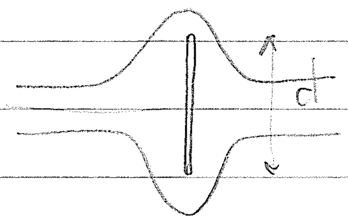
{Displacement thickness from XI}
{Skin friction from XI}
{Momentum loss thickness from XI}

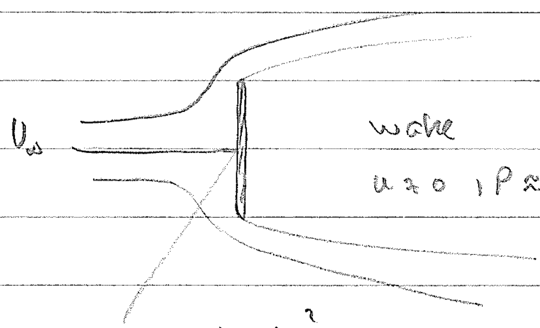
Pressure drag & friction drag {Show fig. 2.21}

Blunt body

irrotational flow

real flow $Re \gg 1$





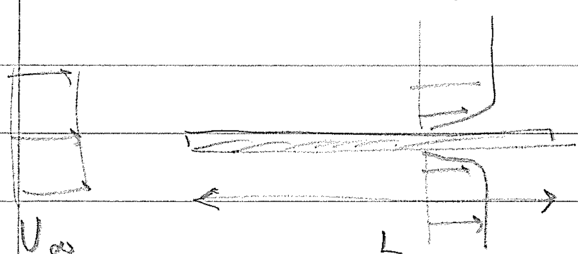
drag $D_1' = 0$

$p_0 = p_\infty + \frac{1}{2}\rho U_\infty^2$

Form drag $D_1' = \frac{1}{2}\rho U_\infty^2\, C_d \cdot d$

drag coefficient ~ 1 for blunted body {show OH C_D}

Streamlined body



Friction drag

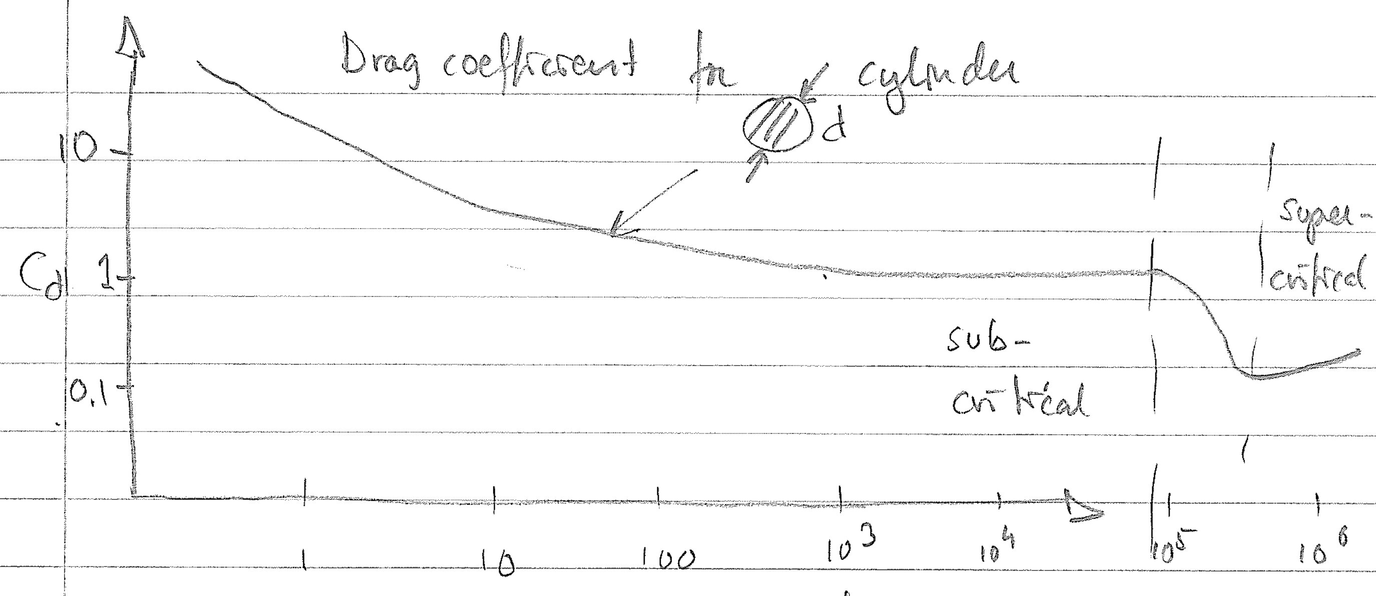
$$D' = \rho U_\infty^2\, 2 \times 0.664 \sqrt{\frac{\nu L}{U_\infty}}$$

