Figure 1:

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Continuum Mechanics
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a) b) The inertia tensor $\mathbf{I}$ with respect to the center of mass of a homogeneous body in the shape of a rectangular prism with sides a, b, c is diagonal and

$$I_{xx} = \frac{m}{12}(b^2 + c^2), I_{yy} = \frac{m}{12}(a^2 + c^2), I_{zz} = \frac{m}{12}(a^2 + b^2).$$

Determine the moment of inertia for the axis, passing the center of mass, with polar directions θ, φ.
The angular momentum $\mathbf{L}$ is given by

$$\mathbf{L} = \mathbf{I}\boldsymbol{\omega}.$$

Write this equation in component notation and calculate the components of the angular momentum for rotation around the given axis, with angular velocity ω. The kinetic energy is given by

$$T = \frac{1}{2}\boldsymbol{\omega} \cdot \mathbf{I}\boldsymbol{\omega}.$$

Calculate the kinetic energy in terms of $m, a, b, c, \omega, \theta, \varphi$.
b) We now consider the bending of a beam, see chapter 5 of Basic Continuum Mechanics. Check that the following two vectors $(\mathbf{e}_1+2\mathbf{e}_2+\mathbf{e}_3)/\sqrt{6}, (2\mathbf{e}_1-\mathbf{e}_2)/\sqrt{5}$ are orthogonal. They have the position (X, Y, Z) in the reference configuration. Find the angle between them in the bent beam. Also find the relative increase of length of the first of the two vectors, when the beam is bent.