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# Interaction between a vertical turbulent jet and a thermocline

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## ABSTRACT

We study the behaviour of an axisymmetric vertical turbulent jet in an unconfined stratified environment by means of well-resolved large eddy simulations. The stratification is two uniform layers separated by a thermocline. We consider two cases: when the thermocline thickness is small and of the order of the jet diameter at the thermocline entrance. The Froude number of the jet at the thermocline varies from 0.6 to 1.9 corresponding to the class of weak fountains. We quantify mean jet penetration, stratified turbulent entrainment, jet oscillations and the generation of internal waves. The mean jet penetration is predicted well by a simple model based on the conservation of the source energy in the thermocline. The entrainment coefficient for the thin thermocline is consistent with the theoretical model for a two-layer stratification with a sharp interface, while for the thick thermocline entrainment is larger at low Froude numbers. We report the presence of a secondary horizontal flow in the upper part of the thick thermocline, resulting in the entrainment of fluid from the thermocline rather than from the upper stratification layer. The spectra of the jet oscillations in the thermocline display two peaks, at the same frequencies for both stratifications at fixed Froude number. For the thick thermocline, internal waves are generated only at the lower frequency, since the higher peak exceeds the maximal buoyancy frequency. For the thin thermocline, conversely, the spectra of the internal waves show the two peaks at low Froude numbers, whereas only one peak at the lower frequency is observed at higher Froude numbers.

## 1. Introduction

This study focuses on the dynamics of an axisymmetric vertical turbulent jet in a stratified fluid. Vertical turbulent jets may serve as models of numerous flows both in nature and industry (see e.g. Turner (1973); List (1982); Hunt (1994)) including effluents from submerged wastewater outfall systems in the ocean (e.g. Jirka and Lee (1994)), convective cloud flows in the atmosphere, pollutant discharge from industrial chimneys, subglacial discharge from glaciers (e.g. Straneo and Cenedese (2015)). The stratification considered is two layers of homogeneous fluids of different temperature separated by a relatively thin layer with a temperature jump - a thermocline. This configuration is a typical model of the upper thermocline layer in lakes, the pycnocline in the ocean, as well as thermal inversions in the atmosphere, when the sharp gradient of the scalar prevails significantly over the scalar change in the layers.

The dynamics of vertical jets is governed mainly by their volume, momentum and buoyancy fluxes, where the buoyancy of a jet is defined by the density difference between the jet and the surrounding medium, normalized by gravity. If the flow density is less than the density of the surrounding medium then the jet is positively buoyant, if heavier the jet is negatively buoyant, while it is neutrally buoyant if the densities are equal. In general, all the examples of turbulent jets in nature and industry mentioned above result from mixed sources of buoyancy and momentum (as a rule they are positively buoyant). However, jets effectively entrain the surrounding fluid, hence, when the source is located far enough from the pycnocline, the density of the flow at the pycnocline entrance is almost equal to the density of the lower layer of stratification. The dynamics of such a flow in the pycnocline can therefore be modelled employing a neutrally buoyant turbulent jet with positive vertical momentum in the lower stratification layer. In other words, an initially buoyant jet in the pycnocline can be modelled employing a neutrally buoyant jet provided they have the same velocity and radius at the entrance of the pycnocline. The turbulent jet considered here results from a momentum source of the same fluid as in the lower layer of stratification. When entering the thermocline, it becomes a negatively buoyant jet, i.e. a fountain.

Stationary regimes of turbulent fountains have been extensively investigated in both homogeneous and linearly stratified media (Turner 1966; List 1982; Bloomfield and Kerr 1998, 2000; Kaye and Hunt 2006; Burridge and Hunt 2012, 2013) revealing the dependency of the mean penetration height and of the entrainment coefficient on the different parameters of the problem. The behaviour of an axisymmetric miscible Boussinesq fountain in a homogeneous fluid is defined by the Reynolds number $Re = U_0 R_0/\nu$ ($U_0$ the inflow velocity, $R_0$ the nozzle radius, $\nu$ the fluid kinematic viscosity), and the Froude number $Fr = U_0/\sqrt{g' R_0}$ (with $g' = g\Delta\rho/\rho_0$ the reduced gravity, $\Delta\rho$ is the density difference between source and ambient fluid). The Reynolds number determines whether the fountain is laminar or turbulent while the Froude number characterizes the ratio between momentum flux $M_0$, buoyancy flux $F_0$ and volume flux $Q_0$ of the fountain. Indeed, it can be rewritten, following Kaye and Hunt (2006), as $Fr \sim M_0^{5/4}/Q_0 F_0^{1/2}$. The Froude number can also be interpreted as the ratio between two length scales: $l \sim M_0^{3/4}/F_0^{1/2}$, known as the momentum jet length (Turner 1966), and $R_0 \sim Q_0/M_0^{1/2}$ corresponding to the initial radius of the jet. Using theoretical considerations and experimental validations, Kaye and Hunt (2006) classified fountains according to their Froude number as: very weak ($Fr \lesssim 1$), weak ($1 \lesssim Fr \lesssim 3$) and forced ($Fr \gtrsim 3$). Later Burridge and Hunt (2012, 2013) extended the classification using more experimental data, further dividing "weak fountains" into weak and intermediate, with a change from weak to intermediate fountains at $Fr \approx 1.7$. The behaviour of forced fountains in a homogeneous fluid is governed by the momentum and buoyancy fluxes and the mean penetration height, here denoted $h_z$, is therefore proportional to the momentum jet length $h_z/R_0 \sim Fr$ (Turner 1966). For weak fountains, instead, all three fluxes are important and dimensional analysis gives a penetration

$h_z/R_0 \sim Fr^2$ (Kaye and Hunt 2006; Burridge and Hunt 2012). Finally, very weak fountains are hydraulically controlled and estimates at large Reynolds numbers give $h_z/R_0 \sim Fr^{2/3}$ (Kaye and Hunt 2006; Burridge and Hunt 2012).

In a linear stratification dimensional considerations yield a penetration height $h_z/R_0 \sim Fr^{1/2}$ for forced fountains with zero initial buoyancy flux (McDougall 1981; Bloomfield and Kerr 1998). In general, however, the rise height of a fountain in a stratified fluid depends on the density profile and requires more complicated numerical models based on the conservation laws for the momentum, volume and buoyancy fluxes of the jet (Morton et al. 1956; Bloomfield and Kerr 2000).

Instabilities are observed for fountains in a homogeneous fluid, and this oscillatory motion has become the object of research only recently (Friedman 2006; Friedman et al. 2007; Williamson et al. 2008; Burridge and Hunt 2013). The dynamics of a fountain in a homogeneous fluid is, analogously to the mean penetration height, fully controlled by the Froude and Reynolds numbers. It has been demonstrated experimentally that weak fountains can undergo oscillations with amplitudes comparable to their heights and well-defined frequencies. The oscillatory dynamics of fountains in stratified fluids is however mostly unexplored. Interestingly, the only experimental investigation in a linear stratification has shown no direct connection between the frequency of the fountain oscillations and the frequency of internal waves (Ansong and Sutherland 2010).

A behavior similar to the oscillatory dynamics of weak fountains has been revealed in pycnocline-like stratified fluids while modelling submerged wastewater outfall systems in the ocean (Troitskaya et al. 2008). Turbulent buoyant plumes discharged horizontally into oceanic salt water gain vertical momentum due to their positive buoyancy while they propagate in the lower layer of stratification. At the same time they are mixing intensively with the surrounding fluid owing to the turbulent entrainment. At the entrance to the pycnocline, these jets have density close to the density of the lower layer of stratification and a non-zero vertical momentum thus forming fountains. These fountains are capable to generate internal waves in a pycnocline through their oscillations. This effect has been demonstrated experimentally, by means of laboratory scale modeling of wastewater outfall systems, and later numerically (Druzhinin and Troitskaya 2012, 2013) both for laminar and turbulent fountains/jets in two-layer stratified fluid with a thin pycnocline (i.e. in the presence of a rather sharp density jump compared to the jet diameter at the pycnocline entrance).

