SG2221 Lab

Disturbance growth in a flat plate boundary layer

(Course on *Wave motions and Hydrodynamic stability*, 7.5 credits)

1 Background and purpose

Laminar boundary layers are subject to a convective wave instability called Tollmien-Schlichting (TS) waves. TS-waves with a given frequency amplify within a range of Reynolds numbers, which can be assessed by linear theory (see Appendix A). When the wave reaches large enough amplitude, non-linear phenomena initiate the transition to turbulence. This is the dominating scenario in a low-disturbance environment, e.g. on an airfoil or a spacecraft under atmospheric conditions. In noisy environments, e.g. on turbine blades, the TS-wave instability competes with other transition mechanisms, which may lead to transition at lower Reynolds numbers (see Appendix A.). This lab will give some practical experience in how to measure the characteristics of the TS wave instability. It will also illustrate some different kinds of disturbances, which may initiate transition in boundary layers, such as excitation of wave packets and turbulent spots, and free stream turbulence.

The experiments will be performed on a flat plate mounted in the MTL wind tunnel, which is a wind tunnel with exceptionally low level of background disturbances. The working section of the plate is 2.2 m long. It is mounted horizontally in the lower part of the test section, and the stagnation line flow is controlled by means of a flap at the downstream end of the plate. The leading edge is designed to give a good approximation to the Blasius boundary layer flow all along the plate.

2 Experimental equipment

2.1 Introducing disturbances into the boundary layer

Different kinds of disturbances will be introduced into the boundary layer:

- regular wave trains at a fix frequency
- localised transient disturbances
- and free stream turbulence

<u>Wave generation.</u> TS-waves are, here, generated by a loudspeaker, which are fed with a signal from a function generator amplified by a hi-fi-system. The waves (by means of periodic blowing and suction) are introduced into the boundary layer through a slot in the plate, and they excite TS-waves with the same frequency.

<u>Localised transient disturbances.</u> These are generated by for instance by a short air pulse introduced through a whole from beneath the plate. This may excite either a TS wave packet or a turbulent spot, depending on the intensity of the disturbance and the Reynolds number.

<u>Free stream turbulence (FST).</u> A grid placed at the entrance of the test section will cause free stream turbulence, which will greatly affect laminar-to-turbulent transition in the boundary layer. The grid used in the lab will give with free stream rms fluctuations of about 2% of the free stream velocity.

2.2 Hot wire anemometry and traversing system

The streamwise velocity and its fluctuations are measured with a hot-wire. The time signal is logged on a computer where the signal is processed and its power spectrum computed. This gives the dominating wave frequency and the wave amplitude at this frequency. The wave can also be observed qualitatively by watching an oscilloscope connected to the hot wire probe.

The hot wire is calibrated against a Prandtl tube placed in the free stream. Note that the anemometer output is a strongly non-linear function of the velocity, which means that the amplitude of velocity fluctuations observed in the oscilloscope will depend on the local mean velocity at different y-positions.

The hot-wire probe can be traversed normal to the plate as well as in the downstream direction. This makes it possible to measure the amplitude profile of the wave across the boundary layer as well as its downstream development.

3 Detailed lab instructions

- Get acquainted with the equipment:
  - (a) how to run the wind tunnel
  - (b) how to control the wave frequency and amplitude
  - (c) how to position the hot wire probe. Make sure you know at what co-ordinate the probe would hit the wall!
  - (d) Connect one channel of the oscilloscope to the signal generator, and the other channel to the hotwire anemometer. Position the hotwire inside the boundary layer close to the wall and watch the waves excited by the generator on the oscilloscope. Watch the effect of changing amplitude and frequency. Move the probe in the downstream direction, while watching the change in phase lag between the wave generator and the hot wire signal.
  - (e) Connect both signals to the computer. In this lab, the calibration is already done beforehand. Make sure the processing gives you the correct frequency of the wave.
- Quick check of the free stream pressure gradient
  - (f) Position the hot wire probe well above the boundary layer at the leading edge, and measure the velocity there with the computer. Traverse the probe downstream to about x=2000, while watching the output voltage of the anemometer. Does it vary monotonously? Measure the free stream velocity at the downstream position. Make a rough estimate of the pressure gradient from these two measurement points. Is it positive or negative? How would this influence the TS-wave instability?
- Determine measurement parameters for the TS-wave
  - (g) Determine a convenient free stream velocity (e.g. 8 m/s) and generator frequency to excite waves with the desired frequency (e.g. $F = 86$). Determine with the help of the stability diagram at which x-positions branch I and II are located. Estimate the wavelength from the stability diagram, Figure 1, in Appendix A.
  - (h) Position the probe at an x-position, which corresponds to branch II according to linear theory. Watch the oscilloscope and the computed spectrum. What happens when the generator amplitude is increased? Can you identify fundamental or subharmonic wave interactions? Watch the time signal on the oscilloscope, and draw a sketch of it for your lab report.
  - (i) Move the probe beyond branch II. Where does the flow become fully turbulent? How long is the transition region?