$$= \rho U_\infty^2\, 1.4 \frac{L}{\sqrt{Re_L}} = \frac{\rho U_\infty^2}{2} \underbrace{\frac{2.8}{\sqrt{Re_L}}}_{C_d} L$$

Plate length for equal drag $\frac{1}{2}\rho U_\infty^2 \cdot d = \frac{1}{2}\rho U_\infty^2 \frac{2.8}{\sqrt{Re_L}} L$

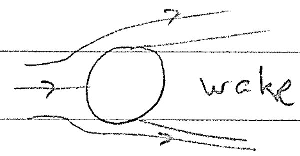
$$\frac{L}{d} = \frac{\sqrt{Re_L}}{2.8} \qquad Re_L \sim 10^6 \Rightarrow \frac{L}{d} = \frac{1000}{2.8} = 357$$

$$Re_d \sim 10^6/357 \sim 3 \cdot 10^3$$

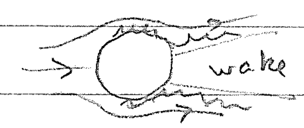


$$Re = \frac{U_\infty d}{\nu}$$

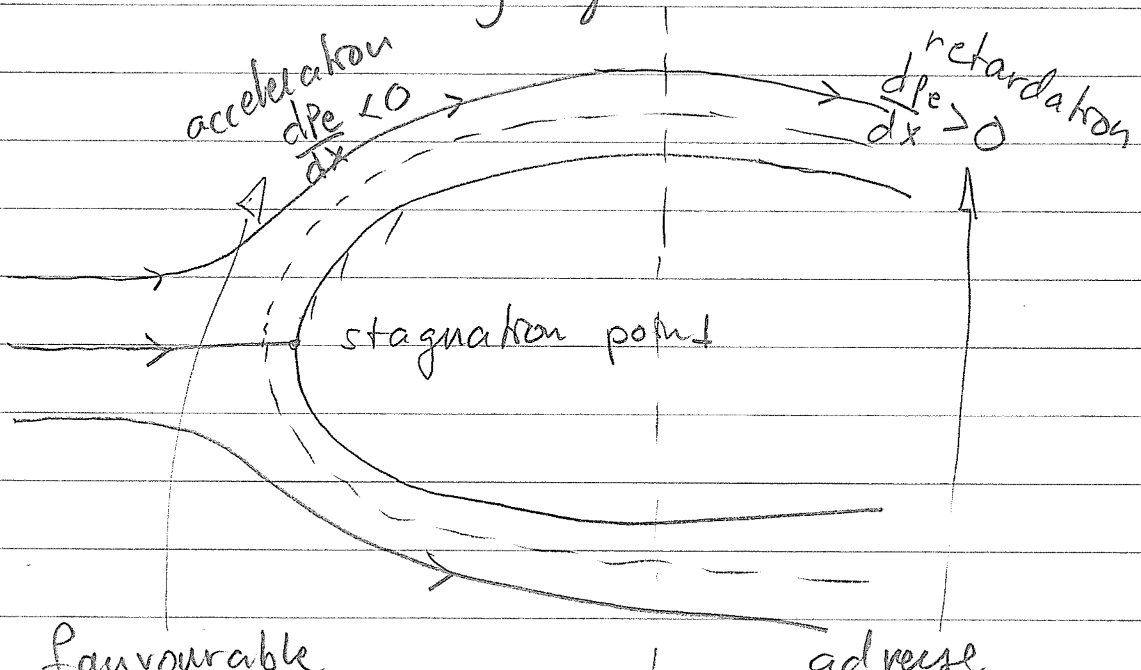
subcritical regime - laminar b.l.



supercritical regime - turbulent b.l.