As mentioned earlier, fountains in a linear stratified fluid do not show pronounced oscillations, while fountains in a two-layers fluid are characterized by strong oscillations. Thus, in addition to the Reynolds and Froude number, with all these parameters taken in the vicinity of the pycnocline, the ratio of the pycnocline thickness to the jet diameter, is expected to play an important role. Therefore, the aim of this paper is to understand the influence of the ratio ”pycnocline thickness/jet diameter” on the dynamics of a turbulent fountain and on the generation of internal waves, using data from well-resolved Large Eddy Simulation (LES). Since the pycnocline is subject to seasonal variability (Kamenkovich and Monin 1978; Knauss 2005; Stewart 2008) this ratio is expected to change throughout the year, making this a relevant question in oceanography.

Previous numerical investigations (Druzhinin and Troitskaya 2012, 2013) investigated a similar configuration but focused on a thin pycnocline in comparison to the jet diameter at the pycnocline entrance. However, field measurements and results of modelling employing nonhydrostatic general circulation model reveal that they are mostly of the same size (Sciascia et al. 2013; Troitskaya et al. 2008). In this paper we compare jet dynamics in two different stratifications: one with a thin thermocline, analogous to Druzhinin and Troitskaya (2013), and the other with a thermocline thickness close to the jet diameter at the thermocline entrance. The latter case, for the thermocline Froude numbers 0.87-1.16, reproduces the conditions of laboratory experiments investigating the generation of internal waves by a turbulent jet (Ezhova et al. 2012). Note, that the parameters of the jet at the entrance to the thermocline in the experiments matched the parameters of the laboratory scale modelling of the real wastewater outfall system in winter conditions (Troitskaya et al. 2008). In summer, convection in the upper layer is weak, governed mainly by the surface wave breaking and mixing due to the wind, and together with the increased temperature difference between upper and lower layers, this results in the sharpening of the pycnocline and its moving closer to the surface. As a result, for the same source location, the radius of the jet at the pycnocline entrance increases and the vertical velocity decreases; some qualitative conclusions about the jet dynamics in these conditions can therefore be drawn from the present results for the thin thermocline and low Froude numbers.

The jet dynamics in the thermocline is relevant for turbulent mixing of the jet with the surrounding media. This important question has been before investigated for a jet in two-layers stratification with a density interface experimentally (Cotel et al. 1997; Lin and Linden 2005) and theoretically (Shrinivas and Hunt 2014, 2015). In this study we investigate the mean flows in the thermocline and compare the entrainment flux of the jet in stratifications characterized by a finite thickness of the thermocline with the results of the theoretical model by Shrinivas and Hunt (2014).

The paper is organized as follows. Section 2 contains the relevant equations and a brief description of the LES model. The test case of a turbulent jet in a homogeneous medium is described and the setup of the simulations for a stratified case discussed. Section 3 is devoted to the results of the simulations: in the first part we investigate the penetration height and turbulent entrainment of the jet in a stratified medium and discuss the dynamics of the jet in the thermocline. The generation of the internal waves is presented in the second part. Our conclusions are given in Section 4.

## 2. Governing equations and numerical method

We consider a jet in an unconfined fluid with a stable thermal stratification. The dynamics of a jet in a stratified fluid is governed by the Navier-Stokes equations for an incompressible fluid with the Boussinesq approximation to model the buoyancy effects and a transport equation for the temperature field. To carry out a parameter study like that presented here, we resort to LES to reduce the computational costs. In a LES, the large turbulent eddies are fully resolved whereas the effect of the smallest scales, those not resolved on the computational mesh, is modelled. A filter is applied to derive an equation for the resolved scales that reads in dimensionless form and in a Cartesian coordinate system

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j^2} + \frac{1}{Fr^2}(T - T_s')\delta_{iz} - \frac{1}{Re}\frac{\partial \tau_{ij}}{\partial x_j}, \tag{1}$$

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \frac{1}{RePr}\frac{\partial^2 T}{\partial x_j^2} - \frac{1}{Re}\frac{\partial \Theta_j}{\partial x_j} \tag{2}$$

$$\frac{\partial u_i}{\partial x_i} = 0. \tag{3}$$

The equations are made dimensionless with the initial jet diameter $D_0$, the jet maximal inflow velocity $U_0$, and the temperature difference between the stratification layers $\Delta T$. We define the profile of stratification as $T_s' = (T_s - T_0)/\Delta T$, where $T_s$ is the undisturbed temperature profile and $T_0$ is the temperature of the lower layer of stratification. The hydrostatic pressure component associated with $T_s'$ is subtracted from the full pressure to get $p$ in our system. We define the Reynolds number $Re = \dfrac{U_0 D_0}{\nu}$, the Froude number $Fr = \dfrac{U_0}{\sqrt{g' D_0}}$, with $g' = \dfrac{g\Delta\rho}{\rho_0} \approx g a_T \Delta T$ the reduced gravity (here $a_T$ is the thermal expansion coefficient), the Prandtl number $Pr = \dfrac{\nu}{\kappa}$ where $\nu$ is the fluid kinematic viscosity and $\kappa$ the thermal conductivity. $\tau_{ij}$ and $\Theta_j$ are the fluxes representing the subgrid Reynolds stresses and turbulent heat transport.

To model the subgrid-scale stresses we employ the dynamic Smagorinsky model (Smagorinsky 1963; Germano et al. 1991) which has been successfully used in the simulations of buoyant flows by several authors, e.g. Pham et al. (2006, 2007). The subgrid-scale stresses are expressed as

$$\tau_{ij} = -2\nu_t S_{ij},\ S_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}), \tag{4}$$

$$\Theta_j = -\frac{\nu_t}{Pr_t}\frac{\partial T}{\partial x_j}. \tag{5}$$

In the spirit of the Prandtl mixing length model, the subgrid-scale viscosity is given by the formula

$$\nu_t = (C_s\Delta)^2|S_{ij}|, \tag{6}$$

where $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ and $C_s$ is the Smagorinsky coefficient, related to the dynamic Smagorinsky constant by $C_s = \sqrt{C_d}$. The idea underlying the dynamic Smagorinsky model is that the small eddies of the large structures that are still resolved in the computations are statistically analogous to the subgrid-scale eddies. Thus an additional filter, the test filter, is used to separate the resolved turbulent spectrum and calculate dynamically the Smagorinsky constant $C_d$ (for more detail see Germano et al. (1991)).

In our simulations, the jet is generated by a round source of diameter $D_0$ with an initial vertical velocity profile

$$U_i = -0.5\tanh\frac{r-0.4}{0.05} + 0.5, \tag{7}$$

where $r = \sqrt{x^2+y^2}$, with $x$ and $y$ the horizontal directions, see Fig. 1. The stratification of the ambient fluid is of a thermocline-type with a temperature jump at the vertical position $z = z_p$. The stratification profile is given by

$$T'_s = \frac{1}{2}(1+\tanh(\gamma(z-z_p))) \tag{8}$$

where $\gamma = D_0/H$ and $H$ is the half-thickness of the thermocline. The temperature of the fluid at the inflow is equal to the temperature of the lower stratification layer.

### *a. Numerical method*

The numerical simulations presented here are performed with the parallel flow solver Nek5000 (Fischer et al. 2008). The dynamic Smagorinsky model is built-in inside this code. Nek5000 is a spectral element code with exponential accuracy within the spectral elements. On each element the flow variables are represented as a superposition of Lagrange polynomials based on Gauss-Lobatto-Legendre quadrature points (GLL points). In the present calculations the spatial discretization is made with polynomials of order 7, which means that each element contains $8 \times 8 \times 8$ grid points or GLL points. Time discretization involves an operator-splitting method using backward differentiation of order 2 for the implicitly treated viscous terms and 2nd order extrapolation for the explicitly treated convective terms (BDF2/EXT2). For stabilization, the highest 2 modes of each element are slightly dampened (5%). The test filter required for the calculations in the framework of the dynamic Smagorinsky model affects the 3 highest polynomial modes with a cut-off of 0.05, 0.5 and 0.95 (Ohlsson et al. 2010).