(j) Choose a generator amplitude, which excites a purely sinusoidal TS-wave at branch II. Record the wave amplitude with the computer - its maximum should not exceed 0.5% of the free stream velocity (U_∞).

(k) Determine the phase velocity of the wave by comparing the oscilloscope signal of the TS wave with that of the triggering mechanism. Then calculate the wavelength.

- <u>Measure the y-profile of the TS-wave amplitude and phase near branch II.</u>

(l) Traverse the probe through the boundary layer and take measurements of the wave amplitude and phase, as well as the mean velocity. Watch the oscilloscope while you traverse and try to detect the phase shift, which occurs approximately in the middle of the boundary layer.

(m) Plot the amplitude and phase as a function of y/d, where $d = (nx/U_\infty)^{1/2}$, x being the downstream distance from the leading edge and n the kinematic viscosity. Compare to results from linear theory.

- <u>Measure the amplification curve $A(x)$ of the TS-wave</u>

(n) Position the probe at a position upstream of branch I. Locate the near wall y-maximum of the wave amplitude and record the wave amplitude A there.

(o) Move the probe downstream by about 100 mm and repeat the same procedure there. Continue until you reach past branch II.

(p) Plot the wave amplitude as $N = \ln(A/A_0)$, where A_0 is the amplitude at branch I, as a function of Reynolds number based on d. Compare to predictions from linear theory.

(q) What effect would a 3D component have on the amplification rate?

- <u>Wave packets and turbulent spots</u>

(r) Trigger transient localised disturbances from the computer. Position the probe near the plate at an x-position near branch II of the wave you studied above. Draw a sketch of the oscilloscope trace for your lab report.

(s) Watch how varying disturbance intensities give rise to TS-wave packets and turbulent spots. Try to estimate the dominating frequency of the wave packet by looking at the oscilloscope trace.

(t) Move the probe in the x-direction and try to determine where the flow becomes turbulent. How long is the transition region?

(u) Use the oscilloscope to determine the propagation speed of the wave packet by comparing phase speed of its front and rear to that of the triggering mechanism.

- <u>Free stream turbulence</u>

(v) Insert the grid at the entrance of the test section. Readjust the free stream velocity to the same value as before.

(w) Position the probe approximately in the middle of the boundary layer and move it in the x-direction. Try to determine where the flow becomes turbulent.

(x) Position the probe near the plate at an x-position upstream of transition. Traverse the probe through the boundary layer taking measurements of the mean and rms velocity.

(y) Watch the oscilloscope time signal and its computed spectrum as you traverse the probe. What can you say about the frequency content in the boundary layer compared to that in the free stream? Record the power spectrum of u near the maximum of u_{rms}.

(z) Position the probe about 1 mm from the plate at an x-position where there are no signs of turbulence. Traverse downstream at steps of about 100 mm, and estimate the intermittence factor by watching the oscilloscope. Plot these values as a function of x and use them to estimate the length of the transition region.

- Turbulent boundary layer

(aa) Position the probe at an *x*-position where the flow is fully turbulent, and traverse through the boundary layer taking measurements of the mean and rms velocity.

Appendix A. Disturbances in a plane boundary layer
– theoretical and experimental background –

Appendix B. Brief introduction to CTA hot wire anemometry

Disturbances in a plane boundary layer

– theoretical and experimental background –

1. Tollmien–Schlichting waves

For low environmental disturbances the transition scenario from laminar to turbulent flow on a flat plate boundary layer is rather well understood. This class of transition starts with the amplification of streamwise traveling two-dimensional waves, so called Tollmien–Schlichting (TS) waves. The first experimental observation of TS–waves was made in a low turbulence level wind tunnel by Schubauer & Skramstad (1947). They were able to determine the critical Reynolds number for different frequencies by using a vibrating ribbon to trigger the TS–waves and hot-wire anemometry to measure the fluctuating velocity signal. The concept of TS–waves is the basis for engineering calculation methods (so called e^N-methods) for transition prediction in simple boundary layer flows.