Separation of boundary layer from solid surface
- connected to <u>pressure gradient</u>, $\frac{dP_e}{dX} > 0$, in the boundary layer



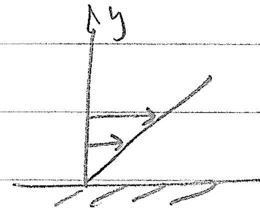
favourable pressure gradient

adverse pressure gradient

Effect on boundary layer profile.

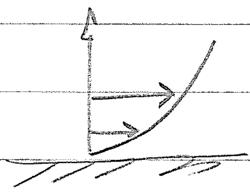
at the wall $$0 = -\frac{1}{\varrho}\frac{d}{dx}p_e + \nu \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0}$$

$$\frac{dp_e}{dx} = 0 \Rightarrow \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = 0$$



$$\frac{dp_e}{dx} < 0 \Rightarrow \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} < 0$$

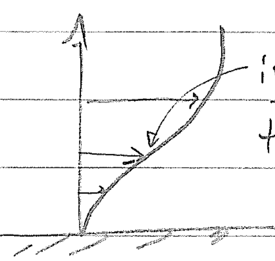
accelerating



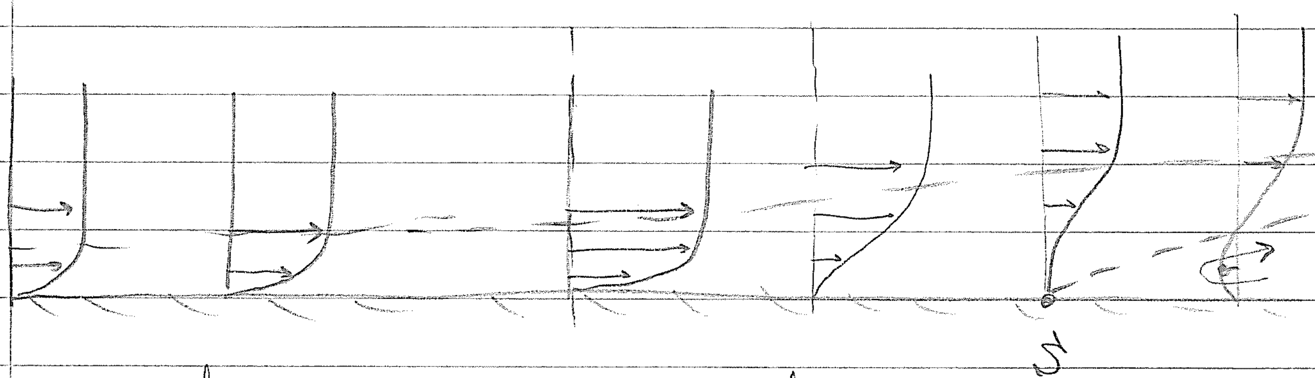
fuller profile, thinner b.l.

$$\frac{dp_e}{dx} > 0 \Rightarrow \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} > 0$$

decelerating

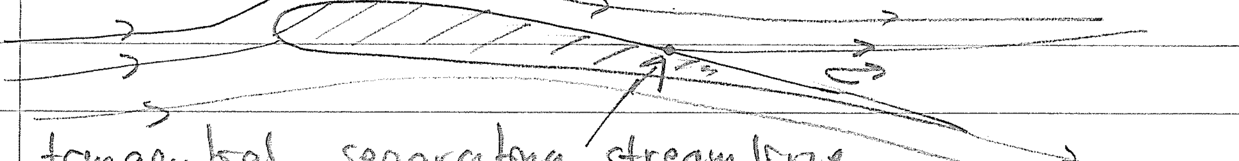


inflection point, thicker b.l.



$\frac{dp_e}{dx} < 0$ | $\frac{dp_e}{dx} > 0$

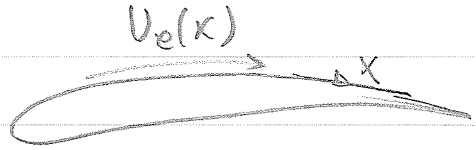
At separation point S $$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$$



tangential separating streamline

Boundary layer theory __not valid__ beyond separation point.