Among the advantages of the spectral element method is the flexibility to construct spatially inhomogeneous meshes. For the particular problem at hand, one needs to resolve the small scales at the jet inflow to accurately reproduce the region of high kinetic energy production and the small scales in the region where the jet impinges on the pycnocline, producing high shears. At the same time, internal gravity waves are characterized by long wavelengths, large-scale motions, so that a lower resolution is enough at larger distances from the jet axis.

### *b. Validation for the turbulent jet in a non-stratified fluid*

To validate the current implementation and be sure to have a fully developed turbulent jet at the thermocline entrance, we perform LES of a turbulent jet in a homogeneous fluid and compare the main flow statistics with the data available in literature, both from experiments and Direct Numerical Simulations (DNS).

The governing equations for a turbulent jet in a homogeneous fluid reduce, after the LES-filtering, to

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j^2} - \frac{1}{Re}\frac{\partial \tau_{ij}}{\partial x_j}, \tag{9}$$

$$\frac{\partial u_i}{\partial x_i} = 0. \tag{10}$$

The jet is generated at the bottom boundary of the computational domain and has a round shape of diameter $D_0$ with the initial velocity profile given in eq. (7). To trigger transition to turbulence, we add to this laminar profile a set of 10 sinusoidal disturbances with frequencies $f$ distributed evenly in the range [0.05:5], wavelengths in $x,y$ directions changing from 4 minimal distances between GLL points ($\Delta x$ = $\Delta y$ = 0.03) to 20 these distances and random phases. The amplitude of the disturbances is about 15% of the base flow velocity at the inflow. The simulations are performed for Reynolds number $Re = 15000$.

We solve the governing equations on a rectangular domain of dimensions $40 \times 40$ along the horizontal $x$ and $y$-axes, and 42 in the vertical direction (Fig. 1). We impose a traction free boundary condition (open boundaries) at the lateral boundaries and the convective boundary condition by Orlansky (1976)

$$\frac{\partial u_i}{\partial t} + c_{zi}\frac{\partial u_i}{\partial z} = 0 \tag{11}$$

at the top of the domain. Here $c_{zi}$ are the components of the phase velocity that are calculated dynamically for each velocity component at the $z$-level adjacent to the upper boundary and filtered over the $xy$-plane by a running average. Negative values of $c_{zi}$ are set to 0.

The mesh used is constructed following the guidelines in Picano and Hanjalic (2012): in the region closest to the jet inflow, $x, y \in [-1.5, 1.5]$, $z \in [0, 12]$, a better resolution is achieved with uniform spectral elements of size $\Delta x = \Delta y = 0.5$ and $\Delta z = 0.6$ (each element containing $8 \times 8 \times 8$ GLL points). From the boundaries of this inner region, we stretch the grid by a factor 1.17 along the horizontal axes and 1.06 along the $z$-axis. The total number of elements used in these validation runs is $30 \times 30 \times 30$ corresponding to $\approx 9$ million grid points. The timestep chosen for the calculation is 0.01, which amounts to keeping the CFL number below 0.25-0.3.

The values of $C_d$ in the model are averaged over the vertical direction in a conical region containing the jet, resulting in a value of the Smagorinsky coefficient $C_s$ in the range between 0 and 0.2, in agreement, for example, with the values obtained in the simulations of buoyant plumes by Pham et al. (2007). The calculations of the statistics start approximately 100 time units after the jet has reached the upper boundary and extend over a time interval of over 500 dimensionless time units corresponding to $\approx 30$ eddy turnover times if the characteristic velocity and the jet diameter are taken at $z = 18$.

Fig. 2a displays the inverse centerline mean velocity $U_c$ versus the vertical coordinate $z$ to show that the velocity follows the $1/z$ dependence that can be derived from the momentum integral for a submerged turbulent jet. The asymptotic behavior starts from $z \approx 12$. The linear fit yields a slope of 0.22, corresponding to 0.165 if recalculated for the initial top-hat velocity profile with the same momentum and volume fluxes. This is in good agreement with the widely assumed values of 0.16-0.17 (see, for instance, Pope (2000)).

Fig. 2b displays the average $z$-velocity profile in the far-field of the jet in self-similar coordinates ($\xi \sim r/(z - z_0)$, $U/U_c$), where $z_0$ denotes the location of the jet virtual origin and $U_c$ corresponds to the maximum velocity at each $z$-level. In practice, we first compute the profiles at each $z$ in self-similar variables and then average over the different profiles in the range $z \in [14, 35]$, following

Picano and Hanjalic (2012) among others. In the figure, we include for comparison the data from 2 sets of DNS for the round and annular jets (Picano and Hanjalic 2012; Picano and Casciola 2007) and 2 laboratory experiments (Panchapakesan and Lumley 1993; Hussein et al. 1994) to confirm the accuracy of the results.

Figs. 2c and d report the turbulent stresses $< u_z'^2 > /U_c^2$ and $< u_r'^2 > /U_c^2$ in the far-field of the turbulent jet together with the data from the experiments and DNS mentioned above. To obtain $< u_r'^2 >$ in the rectangular geometry, we measure the profiles of $< u_x'^2 >$ along the $x$-axis and of $< u_y'^2 >$ along the $y$-axis and then average over the positive and negative $x$ and $y$ directions. We scale the profiles using the self-similar coordinates and average among the different $z$ locations as described before. It can be seen in the figure that the agreement between the different set of data is good. Given these data, we consider a developed turbulent jet from $z$=14 and therefore set, in the calculations in a stratified medium, the thermocline lower boundary at $z = 20$.

### *c. Configuration of the jet in a stratified fluid*

Using the governing equations for a flow in a stratified medium in (1)-(3), we perform 2 series of simulations with the stratification profile in eq. (8). The first set assumes $\gamma = 2$ and $z_p = 20.5$ which corresponds to a relatively thin thermocline since the jet diameter at the entrance to the thermocline, is approximately 4-5, as shown by the simulation of a turbulent jet in homogeneous medium presented in the previous section. Indeed, for $\gamma = 2$ the thermocline is 4-5 times thinner than the diameter of the jet. The second set of simulations assumes $\gamma = 0.5$ and $z_p = 22$, which we will denote as the thick thermocline; in this case, the thermocline thickness is approximately the same as the jet diameter at the thermocline entrance. In both series we perform calculations for 5 different Froude numbers ($Fr = 7, 10, 13, 16, 22$).

A better definitions of the Froude number and $\gamma$ may consider values at the entrance to the thermocline which is defined from the simulations at approximately $z = 18$ (as it will be seen from what follows), corresponding to $Fr_t = 0.6, 0.86, 1.11, 1.37, 1.89$ (here $Fr_t = u_t/\sqrt{g' R_t}$ where $u_t$ and $R_t$ are the mean jet velocity and radius as defined by Shrinivas and Hunt (2014)). We define the ratio of the jet radius at the thermocline to the thermocline thickness, $\gamma_t = R_t/H$. For the thick thermocline $\gamma_t = 1$, while for the thin thermocline $\gamma_t = 4$.

The choice of $Fr_t \sim 1$ is justified by the observations by Burridge and Hunt (2013) of the sudden jump in the amplitude and frequency of the fountain top oscillations in a homogeneous fluid. Note however that the Reynolds number in the experiments of Burridge and Hunt (2013), $Re \approx 1000-3500$, is significantly lower than in our simulations. The experimental investigation of turbulent jets in a stratified fluid by Ezhova et al. (2012) corresponds to $Fr_t \sim 1$ and $Re \sim 10000$. Given that the diameter of the jet in the experiments was comparable to the thermocline width, we in fact reproduce these experimental conditions in the setup with the thick thermocline.