For viscous plane flows an ordinary differential equation can be obtained which describes the development of TS–waves. The normal velocity component v is assumed to have a wave character given by

$$v(x, y, t) = \mathrm{Re}\{\hat{v}(y)e^{i(\alpha x - \omega t)}\} ,$$

where α is the complex streamwise wave number and where the real part gives the wavenumber in the streamwise x-direction and the negative imaginary part is the disturbance amplification, ω is the frequency of the wave, y is the wall normal coordinate and t is time. $\hat{v}(y)$ can be viewed as the complex amplitude function of the normal velocity component. Furthermore, the mean flow is assumed to be parallel $\overline{U} = U(y)\overline{e}_x$, *i.e.* the streamwise development of the boundary layer is not taken into account. Under these assumptions the Navier-Stokes equations may be reduced to the so called Orr-Sommerfeld

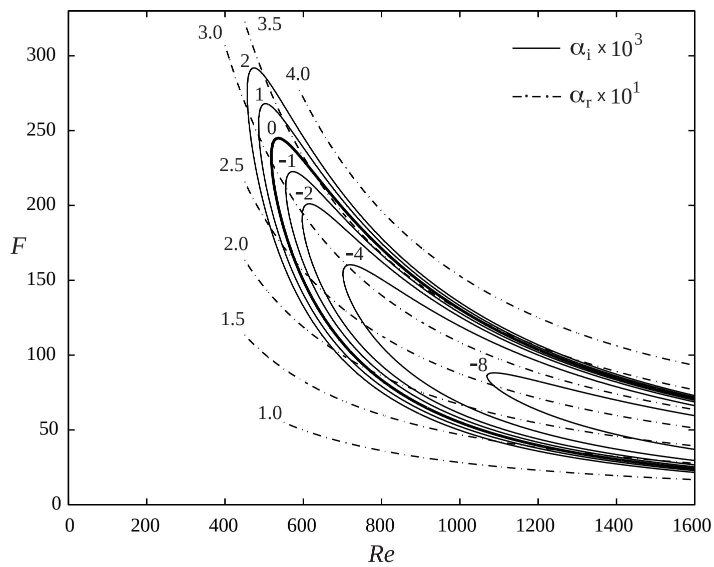

Figure 1: Spatial stability curves for two-dimensional waves in a Blasius boundary layer. Solid lines are for constant imaginary parts of the streamwise wave number (α_i) and dash-dotted for constant real parts (α_r). The bold solid line is the neutral stability curve. The displacement thickness (δ_1) is the characteristic length scale. Figure from Fransson 2003.

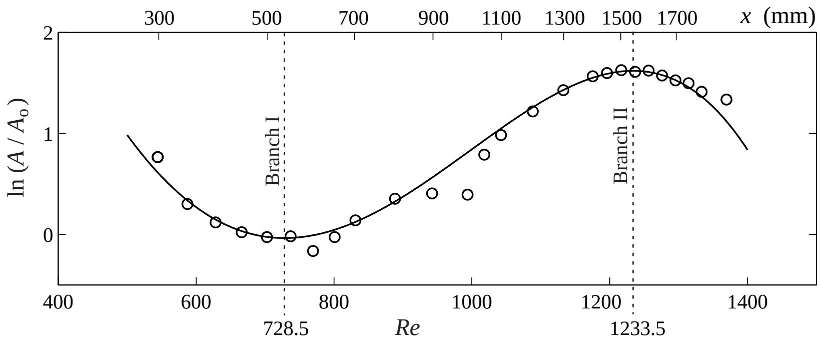

Figure 2: Amplitude evolution of the TS-wave at $F = 100$. Symbols are experimental results, and solid lines are the OS-solution of the Blasius profile. Figure from Fransson 2003.

(OS) equation:

$$\left[(-i\omega + i\alpha U)(\mathcal{D}^2 - \alpha^2) - i\alpha U'' - \frac{1}{Re}(\mathcal{D}^2 - \alpha^2)^2\right]\hat{v} = 0 \ ,$$

where $\mathcal{D} = \partial/\partial y$, $Re = U_\infty \delta^*/\nu$ is the Reynolds number based on the free stream velocity U_∞, ν the kinematic viscosity and $\delta^* = \int_0^\infty (1 - U/U_\infty)dy$ the displacement thickness. For the Blasius boundary layer $\delta^* = 1.72\sqrt{\nu x/U_\infty}$.