Falkner-Skan similarity solutions



For which inviscid flows $U_e(x)$ exist self-similar boundary layers?

$$u(x,y) = U_e(x)\, f'(\eta) \quad ; \quad \eta = y/\delta(x)$$

$$\psi = \int^y U_e(x)\, f'(\eta)\, dy = U_e(x)\, \delta(x)\, f(\eta)$$

$$v(x,y) = -\left(\frac{\partial \psi}{\partial x}\right)_y = -(U_e \delta)' f - U_e \delta f' \left(-\eta \frac{\delta'}{\delta}\right)$$

$\Rightarrow$

$$f''' + \frac{1}{2}(\beta+1)\, f f'' + \beta(1 - f'^2) = 0$$

$$\delta(x) = \sqrt{\frac{\nu x}{U_e(x)}} \quad ; \quad \beta = \frac{x}{U_e}\frac{dU_e}{dx}$$

Possible inviscid flows $U_e(x) = U_L\left(\frac{x}{L}\right)^\beta$ for similarity solution

$$f(0) = f'(0) = 0 \quad , \quad f'(\infty) = 1$$

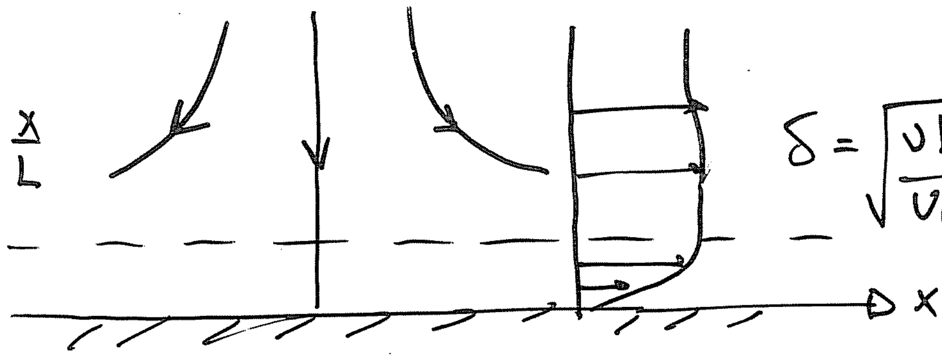
Falkner-Skan flows

$\beta = 0$ Blausius flat plate

$$\delta = \sqrt{\frac{\nu x}{U_\infty}}$$