The coefficient of the dynamic Smagorinsky model, $C_d$, is averaged over the vertical direction from $z = 0$ to the maximum fountain penetration point for $r < 5$ and from $z = 17$ to the upper boundary of the thermocline for $r > 5$, resulting in the same range of the Smagorinsky coefficient $0 < C_s = \sqrt{C_d} < 0.2$ as for the test case with a jet in homogeneous fluid (sec. b). We use open boundary conditions on all the boundaries except the inflow where we impose the velocity profile of eq. (7). On the lateral boundary, we also use a sponge layer to damp the vertical velocity component and the temperature fluctuations. The length of the sponge layer is 5 in the simulations with the thin thermocline and 7 in the simulations with the thick thermocline.

The mesh used for the stratified case has the same stretching as in the test case of a jet in homogeneous fluid in the $x$ and $y$ directions, though in a wider domain to be able to capture the internal waves propagating in the thermocline. However, we refine the mesh and increase the vertical resolution at the thermocline and in the upper layer of the stratification approximately up to the penetration height of the fountain to maintain a well-resolved LES. The parameters pertaining to all simulations are summarized in Table 1, where we also report the case used for the validation with increased resolution discussed in the Appendix (denoted as 'test'). The resulting flow is displayed in Fig. 4 for the thick thermocline at $Fr = 22$.

Validation of our LES model ($Fr = 13$, thick thermocline) against the data on weak fountains in a homogeneous fluid by Lin and Armfield (2000) and experiments on turbulent jets in a stratified fluid by Ezhova and Troitskaya (2012) is shown in Fig. 3. Fig. 3a shows the decay of the axial vertical velocity of the jet in the thermocline versus that of the weak fountain in a homogeneous fluid (Lin and Armfield 2000). Fig. 3b shows several LES profiles of the vertical velocity in the thermocline and compares them to the experimental data by Ezhova and Troitskaya (2012). We do not include DNS data for the fountains in this figure since Lin and Armfield (2000) used an initial parabolic vertical velocity and the vertical velocity profiles tend to keep the parabolic form in weak fountains; as shown in the figure, the experimental and LES profiles are closer to Gaussian. Thus, the LES model presented here captures the properties of the mean velocity fields of the weak fountains.

For each simulation we gather statistics (the mean values and the rms of the fluctuations of all quantities) and save time histories to analyse the jet oscillations and the main features of the internal waves at specific locations in the flow. We collect statistics approximately 100 time units after the perturbations at the thermocline have reached the lateral boundary of the computational domain. This time changes from approximately 900 time units for the thin thermocline and small Froude number to about 2100 time units for the thick pycnocline and large Froude number. The duration of the sampling changes from 1200 time units to 4100 time units, with intervals 0.25 time units for the time histories. To investigate the dynamics of the fountain at the thermocline and the internal waves we examine the oscillations of the isotherm $T = 0.5$. The jet oscillations are characterized by the isotherms at the center of the jet and at 4 points at distance $r = 1.5$ from the jet axis, while internal waves are studied by the isotherms corresponding to 2 sets of points located further away, at $r = 20$ and 25.

## 3. Results

We shall first examine the statistics of the flow, and, in the following section, consider the internal waves generated by the interaction between the jet and the thermocline.

### *a. Jet impingement and entrainement*

Fig. 5 shows cross-sections of the absolute value of the mean velocity from our simulations. The first observation is that the higher the Froude number, the higher the jets penetrate into the thermocline and eventually into the upper layer of stratification. For the lowest $Fr$ and the strongest stratification (thin thermocline), the mean flow is reminiscent of a jet impinging on a wall. In the other cases, the flow has a more complicated structure and a counterflow appears in the thermocline and upper stratification layer to form a fountain. This counterflow is more evident when increasing the Froude number and decreasing the thickness of the thermocline. The higher the jet penetrates, the higher the counterflow velocity is and the deeper the annular flow surrounding the jet propagates into the lower layer. Mixing, in turn, makes the fluid in the counterflow lighter than the lower layer of stratification, so that it bounces back to the thermocline where it finally spreads at the level of neutral buoyancy. This structure is characteristic of a two-layer stratification (Cotel et al. 1997; Ansong et al. 2005) as compared to fountains in homogeneous and linearly stratified media, where the counterflow simply protrudes to the bed or to the level of neutral buoyancy (Bloomfield and Kerr 1998).

To quantify the jet penetration into the thermocline, we report the mean axial jet velocity for all the stratified cases and for the turbulent jet in homogeneous medium in Fig. 6a. The evolution in the stratified media follows that in a homogeneous medium to $z \approx 18$, before the typical behavior of a fountain is observed.

Fig. 6b reports the penetration heights from the LES defined as the location where the jet velocity falls below 1% of the initial velocity.

The Froude numbers calculated at the thermocline entrance are characteristic of weak fountains and the rising height can be estimated from the conservation of energy (Kaye and Hunt 2006), so that the source kinetic energy of the flow is converted into potential energy. This implies:

$$\frac{U_m^2}{2} \sim \int_0^{h_z^*} g a_T (T_s - T_0) dz^*, \tag{12}$$

where $U_m$ is the centreline jet velocity at the level where the fountain is formed (we take $z* = 18D$), $h_z^*$ the penetration height. Normalizing eq. (12) with $D_0$, $U_0$ and $\Delta T$ we finally obtain

$$(\lambda u_m)^2 \frac{Fr^2}{2} = \int_0^{h_z} T_s' dz, \tag{13}$$

where $u_m = 0.22$ at $z = 18$ and $\lambda$ is a constant of order one which we find from the best fit of the LES data. Fig. 6b displays the theoretical dependence of the penetration height obtained by integrating eq. (13) with $\lambda = 0.8$. The comparison with the LES results indicates that the penetration height is well predicted at low Froude numbers but overestimated at the largest $Fr$. To explain this, we recall that the rising height of weak fountains in a homogeneous fluid scales as $Fr^2$ whereas that of forced fountains (where the turbulent entrainment is taken into account) scales with $Fr$. The largest Froude numbers investigated here correspond to the transition between the weak and forced regime, and therefore eq. (12) appropriate for the weak regime overestimates the penetration height at these Froude numbers.

Note that the theory based on the conservation equations by Morton et al. (1956) is not expected to be valid for weak fountains near the thermocline because the basic assumptions of the model about self-similarity and constant turbulent entrainment do not hold. Our calculations show that this model significantly underestimates the penetration heights from the LES.

The dashed lines in Fig. 5 indicate the boundaries of the thermocline (they correspond to 10% and 90% of the temperature jump) obtained from the average temperature field. It can be seen that for the thin thermocline and small Froude number (Fig.4a) the temperature jump is deformed as an entire structure reminiscent of a thick membrane, with variations of the height of the upper and lower boundaries only in the region of the jet impingement: a strong stratification dampens turbulence and inhibits mixing. For the higher Froude numbers (Fig. 5b-c) the thin thermocline is significantly deformed, revealing a toroidal well-mixed region adjacent to the jet. The size and depth of this well-mixed region grow with the Froude number.

Two observations can be made here. Firstly, this behaviour is consistent with the experimental observations for a turbulent jet impinging on a stratified interface (see, for instance, Shy (1995); Cotel et al. (1997)) where the formation of a large toroidal vortex was observed immediately after the jet impingement and related to the generation of baroclinic vorticity which tends to push back the interface to the unperturbed state. Secondly, which might be more relevant to our system, we report that the turbulent regime of weak fountains ($1 \lesssim Fr_t \lesssim 1.9$), forming at the thermocline, is characterized by vertical oscillations of the jet. Here, the fluid falls down quasi periodically from the top (Troitskaya et al. 2008; Burridge and Hunt 2012; Druzhinin and Troitskaya 2013); these oscillations are not necessarily axisymmetric, although their average is. The fluid falling from the top forms the vortical structures adjacent to the jet at the lower boundary of the thermocline. These structures, together with the small-size eddies on the jet shear layers crossing the thermocline, are responsible for the turbulent mixing.These large structures and the small eddies on the shear layer are illustrated by the instantaneous fields of temperature and vertical velocity shown in Fig. 7 for both stratifications and $Fr = 7$ and 22 (i.e. $Fr_t$=0.6 and 1.9).

The effect of the fountain oscillatory dynamics on the generation of internal waves in the thermocline will be discussed in the next section.

