COMMENTS

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Comment on ''A note on the intermediate region in turbulent boundary layers'' [Phys. Fluids 12, 2159 (2000)]

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The possibility of a power-law scaling in the overlapping part of the inner and outer regions of wall-bounded turbulent flows was first considered by Millikan[1] and later analyzed by George *et al.*,[2] George and Castilio,[3] and Barenblatt and co-workers (Refs. 4–8). However, there are some differences between the different approaches taken in these studies and this type of power-law should not be confused with the power-law used to represent the profile of the entire wall-bounded layer in engineering approximations. In Refs. 4–5 pipe flow was considered. In recent work[6–8] the authors have extended their approach to zero-pressure gradient turbulent boundary layers. Using results from measurements made on the test-section floor of the NDF facility at IIT by Hites,[9] they claim good agreement.[7] Recent analysis of their results by Ron Panton (private communication), reveals inconsistencies in the length scale extracted from the two relations defining their power law that are of the order of 25%–30%.

In the recent manuscript by Barenblatt *et al.*,[6] the authors use the data obtained from measurements in a zero pressure-gradient turbulent boundary layer by Österlund[10] and made available on the internet.[11] The authors claim that the conclusions obtained by Österlund *et al.*[12] are incorrect and that ''Properly processed these data lead to the opposite conclusion.'' The authors key arguments in the manuscript[6] are:

(1) Plots of the mean velocity profiles in log–log scaling show a straight line in the overlap region, and thereby imply that a power-law is more appropriate than the log-law in representing the data.[10,12]
(2) The low value of $\kappa = 0.38$ found by Österlund *et al.*[12] compared to the ''standard'' values is by itself an indication that a universal log-law does not exist.
(3) The existence of the Reynolds number dependent power-law is proven by plotting a ''universal'' form of the measured velocity profiles in the overlap region.
(4) The behavior of the function Γ as presented in Österlund *et al.*[12] is a result of incorrect processing of the data.

The following comments address each of the above four statements in the same order they are listed above.

Power- or log-law behavior. In Fig. 1 of Ref. 6, the authors show some selected mean velocity profiles from the data-base of Österlund[11] in a log–log plot. Barenblatt *et al.*[6] make the statement: ''All 70 runs corresponding to different Re_θ yield the same pattern: In the intermediate region between the viscous sublayer and free stream the average velocity distribution consists of two straight lines.'' As was demonstrated by Österlund *et al.*,[12] the Reynolds number is too low in the top diagram of Fig. 1 of Ref. 6 and no universal overlap region exists. Therefore it will be excluded from the following discussion. Also, in the diagrams of Fig. 1 of Ref. 6 two straight lines are shown representing two different power-law relations. The one adjacent to the free stream [Ref. 6 Eq. (2)] lies in the outer or wake region. Since our paper[12] is about the overlap region and not about the wake part, this outer power law is also excluded from the following discussion. However, it is important to note that properly scaled the wake part shows no significant Reynolds number dependence, see e.g. Fig. 3.4 on p. 15 and Fig. 9 on p. 49 in Österlund.[10]

Instead we proceed to look at the main argument for the power-law [Ref. 6, Eq. (1)], the inner straight line of the middle and low parts of Fig. 1 in Barenblatt *et al.*[6] The authors claim that the measured profiles lie, within the experimental accuracy, along the straight lines and, thereby, implying the existence of a power law. From this figure one may get this impression, but a closer scrutiny and comparison with the log-law profile, yields that the log-law actually gives a better description than the power-law. One should make both a log–log and a lin–log plot and compare the two. A power-law would give a straight line in the former whereas a straight line in a lin-log plot corresponds to a log-law.