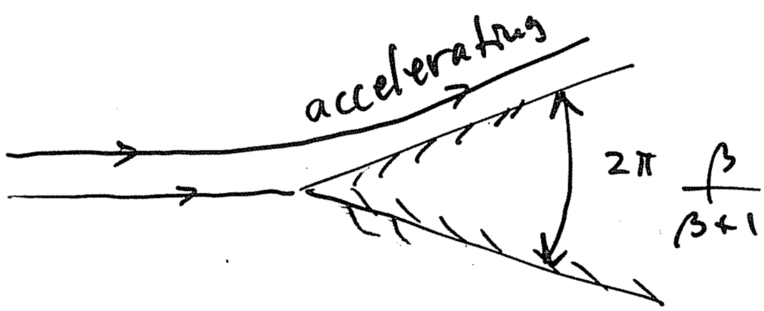
$\beta = 1$ Stagnation point flow

$$U_e = U_L \frac{x}{L}$$



$$\delta = \sqrt{\frac{\nu L}{U_L}} = \text{const.}$$

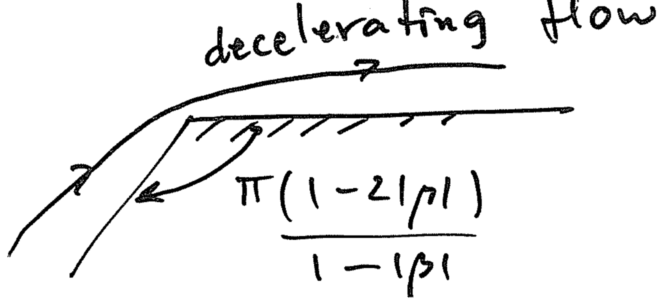
$0 < \beta < 1$: Flow towards wedge



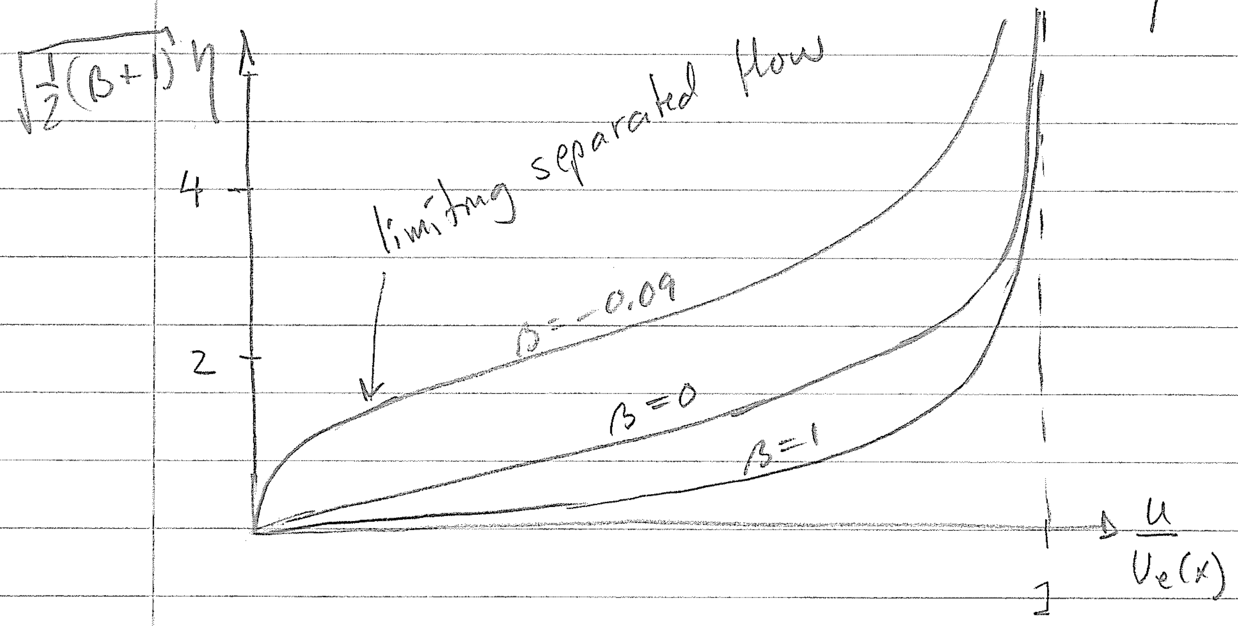
$$\delta = \sqrt{\frac{\nu L}{U_L}} \left(\frac{x}{L}\right)^{\frac{1-\beta}{2}}$$

$\beta < 0$ Flow around corner

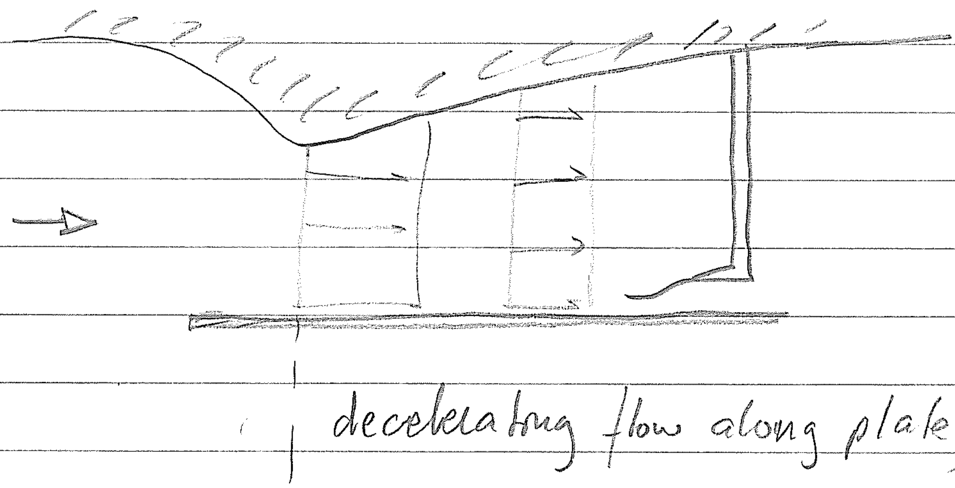
$$\delta = \sqrt{\frac{\nu L}{U_L}} \left(\frac{x}{L}\right)^{\frac{1+|\beta|}{2}}$$



Numerical solution exist for $-0.0904 < \beta$



<u>Experimental</u> investigation of laminar boundary layers with and without pressure gradients.



decelerating flow along plate, $\frac{dp_e}{dx} > 0$

Measure : * pressure distribution along plate

* boundary layer velocity profiles, 2 stations

Comparison with "equivalent" Falkner-Skan solution.