To substantiate these observations we study the change of the level of neutral buoyancy with increasing Froude number, i.e the level where the jet spreads horizontally forming a gravity current. This is illustrated by the mean horizontal velocity profiles at the distance $r = 20$ from the jet center (Fig. 8a). The vertical coordinate in this picture is $\eta = \gamma(z - z_p)$, so that the origin is moved to the thermocline center and $\eta = \pm 1$ correspond to the thermocline boundaries. Note that the width of the horizontal flow is determined by the radius of the jet at the thermocline entrance which is the same for the two values of thermocline thickness used. In other words, the different width of the flows in Fig. 8a reflects the different ratio between the thermocline width and the radius of the impinging jet. For small Froude numbers, the level of neutral buoyancy is below the lower thermocline boundary while it is moving higher up into the thermocline for larger Froude numbers, indicating a better mixing with the fluid in the thermocline and from the upper layer of stratification.

To quantify mixing, we calculate the mean temperature of the horizontal flow through the cylindrical surface of radius $r = 20$ surrounding the jet; this distance is chosen so that the control volume is far enough from the mixing region adjacent to the jet (see Fig. 5, $15 < z < 25$). Using the mean volume and mass fluxes of the gravity current, we obtain the following expression for the averaged temperature of the gravity current

$$T_{gr} = \frac{\int T U_{hor} dz}{\int U_{hor} dz} \tag{14}$$

where we perform the integration over the region characterized only by positive values of $U_{hor}$, i.e. we consider the flow propagating outwards from the jet (detrainment) and do not account for entrainment. The values obtained are displayed in Fig. 8b to demonstrate that the average temperature of the horizontal flow increases with the Froude number, again indicating a better mixing with the fluid in the thermocline and the upper layer of stratification.

To study the mixing at the thin and thick thermoclines we introduce the entrainment flux $E_i = Q_e/Q_{in}$, similarly to the definition used for investigations of turbulent entrainment by jets and plumes in two-layer (sharp interface) stratified fluid (Shy 1995; Cotel et al. 1997; Shrinivas and Hunt 2014, 2015). When the jet penetrates into the upper layer of stratification, it forms a dome-like structure which entrains the ambient fluid. Thus $Q_e$ in the definition above is the volume flux of the fluid entrained by the jet top and $Q_{in}$ is the volume flux of the fluid in the jet at the interface between the two layers of stratification. This dome-like structure is reported in Fig. 9, where we show the mean horizontal velocity where the jets interact with the thermocline for both stratifications under consideration. Since we have a smooth change of temperature between the two layers, the 'dome' over which the fluxes are computed is depicted by the black lines in Fig. 9: we consider a closed surface consisting of a circular cylinder cut on the lower side along the surface of the fountain. As the total volume flux is equal to 0, we can estimate the flux through the dome perimeter $Q_e$ as the sum of the fluxes through the cylinder top and side $Q_{cyl}$:

$$Q_{cyl} = 2\pi R \int_{z_1}^{z_2} u_{side}\, dz + 2\pi \int_0^R u_{top} r\, dr, \tag{15}$$

where $u_{side}$, $u_{top}$ are the velocities normal to the side and top surfaces of the cylinder respectively, $z_1, z_2$ are the vertical coordinates corresponding to the bottom and top of the cylinder, $R$ is the radius of the base of the cylinder.

We define the inflow volume flux $Q_{in}$ as the volume flux of the jet at the level $z = 18$ where it is the same for all the cases considered here (see Fig. 6a)

$$Q_{in} = 2\pi \int_0^\infty u_{in} r \, dr \tag{16}$$

with $u_{in}$ the vertical velocity. The entrainment flux can finally be written as

$$E_i = \frac{R \int_{z_1}^{z_2} u_{side} \, dz + \int_0^R u_{top} r \, dr}{\int_0^\infty u_{in} r \, dr}. \tag{17}$$

The dependence of the entrainment flux on the Froude number at the thermocline entrance is displayed in Fig. 8c where the dashed curve indicates the theoretical entrainment flux for a jet in an unconfined medium in the limit of small Froude numbers ($Fr_t < 1.4$) and a sharp interface, $E_i = 0.24 Fr_t^2$, together with the approximation of the theoretical curve for larger Froude numbers, both taken from Shrinivas and Hunt (2014). This power law is obtained from an energy balance: a fraction of the kinetic energy supplied by the jet at the interface per time unit is expended into work (per time unit) against the gravity force to entrain fluid from the upper stratification over a distance of the order of the jet scale at the thermocline entrance, yielding $Q_e/Q_i \sim u_t^2/R_t \Delta g \sim Fr_t^2$ (here $u_t$ and $R_t$ are the mean jet velocity and radius taken at the level of the density interface). The value of the constant $A = 0.24$ is obtained theoretically by Shrinivas and Hunt (2014).

Our data follow the quadratic law obtained in Shrinivas and Hunt (2014) for the thin thermocline, however, the entrainment rate is slightly higher. Owing to the smoother temperature change in the thermocline, the turbulent transfer is expected to be more active in this case than for the sharp interface.

At small Froude numbers, the fountain in the thick thermocline entrains more fluid although the average temperature of the horizontal gravity current is lower. This is due to the fact that the jet does not penetrate through the thermocline up to the warm upper layer. At the same time the stratification is weaker which results in a larger surface of the dome and more efficient turbulent transfer. At higher Froude numbers, when the jet penetrates through the thermocline, the entrainment fluxes are rather close for the two cases, but the average temperatures for the thick thermocline are lower.

We explain this difference by the presence of a horizontal flow towards the jet in the upper part of the thick thermocline. In fact, we recall that entrainment velocities (denoted as secondary flows Shrinivas and Hunt (2015)) play an important role in the process of confined entrainment at small Froude numbers. As shown in Fig. 9a, the flow above the dome in the thin thermocline looks similar to the model of a thin ”vortex sheet” on the dome perimeter for unconfined entrainment in a two-layer stratification (Shrinivas and Hunt 2014). Interestingly, a horizontal secondary flow appears in the thick thermocline. Fig. 9b shows a two-layer horizontal flow in the thermocline. In this case, stratification inhibits vertical turbulent transfer and the jet entrains the fluid from the upper thermocline forming a well-pronounced horizontal secondary flow over the initial gravity current. Even for the largest penetration heights of the jet investigated here, the structure of the horizontal flow essentially does not change and the jet entrains fluid mostly from the thermocline, not from the upper stratification layer as in the case of the thin thermocline.

Finally, note that the thick thermocline conditions correspond to the experimental setup used in Troitskaya et al. (2008) and Bondur et al. (2010) to investigate turbulent jets and plumes in thermocline-like stratified tank. The horizontal velocity profiles measured in the experiments at a distance $24R_t$ from the jet center display a back-flow from 6 to 15% of the maximal velocity of the gravity current in the upper thermocline. Our simulations give a magnitude of 15-20%, at a closer distance of $10R_t$.

*b. Generation of internal waves*

In all the simulations, as in the experiments of Troitskaya et al. (2008) and Ezhova et al. (2012) we observe oscillations of the jet top at the thermocline, which results in the generation of internal waves. An example of the instantaneous temperature field at the center of the thermocline and the corresponding isotherms at the distance $r = 20$ from the jet center is shown in Fig. 10 for the case of the thin thermocline, $Fr = 10, 22$ ($Fr_t = 0.86, 1.9$). The top figure, pertaining the lower Froude number, displays rather regular waves emanating from the jet and almost sinusoidal isotherms. The plots for the larger Froude number show a more chaotic behaviour, the isotherms displaying signs of wave breaking in the thermocline.

The analysis of the dynamics of the jet in the thermocline and of the internal waves is based on the power spectra of the temperature oscillations. We consider the isotherm at the center of the undisturbed thermocline, $T = 0.5$, and investigate its displacement at several points close to the jet center and far from it. The spectra of the jet oscillations, $z - z_p$ with $z_p$ the average height of the thermocline $T = 0.5$, are obtained by averaging data from 5 locations: one in the center of the jet and 4 from the points on the circle of radius 1.5 (see Sec. 2c). The spectra of internal waves, instead, are obtained by averaging spectra from 8 locations at distance $r = 20$ from the jet center. The spectra for Fr=13, thin and thick thermoclines are shown in Fig. 11b, c as an example. It can be seen that the jet generates internal waves with pronounced spectral peaks.