The same data used in Fig. 1 of Barenblatt *et al.*[6] are replotted here in Fig. 1 on a log–log graph and in Fig. 2 using lin-log scaling with compatible ranges of axes. The

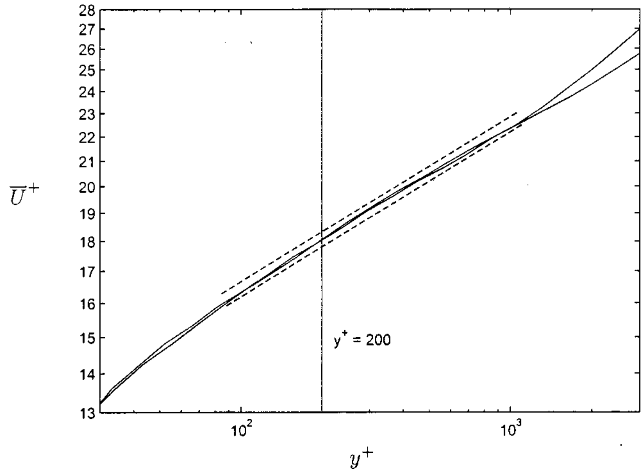

FIG. 1. Mean velocity distribution for $\mathrm{Re}_\theta=\{14207,26612\}$ in log–log scaling. The dashed straight lines are intended to reveal the curvature of the profiles.

data in the lin-log plot show a good agreement with a straight line, whereas the data in the log–log plot display a slight curvature in the region of interest, y^+ larger than about 100. Therefore, careful examination and comparison between the two possibilities for the overlap velocity distribution actually reveals that the log–law is the preferable one and that the main argument in the manuscript by Barenblatt *et al.*[6] is not valid. It is based on an insensitive representation of data on log–log plots. In fact the subtle differences in the mean velocity distributions and their comparison with the various relations is more clearly revealed with the aid of the more sensitive comparison based on a normalized slope of the mean profile, as will be discussed later.

The value of κ. In our recent work[12] we systematically derive inner and outer limits for the overlap region with the aid of figures such as 2 and 3 (of Ref. 12). In particular we arrive at $M_i=200$ as the inner limit of the overlap region and $M_0=0.15$ as its outer limit (compared to the "traditional" limits of $M_i=50$ and $M_0=0.15$). Subsequently in Fig. 5 of the same article, the von Karman constant is evaluated by fitting a log-law to the data in this carefully established overlap region, that is assumed to be governed by a log-law. The

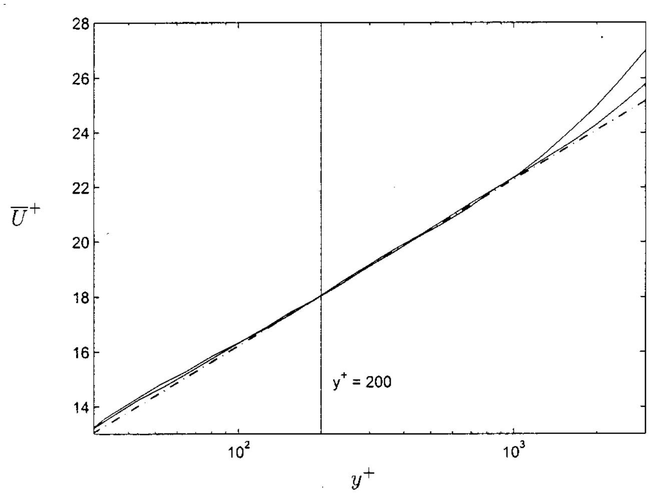

FIG. 2. Mean velocity distribution for $\mathrm{Re}_\theta=\{14207,26612\}$ in lin–log scaling. Dashed–dotted straight line is the log-law using $\kappa=0.38$ and $B=4.1$.

authors of Ref. 6 make a remark about this and seem to misinterpret the importance of the inner limit of the overlap region M_i.

Figure 5 of Österlund *et al.*[12] leads to several important conclusions. First, by using the traditional limits we get a Reynolds number dependence of κ as a result of using a part of the mean velocity profile that is outside the overlap region. Second, by increasing the inner limit of the outer layer to 200 the Reynolds number dependence disappears. Again, recall that the limits were found independently, from Figs. 2 and 3 of (Ref. 12). Third, evaluating κ using the low range of Re_θ around 5000–7000 (a region where many previous experiments were performed) we obtain a value of κ near 0.41. This explains the difference between the newly determined value of κ and the value found in previous experiments. The large scatter in the low Re_θ range using the new inner limit is expected, since fitting the log-law to a much smaller range of data introduces large uncertainties. With increasing Reynolds number the scatter decreases (solid symbols) as a result of the growing overlap range. Also, one can conclude that no universal overlap region exists for $\mathrm{Re}_\theta<6000$ (and no fitting is possible). In conclusion Fig. 5 strongly supports the conclusions of Figs. 2 and 3 of the same article.[12] Also shown in the same Fig. 5 are the results from measurements made on a cylindrical body in the NDF facility at IIT by Hites[9] (shown as squares). These carefully documented results also lead to the identical conclusions as drawn from the KTH data.