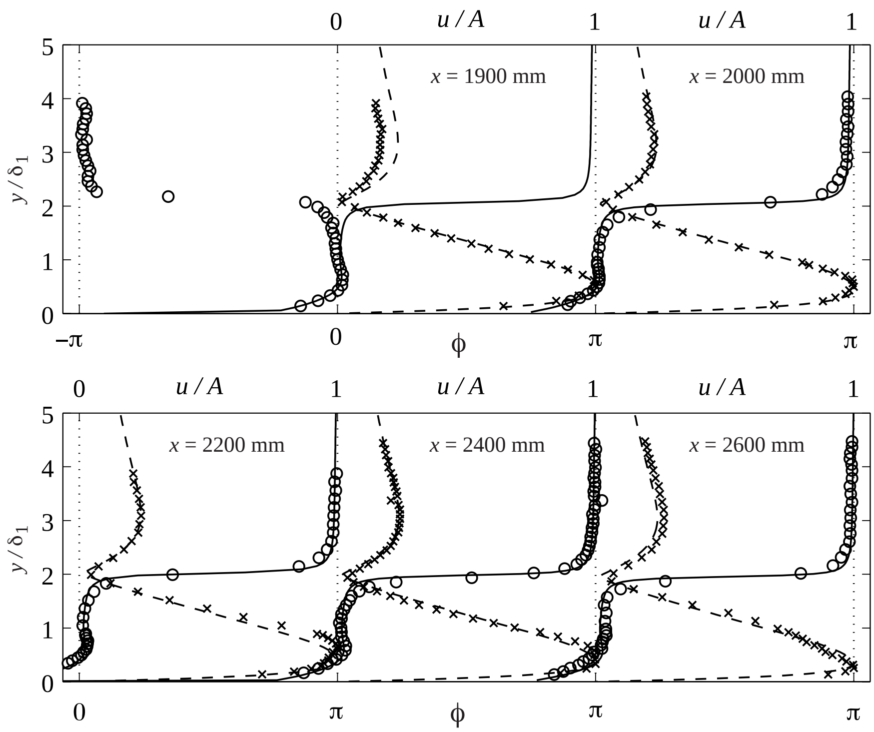

Figure 3: Amplitude- and phase distribution profiles for $F = 59$ at different x-positions. ($\times$)-symbols and dashed lines correspond to measured and theoretical amplitude profiles respectively. ($\circ$)-symbols and solid lines are the corresponding phase profiles. Figure from Fransson 2003.

A wave traveling in the streamwise direction may be amplified only in a certain region of the wave frequency-Reynolds number (F-Re) parameter plane, see figure 1. Here, $F = 2\pi f \nu \cdot 10^6 / U_\infty^2$ is a non-dimensionalized frequency and $Re = 1.72\sqrt{Re_x}$ is the Reynolds number, where $Re_x = U_\infty x/\nu$ and x is the distance from the leading edge.

The downstream development of the TS–wave can be seen in figure 2. The wave is damped downstream of the TS–wave generation slot, until it enters the unstable region and begins to amplify, and finally decreases again. Neutral points can be determined from the downstream evolution of the TS–wave amplitude maximum. The minimum and maximum amplitude positions correspond to branch I and branch II, respectively (see figure 1).

Schubauer & Skramstad (1947) found in their experiment that the amplitude distribution of the disturbance has two maxima, the largest close to the wall and the second maximum at the boundary layer edge, see figure 3. In the same figure the corresponding phase distribution profiles are also plotted, and they clearly show the phase shift of π radians which can be shown to appear where $\partial\hat{v}/\partial y$ changes sign, i.e. at the wall-normal amplitude ($\hat{v}$) maxima.