We first note that all the spectra of the jet oscillations in both stratifications have two peaks. This is consistent with the observation of fountains in a homogeneous fluid where 2 peaks have been reported for all cases by Burridge and Hunt (2013). Moreover, for fixed $Fr$, the spectra of the jet oscillations have peaks at similar frequencies in different stratifications, an shown in Fig. 11a. Thus, the frequencies of the oscillations do not depend on the thermocline thickness for the parameters chosen in the simulations. Note, however, that one expects differences in frequencies when the jet does not penetrate through the thermocline since its effective Froude number is defined by the temperature difference between the lower stratification layer and the level to which jet penetrates, rather than by the difference between upper and lower stratification. In our case this difference is probably too small to be detected. Simulations at even lower $Fr$ may possibly reveal this effect.

The frequencies of the spectral peaks for jet oscillations and internal waves are summarized in Table 2 and displayed in Fig. 12. Since the peaks in the spectra are rather wide we used the following expression to define the main frequencies in the spectra:

$$\hat{f} = \frac{\int_{f_{\min}}^{f_{\max}} f \mathscr{S}(f)\, df}{\int \mathscr{S}(f)\, df}, \tag{18}$$

where $f_{\min}$ and $f_{\max}$ denote the range of frequencies corresponding to each spectral peak. The figure shows a decrease in the frequency of the oscillations with the Froude number in agreement with the fountains in a homogeneous medium (Burridge and Hunt 2013).

For the 3 smallest Froude numbers, the spectra of jet oscillations have a pronounced large peak and a second small peak at approximately double the frequency. For the 2 highest Froude numbers investigated, the peaks have approximately equal magnitude. The spectra of internal waves are different at lower Froude numbers, with two peaks in the thin thermocline, and one peak in the thick thermocline, primarily due to the difference in the maximal buoyancy frequencies as explained below.

Indeed, the thickness of the two thermoclines considered in this paper corresponds to a factor 2 difference in the maximal buoyancy frequency. The dimensionless buoyancy frequency, $N^2 = g a_T \dfrac{dT_s}{dz}$, can be re-written in our case as

$$N^2 = \frac{0.5\gamma}{Fr^2} \frac{1}{\cosh^2(\gamma(z - z_p))}. \tag{19}$$

Thus, $N_{\max} = 1/Fr$ and $N_{\max} = 0.5/Fr$ for $\gamma = 2$ and $\gamma = 0.5$, respectively. The spectra of jet oscillations and internal waves for the same $Fr = 13$ and different stratifications are shown in Fig.11b, c. The spectra of jet oscillations have two distinct peaks, the higher one possibly corresponding to the harmonics of the lower. The thin thermocline has a larger maximal buoyancy frequency which allows propagation of waves of both frequencies (first and second peak) while only the lowest frequency perturbation can generate internal waves at the thick thermocline. Fig.12b clearly indicates the frequency cutoff due to the smaller maximal buoyancy frequency, since the second frequency in the spectra of the jet oscillations is always higher than the maximum buoyancy frequency for the thick thermocline.

For the 2 higher Froude numbers the spectra of internal waves have one pronounced peak close to the lower peak of the jet oscillations, which is surprising in case of the thin thermocline where one expects propagating waves at both frequencies. The simulations for these cases, when the jet penetrates far enough through the thermocline, show that fluid falling from the jet top loses axial symmetry, in contrast to the cases at smaller Froude numbers, and the jet undergoes 'tilting' from one side to the other. This may explain why internal waves propagate only at the low frequency. Moreover, the fluid falling from the fountain goes deep to the lower layer of stratification and then bounces back creating additional disturbances in the thermocline which might result in a frequency shift. This is more relevant for the thin thermocline where we see a more pronounced shift of the frequency of the internal waves from the lower peak in the spectra of the jet oscillations (Fig. 12a).

In case of the thin thermocline, the frequency of the higher peak in the spectra of the internal waves decreases from $0.5N_{\max}$ to $0.3N_{\max}$. For the thick thermocline the peak in the spectrum of the internal waves corresponds to the lower peak in the jet oscillations spectrum and is close to $0.7N_{\max}$ for all the simulations. The latter is consistent with the results of the experiment by Ezhova et al. (2012) where the oscillations of a turbulent jet in a stratified fluid and the corresponding internal waves have been investigated. As mentioned before, the jet diameter at the thermocline entrance was of order of the thermocline thickness in these experiments corresponding to our simulations with the thick thermocline, $Fr_t \sim 1$. In the experiments, the jet oscillations are characterized by pronounced peaks close to $0.7N_{\max}$ and at the frequency close to the maximum buoyancy frequency. Internal waves have been revealed at the frequencies $0.7N_{\max}$ in agreement with our simulations.

The root mean square $\sigma$ of the isotherms both for jet oscillations and internal waves, i.e. close and far from the jet axis, is obtained from the power spectra, $\mathscr{S}(f)$,

$$\sigma = (\int \mathscr{S}(f)\, df)^{1/2}, \tag{20}$$

and used to characterize the amplitudes of the oscillations. The amplitudes of the jet oscillations and internal waves are displayed in Fig.13a versus the Froude number for both stratifications. Interestingly, the amplitudes of the jet oscillations and of the internal waves are close to each other in both cases, although the work against gravity force to obtain the same amplitude is larger in the thin thermocline as the density gradient is higher. This suggests that the waves are transmitted more effectively in the case of the thin thermocline, probably due to the fact that for the thick thermocline the wave frequency is close to the maximal buoyancy frequency.

The amplitude of the jet oscillations follows the stationary solution to the Landau equation

$$\frac{d\sigma}{dt} = \sigma(\mu(Fr_t - Fr_{t0}) - \beta\sigma^2) \tag{21}$$

describing the soft excitation of self-sustained oscillations ($\mu$ and $\beta$ are free parameters here) with $Fr_{t0} = 0.4$, $\mu/\beta = 0.42$ based on the best fit of the experimental data. This is consistent with the experimental and numerical results obtained for a jet interacting with a pycnocline (Troitskaya et al. 2008; Druzhinin and Troitskaya 2013). The investigation of the stability of the experimentally measured velocity profiles of the fountain in the pycnocline by Troitskaya et al. (2008);

Ezhova and Troitskaya (2012) reveals a finite region of absolute instability along the jet, thus fulfilling a necessary condition for self-sustained oscillations of the flow. It has been demonstrated that the frequency of self-sustained oscillations is in agreement with the results of the linear stability analysis of the flow in the thermocline. The present simulations for the thick thermocline follow the experimental setup of Ezhova and Troitskaya (2012) and the LES results are consistent with the experiment. Hence, we can conclude that the generation of internal waves results from the self-sustained excitation of the jet oscillations in the thermocline.

We investigate the vertical structure of the internal waves and quantify their energetics. The energy flux of the internal waves in the presence of an inhomogeneous horizontal flow, as we have in this case because of the horizontal gravity flow, is calculated following Kamenkovich and Monin (1978). The equations of motion linearised around a horizontal mean flow in cylindrical coordinates are:

$$\frac{Du'_r}{Dt} + u'_z \frac{dU_{hor}}{dz} + \frac{1}{\rho_0}\frac{\partial p'}{\partial r} = 0, \tag{22}$$
$$\frac{Du'_\phi}{Dt} + \frac{1}{\rho_0}\frac{1}{r}\frac{\partial p'}{\partial \phi} = 0, \tag{23}$$
$$\frac{Du'_z}{Dt} + g\frac{\rho'}{\rho_0} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} = 0, \tag{24}$$
$$g\frac{D(\rho'/\rho_0)}{Dt} - N^2(z)u'_z = 0, \tag{25}$$
$$\nabla \cdot \vec{u'} = 0, \tag{26}$$

where $D/Dt = \dfrac{\partial}{\partial t} + U_{hor}\dfrac{\partial}{\partial r}$.