The "universal" form of the (power) scaling law. Barenblatt *et al.*[6] check the applicability of their Reynolds number dependent power-law by rewriting it into what they call a "universal" form,

$$\Psi=\frac{1}{\alpha}\ln\left(\frac{2\alpha\overline{U}^+}{\sqrt{3}+5\alpha}\right)=\ln y^+. \tag{1}$$

Then, they claim that when plotted in this universal form, the data support their power-law by their collapse with sufficient accuracy onto the bisector of the figure. They attribute the differences found to higher order terms or to the form of the skin friction law used by us.[12] In fact, the discrepancies are completely explained by assuming that the mean velocity profile is actually a log-law, and inserting that into the universal form obtained by them and tabulated in Barenblatt *et al.*[8] In this procedure, the log-law will be Reynolds number dependent in this "universal" form and is shown here in Fig. 3 for two values of $\mathrm{Re}_\theta=\{14207,26612\}$ together with the measurements in the same Re_θ range.

It is abundantly clear from the figure that the experimental data lie within the area bound by the log-laws corresponding to the maximum and minimum Reynolds number values of the data and that the log-law is definitely a better representation of the data. Again, it is demonstrated here how insensitive log-log plots are since the power- and log-laws are very close together in this figure. Instead, as we stated earlier, it is preferable to use the more sensitive normalized slopes.

The Γ-*function*. Österlund *et al.*[12] demonstrated that the diagnostic function for a log law,

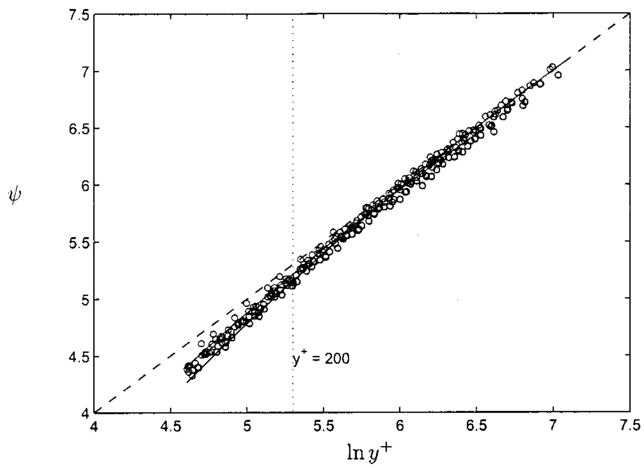

FIG. 3. Power scaling law in ''universal form.'' ○, Measurement $15000 < Re_\theta < 27300$, and $100 < y^+ < 0.2\delta^+$. Dashed line, Reynolds number dependent power-law [Eq. (1)]. Solid lines, Log-law for $Re_\theta = \{14207, 26612\}$.

$$\Xi = \left(y^+ \frac{d\overline{U}^+}{dy^+} \right)^{-1}. \tag{2}$$

is very close to a constant equal to 0.38 in the region $200 < y^+ < 0.15\delta^+$, see e.g., Figs. 2 and 3 in Ref. 12. They also showed that the diagnostic function for a power-law,

$$\Gamma = \frac{y^+}{\overline{U}^+} \frac{d\overline{U}^+}{dy^+} \tag{3}$$

is decreasing in the overlap region and that a power-law therefore is a less accurate than a log-law in describing the mean velocity in the overlap region.

In the recent work of Barenblatt *et al.*[6] they argue that incorrect processing of the data is the cause for the decreasing behavior of Γ in Ref. 12. The function Γ as presented in Österlund *et al.*[12] was averaged over the whole ensemble of measurements, while removing the outer-flow parts where $y/\delta > 0.15$. If the data would adhere to the Reynolds number dependent power-law proposed in Ref. 6 an ensemble averaging over profiles of different Reynolds numbers would result in a decreasing Γ-function. However, this is actually not the reason for the decreasing Γ found in figure 6 of Österlund*et al.*[12] One indication is given by the error-bars which would have been larger in the low y^+ end. However, the best check is to look at the individual Γ curves for separate Reynolds numbers.