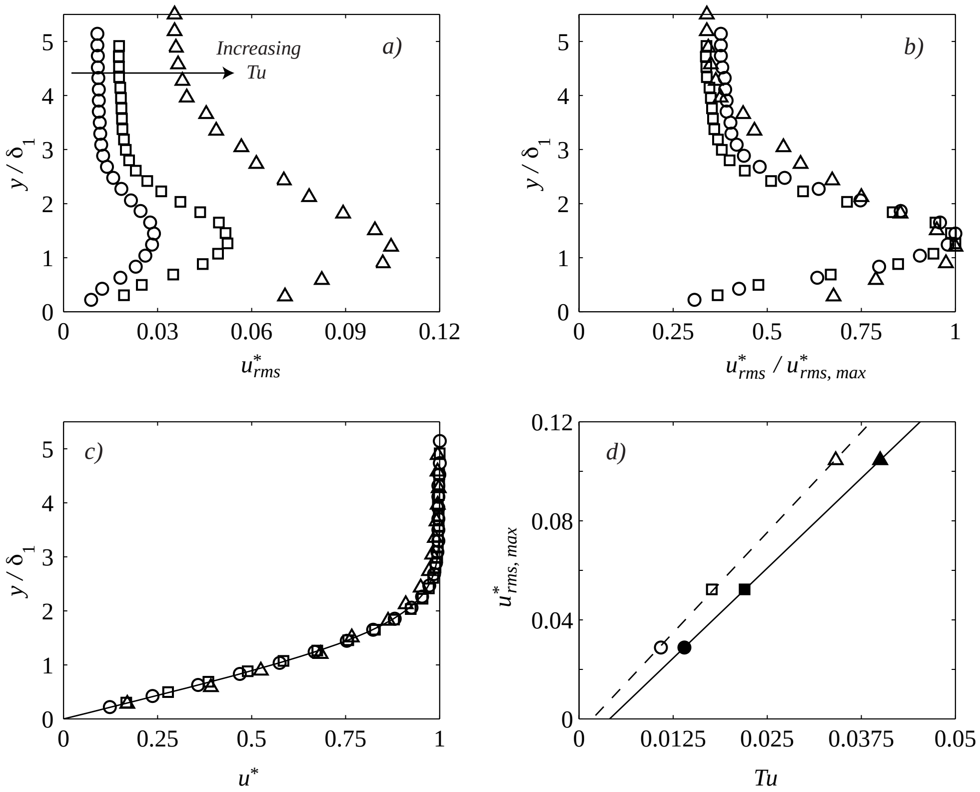

Figure 4: Wall normal perturbation- and mean velocity profiles for different Tu-levels at $Re = 544$ (based on δ_1) or $Re_x = 10^5$, $u^* = u/U_\infty$ and $u^*_{rms} = u_{rms}/U_\infty$. a) and c) show the perturbation- and the mean velocity profile for different Tu ($= u_{rms}/U_\infty$, in the free stream). b) correspond to the data in a) but normalized to unity and d) shows the normalization value versus the local (open symbols) and the leading edge (filled symbols) Tu-value, respectively. ($\bigcirc$) $Tu = 1.4\%$, ($\square$) $Tu = 2.2\%$, and ($\triangle$) $Tu = 4.0\%$. Figure from Fransson 2003.

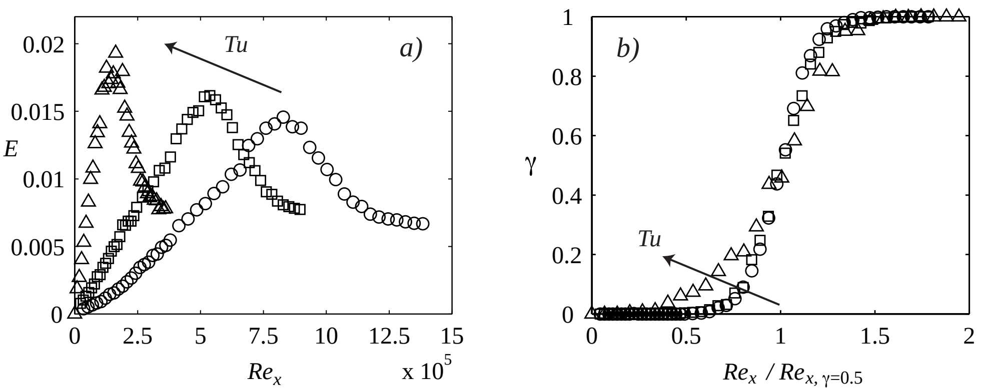

Figure 5: a) Energy growth ($E = u_{rms}^2/U_\infty^2$) as function of Re_x for three different Tu-levels. Measurements are made at y/δ_1=1.4. b) The corresponding intermittency distribution of the data in a) versus Re_x normalized with ditto for $\gamma = 0.5$. (◯) $Tu = 1.4\%$, (□) $Tu = 2.2\%$, and (△) $Tu = 4.0\%$. Figure from Fransson 2003.