The equation for energy conservation can be obtained by multiplying Eq. (22) with $u'_r$, Eq. (23) by $u'_\phi$, Eq. (24) by $u'_z$ and summing. From Eq. (25), taking into account that $u'_z = \dfrac{D\xi}{Dt}$, we find that $g\rho' = \rho_0 N^2(z)\xi$ (where $\xi$ is the vertical displacement of a fluid particle).

Finally, the equation of the wave energy conservation reads:

$$\frac{\partial E}{\partial t} + \nabla \vec{F} = -I, \tag{27}$$

where the wave energy $E$, the energy flux $\vec{F}$ and the production/dissipation term $I$ are

$$E = \frac{1}{2}\rho_0(\vec{u'}^2 + N^2\xi^2), \tag{28}$$

$$\vec{F} = \vec{U}_{hor}E + \vec{u'}p', \tag{29}$$

$$I = \rho_0 u'_r u'_z \frac{dU_{hor}}{dz}. \tag{30}$$

The term $I$ describes the interaction of the mean flow with the wave. From equation (27) it follows that the integral wave energy flux is not conserved due to this term. In the present configuration, waves can grow or decay in space where $\dfrac{\partial E}{\partial t}$ is zero at statistically steady state.

The surface-integrated value of the wave energy flux at the distance $r$ from the jet axis is normalized with the energy flux of the jet at the thermocline entrance,

$$\frac{F}{F_{jet}} = \frac{R\int_{z_1}^{z_2}\left(\frac{1}{2}\left(\vec{u'}^2 + N^2\xi^2\right)U_{hor} + \frac{p'u'_r}{\rho_0}\right)dz}{\frac{1}{2}\int_0^\infty {U_t}^3 r dr}. \tag{31}$$

We measure the profiles of the energy flux at 4 radial points and averaged them to get the final profile. The inflow energy flux is taken at the level $z = 18$. The profiles $\dfrac{F}{F_{jet}}$ pertaining the thick and thin thermocline at the distances $r = 20$ and $r = 25$ are shown in Fig. 14. It can be seen that the energy does not only decay with the distance from the jet centre, but the profiles are also deformed, especially in the areas affected by the shear due to the horizontal flow, presumably due to the enhanced decay resulting from the interaction with the mean flow.

The surface-integrated wave energy flux normalized with the jet energy flux is displayed in Fig. 15 versus the Froude number. The difference between the values at $r = 20$ and $r = 25$ illustrates the difference in the decay of the energy of the internal waves due to the interaction with the mean flow. Note that the wave energy flux is around 4-5% of the energy of the jet for the thin thermocline, and is almost half for the thick thermocline. This can be partly explained by the fact that the counterflow in the upper thermocline transfers energy in the opposite direction, i.e. towards the jet. The whole flux, however, is always positive. Note also the jump in the energy flux for the largest Froude number when the horizontal flow occupies more space in the thermocline preventing the transfer of energy towards the jet.

We finally comment on the difference in the velocities of wave propagation. As explained above, transients in the simulations are very different when changing the thickness of the thermocline, and the difference in the domain size (40 from the jet center to the lateral boundary along the axes for the thin thermocline, and 47 for the thick thermocline) is too small to explain this. The fact that the internal waves are significantly slower in the thick thermocline can be related to the dispersion properties of the internal waves. The dispersion relation for the waves $\Psi \sim \psi(z)e^{-i\omega t+ikr}$ in a stratified medium is obtained from the solution of the eigenvalues of the Taylor-Goldstein equation

$$\frac{d^2\psi}{dz^2}+\left(\frac{N^2}{(U_{hor}-c)^2}-\frac{(U_{hor})''_{zz}}{U_{hor}-c}-k^2\right)\psi=0, \quad \psi(H_d)=\psi(H_u)=0, \tag{32}$$

where $\psi$ is a stream function, $N$ the buoyancy frequency, $U_{hor}$ the mean horizontal velocity which depends on the vertical coordinate $z$, $c=\omega/k$ the wave phase velocity ($\omega$ is the wave frequency, $k$ is the wave number), and $z=H_u,H_d$ denote the locations of the upper and lower boundaries (eq. (32) is made non-dimensional with $U_0$ and $D_0$). This eigenvalue problem is solved numerically for the stratification profiles and the horizontal velocities extracted from the simulations.

The group velocity, $c_g=d\omega/dk$, of the first (fundamental) mode of the internal waves is displayed in Fig.13b for the Froude numbers $Fr=0$ (no flow) and $Fr=13$ (with horizontal flow) and both stratifications. Similar conclusions apply to higher modes. Note the significant change in the group velocities in the presence of the horizontal flow due to the jet intrusion at the level of neutral buoyancy. In particular, we find waves with frequencies higher than the maximal buoyancy frequency that propagate with group velocities approaching the maximal velocity of the horizontal gravity flow. The largest fluctuations associated to these modes are localized in the flow as compared to the lower frequency modes localized in the thermocline. For the internal waves at $Fr = 13$ (see the frequencies in Table 2) we estimate the group velocities of the fundamental modes to be $c_{gr} = 0.04$ for the thick thermocline and $c_{gr} = 0.08$ for the thin thermocline, which explains the difference in time needed to reach a statistically steady state, $t_{d,thin} = 1000$ and $t_{d,thick} = 1800$.

## 4. Conclusion

We have presented the results of numerical simulations of a turbulent jet interacting with a thermocline in an unconfined stratified medium. Two stratifications have been modelled: a thin and a thick thermocline, with thickness smaller and of the order of the jet diameter at the thermocline entrance. The simulations have been performed for 5 Froude numbers in each stratification, ranging between 0.6 and 1.9, values typical of engineering and geophysical flows, such as submerged buoyant jets from wastewater outfalls.

We show that the jet mean penetration height can be well predicted from the conservation of the source energy of a turbulent jet in a thermocline (Kaye and Hunt 2006), valid for weak fountains.

The entrainment flux in the thin thermocline, related to the turbulent mixing of the jet with the surrounding medium, is consistent with the theoretical model developed for the case of a jet impinging at a sharp interface. At small Froude numbers, the entrainment is more effective in the thick thermocline, but already at $Fr_t \approx 1$ the fluxes become equal for both stratifications. There is an important difference, however, in the average secondary flows for the two stratifications. For the thin thermocline the entrainment velocity is approximately the same around the 'dome' formed by the jet penetrating through the thermocline. The entrainment in the thick thermocline, instead, is mostly from the sides of the 'dome' due to a pronounced horizontal flow in the upper thermocline, with only a small part of fluid coming from the upper stratification layer. This difference is observed over the whole range of Froude numbers investigated here, even in the case when the jet penetrates through the thick thermocline.

The fountain formed by the jet penetrating into the thermocline oscillates generating internal waves. The amplitudes of the jet oscillations grow with the Froude number as $Fr_t^{1/2}$ corresponding to the regime of soft self-excitation of the flow. We find two peaks in all the spectra: for the smaller Froude numbers (up to $Fr_t \approx 1$) the peak at the higher frequency is rather weak as compared to the second one, while at the larger Froude numbers they are comparable. The frequencies of the jet oscillations at fixed $Fr_t$ are basically the same for both stratifications. These oscillations generate internal waves. The frequencies of the internal waves depend also on the dispersion properties of the stratified medium and oscillations at frequency exceeding maximal buoyancy frequency are not found in the spectra of internal waves. Therefore, at the lower Froude numbers both peaks are present in the spectra of internal waves in case of the thin thermocline while only one peak in the thick thermocline.

At the higher Froude numbers there is one pronounced peak in the spectra of internal waves corresponding approximately to the lower peak in the spectra of jet oscillations. This is consistent with the results of the laboratory experiments of Troitskaya et al. (2008) and Ezhova et al. (2012), corresponding to our simulations with the thick thermocline at $Fr_t \approx 1$.