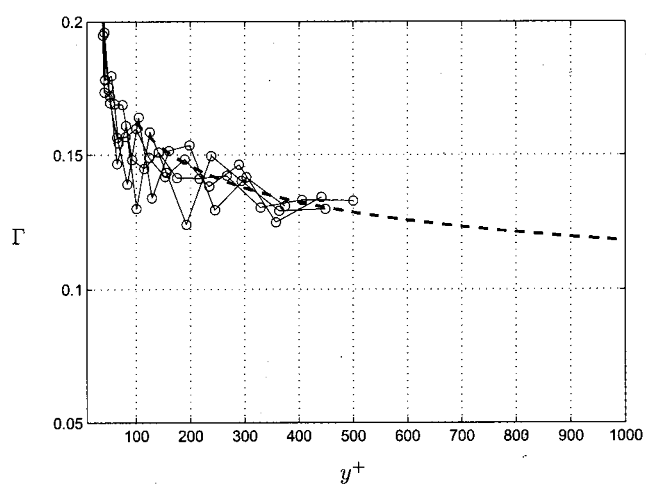

FIG. 4. Diagnostic function for power scaling laws for individual profiles in the range $13000 < Re_\theta < 15000$, and $y/\delta < 0.15$. Dashed line corresponds to the log law using $\kappa = 0.38$ and $B = 4.1$.

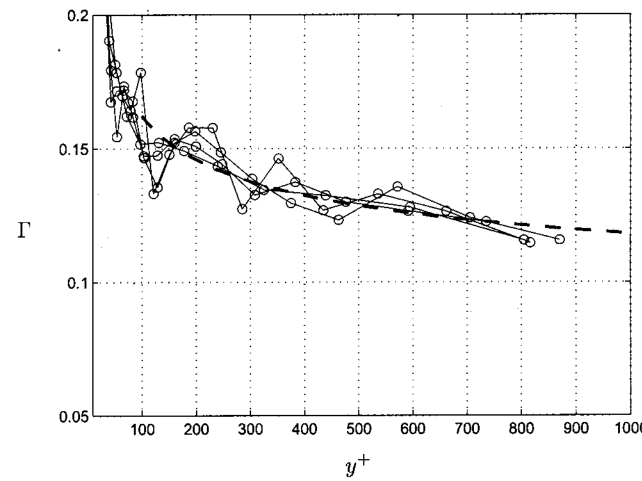

FIG. 5. Diagnostic function for power scaling laws for individual profiles in the range $25000 < Re_\theta < 27300$, and $y/\delta < 0.15$. Dashed line corresponds to the log law using $\kappa = 0.38$ and $B = 4.1$.

In Fig. 4, measurements of individual boundary layer profiles in the range $13\,000 < Re_\theta < 15\,000$ are represented, while in Fig. 5 the measurements in the range $25\,000 < Re_\theta < 27\,300$ are shown. These ranges of Re_θ were selected to be compatible with the two cases used in the middle and bottom parts of Fig. 1 of Barenblatt *et al.*[6] From Figs. 4 and 5 here one can easily conclude that the individual Γ curves display the same behavior without any significant dependence on the Reynolds number. Figures 4 and 5 also demonstrate that the averaged curve in Ref. 12 accurately describes the ensemble of measurements and that the individual profiles closely conform to the log-law found in Ref. 12, and shown here by the dashed line. Therefore, we conclude again that the diagnostic function Γ decreases with increasing wall distance y^+ and that the functional behavior is different from that of a power law. Although the authors of Ref. 6 make a strong point about this issue, one can clearly conclude from the above results that their statement is erroneous.

Conclusions. The main arguments put forward in the work of Barenblatt *et al.*,[6] regarding a power-law behavior in the overlap region of turbulent boundary layers, have been systematically analyzed. An objective scrutiny of the data clearly demonstrates that, for sufficiently high Reynolds numbers, the mean velocity distribution in the overlap region of turbulent boundary layers is very accurately described by *the Reynolds number independent log-law*, given here in inner scaling,

$$\overline{U}^+ = \frac{1}{\kappa} \ln y^+ + B, \tag{4}$$

where $\kappa = 0.38$ and $B = 4.1$. The Reynolds number dependent power-law proposed by Barenblatt *et al.*[6] yields a substantially less accurate representation of the data.

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