2. Free stream turbulence and algebraic growth

It is well known that for the Blasius boundary layer free stream turbulence (FST) induces disturbances into the boundary layer which give rise to streamwise oriented structures of low and high speed fluid. These structures grow in amplitude and establish a spanwise size which is of the order of the boundary layer thickness far away from the leading edge. When the streaks reach a certain amplitude they break down to turbulence, probably through a secondary instability mechanism. This type of boundary layer disturbance was originally called the breathing mode, since the wall-normal disturbance profile resembles that which would result from a locally continuous thickening and thinning of the boundary layer edge. However, this mode is nowadays recognized as the Klebanoff mode which was proposed by, and can be viewed as one scenario of by–pass transition. It is a relatively rapid process by–passing the traditional TS-wave dominated transition scenario resulting in breakdown to turbulence at subcritical Reynolds numbers when compared with the predicted value by traditional theory.

In figure 4 streamwise disturbance and mean velocity wall normal distributions are plotted for different levels of Tu ($= u_{rms}/U_\infty$, in the free stream). In figure 4a) it is clear that the presence of higher FST intensity causes a higher disturbance level inside as well as outside the boundary layer, without affecting the mean velocity (cf. figure 4c). It is both the Reynolds number and the Tu-level that sets the state, i.e. whether the flow is in the sub-transitional, transitional, or in the post-transitional state. At least up to the transitional state one can expect a self similar disturbance profile through the boundary layer. Thereafter, the disturbance peak moves towards the wall and the disturbance level spreads out more in the entire boundary layer, this may be observed in figure 4b). An interesting observation is that the level of the disturbance peak inside the boundary layer increases linearly with Tu which is shown in figure 4d), where solid lines are curve fits to the data (see caption for more information).

In figure 5a) the energy distribution versus the downstream distance for the three different Tu-levels are shown. It is seen that the disturbance (u_{rms}/U_∞) reaches levels around 14% for Tu=4.0% before it starts to decrease, which is connected to the transitional nature of the boundary layer. What is observed in figure 5a) is the algebraic growth followed by transition. That the peak becomes smaller with decreasing Tu is probably connected to the relation between turbulent scales in the free stream (that are different for all three cases), the disturbance level, the streak spacing, and the boundary layer thickness.

Note, also that the energy seems to asymptote to a constant level around $E = 0.007$ independent of the Tu-level after reaching the maximum value. The intermittency function for the three different cases in a) are shown in figure 5b). The maximum (in figure 5a) is closely related to the point of γ=0.5, i.e. the point where the flow alternatively consists of laminar portions and turbulent spots which explains the high u_{rms} value. In a) it is seen that the higher the Tu the smaller the Re_x for which the maximum occurs, and figure 5b) shows that the relative extent of the transitional zone is larger for Tu=4.0% than the other two.

References

- Fransson J. H. M. 2003 Flow control of boundary layers and wakes. PhD thesis, KTH, Stockholm, TRITA-MEK Tech. Rep. 2003:18.

- Schubauer, G.B. & Skramstad, H.K. 1947 Laminar boundary layer oscillations and transition on a flat plate. *J. Res. Nat. Bur. Stand.*, **38**, 251-292.

Brief introduction to CTA hot-wire anemometry

Constant–Temperature Anemometry (CTA) for hot-wires is based on convective heat loss from a heated resistance sensor. In CTA the hot-wire sensor is held at a constant temperature. If the fluid properties remain constant the instantaneous heat loss is a measure of the instantaneous velocity. A hot-wire is simply a thin wire, usually made of platinum and in the range 0.5–5 μm in diameter, mounted between two prongs. The length to diameter ratio of the sensor should be at least 200 in order to have large heat loss to the fluid as compared to the heat conduction to the prongs. With CTA the sensor temperature is constant by means of a highly amplified feed-back loop of a servo system. The anemometer essentially consists of a Wheatstone bridge and a servo amplifier. The sensor resistance, typically around 10 Ω, forms one of four legs of the bridge. By setting the resistance of one of the other legs to a chosen value, the sensor resistance at operating conditions will be given. The difference between the sensor resistance at operating conditions and the cold resistance (at ambient temperature) determines the temperature difference between the sensor and the fluid. This overheat is typically 150 degrees in air flows, which make the system insensitive to small changes in air temperature. The electrical current needed to keep the sensor at constant temperature, and thereby the bridge in balance, is measure indirectly by measuring the voltage between the bridge ”top” and ground. This voltage increases with increasing velocity, and rapid changes in velocity can be measured. The frequency response of the system is usually of the order of 10-100 kHz.

The relation between output voltage E and velocity U is given by King's Law

$$E^2 = A + BU^n \ ,$$

where E is the anemometer output voltage at the velocity U, and A, B and n are constants to be determined by calibrating the wire against a set of known velocities. The value of n is typically close to 0.5 and A should be close to the square of the output voltage at zero velocity, $A \approx E_0^2$.