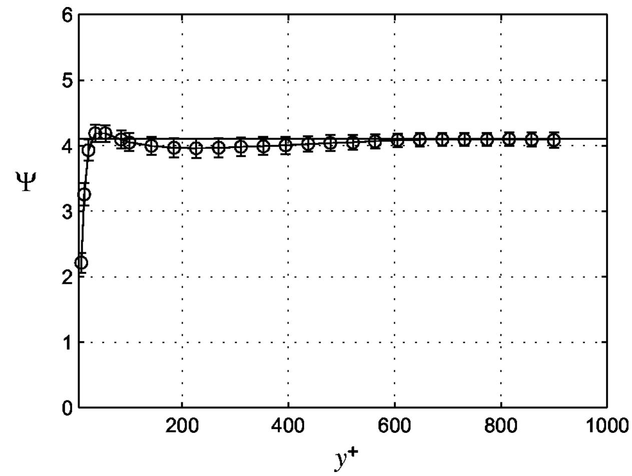

FIG. 4. Deviation from the logarithmic function using $\kappa = 0.38$; the horizontal line corresponds to $B = 4.1$, and only the part of the profiles in which $\eta < 0.15$ was used.

Next, the additive constant B was investigated by looking at the deviation of the mean velocity from the log function with the aid of the variable Ψ, where

$$\Psi = \overline{U}^{+} - \frac{1}{\kappa}\ln y^{+}. \tag{8}$$

The variable Ψ is also constant in a region governed by a logarithmic law. The average of the value of Ψ at a constant wall distance is taken for all Reynolds numbers while omitting the part of the profile where $\eta > M_o$. In Fig. 4, Ψ averaged over all Reynolds numbers for the KTH data is shown with error bars corresponding to a 95% confidence interval. A constant value is found over a wide range in y^+, again indicating a log layer. Calculating the average of Ψ within the proposed limits, M_i and M_o, gives $B = 4.1$. A slight under-shoot can be seen around $y^+ = 200$; this was found to be caused by a slight Reynolds number variation of B in the lower part of the range. Using only Reynolds numbers above $8-10\times10^3$ eliminates the small under-shoot.

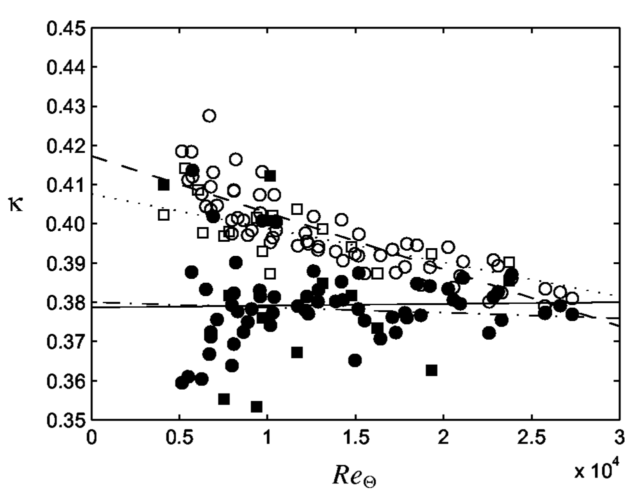

FIG. 5. The von Kármán constant determined by a least-squares fit, with the outer limit fixed at $\eta = 0.15$ and the inner limit at M_i; ○: KTH, $M_i = 50$. ●: KTH, $M_i = 200$. Dashed line: KTH, linear fit, $M_i = 50$. Solid line: KTH, linear fit, $M_i = 200$. □: IIT, $M_i = 50$. ■: IIT, $M_i = 200$. Dotted line: IIT, linear fit, $M_i = 50$. Dash dotted line: IIT, linear fit, $M_i = 200$.

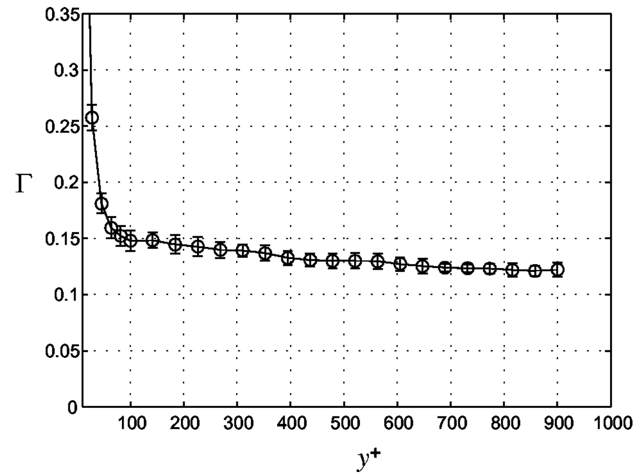

FIG. 6. The power-law diagnostic function Γ; only the part of the profiles in which $\eta < 0.15$ was used.

In addition to using the above described method to determine the log-law constants, we used the traditional procedure to determine κ and B by performing a least-squares type of fit to the mean velocity profiles. In Fig. 5, κ was calculated by fitting a log-law relation for each profile using the following traditional limits of the fit: $M_i = 50$ and $M_o = 0.15$. The process was also repeated with the newly established limits of $M_i = 200$ and $M_o = 0.15$. The value of κ obtained when using the traditional limits varies with Reynolds number and gives about the commonly used value of 0.41 at low Reynolds numbers. Using the new limits, which are more representative of the logarithmic law, again yields a value of $\kappa \approx 0.38$ independent of Reynolds number.

To investigate the existence of a power law as proposed recently by several authors,[2,3] the following diagnostic function averaged for the KTH data is shown in Fig. 6:

$$\Gamma = \frac{y^{+}}{\overline{U}^{+}} \frac{d\overline{U}^{+}}{dy^{+}}. \tag{9}$$

The function Γ should be a constant in a region governed by a power law. However, no region of constant Γ is depicted in Fig. 6, in particular when compared to Figs. 2 and 3. This clearly indicates that a power-law relation is less representative of the entire region of overlap between M_i and M_o.

Therefore, based on analysis of data from two recent experimental investigations it can be concluded that a logarithmic overlap region, between the inner and outer parts of the mean velocity profiles, exists for $\mathrm{Re}_\theta > 6{,}000$. Establishing, based on the analysis of the data, an inner limit of the region at about $y^+ = 200$ and an outer limit at $\eta = 0.15$ demonstrated the validity of the logarithmic relation with $\kappa = 0.38$, $B = 4.1$ and $B_1 = 3.6$ ($\delta = \delta_{95}$). The data will be made available in electronic form (http://www.mech.kth.se/~jens/zpg/).

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