The energy flux of internal waves at the thermocline entrance is estimated to be around 4-5% of the jet energy for the thin thermocline at the distance $r = 20$ from the jet center, and almost half for the thick thermocline, except for the largest Froude number, $Fr_t = 1.89$, when the fluxes are equal. The energy profiles and estimates of the energy flux at the distance $r = 25$ show that internal waves are significantly influenced by the horizontal gravity flow.

We finally make some remarks regarding a possible application of the present numerical results to the wastewater outfall system. As in the scale laboratory modelling of the real system (Troitskaya et al. 2008), we have observed the jet oscillations resulting in the generation of internal waves at a frequency close to $0.7N_{\max}$ for the thick thermocline cases. These waves can be rather strong with an average amplitude up to 20% of the thermocline thickness (40% peak-to-peak) at the distance of 5 thermocline thicknesses from the source. The seasonal change of the pycnocline, as we briefly discussed in the Introduction, is characterized primarily by its sharpening and its transition closer to the surface. Therefore, at the entrance to the thermocline, the diameter of the jet increases and the vertical velocity decreases, i.e. increasing $\gamma_t$ and decreasing the Froude number at the thermocline entrance. Hence we expect that in summer, due to the lower $Fr_t$ and amplitude, the internal waves, albeit closer to the free surface, generate less mixing and the entrainment at the top of the jet be less effective than in winter. The waste water effluent will be located closer to the free surface and its dilution will be reduced in summer, presenting a larger threat than in winter when a more effective entrainment and larger amplitude internal waves will contribute to the dilution of the effluent trapped further away from the free surface. Better dilution is expected either due to the possible wave breaking or due to the effect of enhancement of turbulence in the field of a non-breaking internal wave (Matusov et al. (1989), Druzhinin and Ostrovsky (2015)).

This study focuses on turbulent jets generated from a momentum source of fluid of the same density as the surrounding ambient fluid. The investigation of the applicability of the results discussed in this paper to a turbulent plume with a finite buoyancy flux is underway. However, if the results presented hold for a plume with a finite buoyancy flux, we expect that the different stratification observed in summer and winter in tidewater glaciers in Greenland (Straneo et al. 2011) will influence dramatically the formation and propagation of internal waves in this setting. In particular, in winter, the interface between the top and bottom layers in some of Greenland's fjords is sharp and thinner than in summer (Straneo et al. 2011) and the buoyant plumes forming at the glacier face due to submarine melting are weaker due to a low (or absent) subglacial discharge. Hence, we expect the lower $Fr_t$ and thinner interface observed in winter to generate low amplitude internal waves with two spectral peaks and the entrainment at the top of the plume to be less effective than in summer when the Froude number is larger. Additionally in summer, given the larger $Fr_t$ and thicker interface, the buoyant plumes interacting with the interface are expected to generate large amplitude internal waves which can possibly break and contribute to the dilution of the meltwater plume intruding horizontally at the interface.

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# APPENDIX

## Convergency test and additional validations

We investigate the sensitivity of the simulations to the size of spectral elements. For this aim we perform an additional simulation for $Fr = 13$, thin pycnocline, with an increased resolution. We left the size of the well-resolved initial region at the inflow ($4 \times 4 \times 10$ spectral elements) unaffected in order to keep the same velocity perturbations and obtain the turbulent jet with the same characteristics. However, we reduce stretching factor to have 2 times smaller elements at

$|x| = |y| = 10$ and reduced the stretching factor along $z$-axis to have twice more elements in the thermocline. The meshes for both cases are shown in Fig. A1a,b displaying as an example the instantaneous temperature fields for both simulations.

Fig. A1d,e shows the average velocity fields together with the thermocline boundaries for both cases indicating good correspondence. The jet centerline velocity as the function of the vertical coordinate illustrating mean jet penetration is shown in Fig. A1c. The entrainment flux for the test case is $E_{i,test} = 0.36$ as compared to the regular grid with $E_i = 0.37$. Thus we may conclude that the simulations converge and the calculations are resolved enough to get reliable results.

One can investigate the influence of reflections from the boundaries comparing the internal waves measured at the distances $r = 20$ and $r = 25$. We expect to get the weaker signal at $r = 25$ and delay with respect to $r = 20$.

The examples of the isotherms for the largest Froude numbers, as the most critical case for reflections, are shown in Fig. A2 for both stratifications. Fig. A2 displays also the averaged spectra of internal waves measured at $r = 20$ and $r = 25$ (averaging performed over 8 realizations as explained in Sec. 2 b). It can be seen that the signals at $r = 25$ follow the signals at $r = 20$. The average spectra of the isotherms are similar but the peak for $r = 25$ is lower thus confirming the absence of reflections from the boundaries at $r = 20$ where we measure internal waves.

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TABLE 1. Parameters of the simulations of a jet impinging on a thin or thick thermocline. The nominal Froude number is $Fr = U_0/\sqrt{g'D_0}$, while the thermocline Froude number $Fr_t = u_t/\sqrt{g'R_t}$ uses the jet mean radius and velocity at $z = 18$ (for comparison with Shrinivas and Hunt (2014, 2015)), $\gamma = D_0/H$ indicates the inverse thickness of the thermocline.

| $Fr$, $Fr_t$ | $\gamma = D/H$ | Domain Size | N. Spectral Elements | N. of Grid Points |
|---|---|---|---|---|
| 7 (0.60) | 2 | $80 \times 80 \times 31$ | $38 \times 38 \times 36$ | 26 615 808 |
| 10 (0.86) | 2 | $80 \times 80 \times 31$ | $38 \times 38 \times 36$ | 26 615 808 |
| 13 (1.11) | 2 | $80 \times 80 \times 32$ | $38 \times 38 \times 38$ | 28 094 464 |
| test 13 (1.37) | 2 | $80 \times 80 \times 32$ | $48 \times 48 \times 45$ | 53 084 160 |
| 16 (1.37) | 2 | $80 \times 80 \times 33$ | $38 \times 38 \times 42$ | 31 051 776 |
| 22 (1.89) | 2 | $95 \times 95 \times 37$ | $40 \times 40 \times 52$ | 42 598 400 |
| 7 (0.60) | 0.5 | $95 \times 95 \times 32.5$ | $40 \times 40 \times 33$ | 27 033 600 |
| 10 (0.86) | 0.5 | $95 \times 95 \times 33.5$ | $40 \times 40 \times 35$ | 28 672 000 |
| 13 (1.11) | 0.5 | $95 \times 95 \times 34.5$ | $40 \times 40 \times 36$ | 29 491 200 |
| 16 (1.37) | 0.5 | $95 \times 95 \times 35.5$ | $40 \times 40 \times 37$ | 30 310 400 |
| 22 (1.89) | 0.5 | $95 \times 95 \times 40.5$ | $40 \times 40 \times 45$ | 36 364 000 |

TABLE 2. Frequencies of jet oscillations and internal waves in the thin and thick thermoclines.

| $Fr(Fr_t)$ | $\hat{f}_{1jet}$ thin | $\hat{f}_{2jet}$ thin | $\hat{f}_{1jet}$ thick | $\hat{f}_{2jet}$ thick | $\hat{f}_{1IW}$ thin | $\hat{f}_{2IW}$ thin | $\hat{f}_{1IW}$ thick |
|---|---|---|---|---|---|---|---|
| 7 (0.60) | 0.0096 | 0.0179 | 0.0098 | 0.0177 | 0.0100 | 0.0173 | 0.0083 |
| 10 (0.86) | 0.0072 | 0.0146 | 0.0068 | 0.0152 | 0.0078 | 0.0135 | 0.0063 |
| 13 (1.11) | 0.0038 | 0.0082 | 0.0042 | 0.0084 | 0.0040 | 0.0073 | 0.0045 |
| 16 (1.37) | 0.0024 | 0.0055 | 0.0026 | 0.0054 | 0.0037 | - | 0.0030 |
| 22 (1.89) | 0.0017 | 0.0048 | 0.0021 | 0.0048 | 0.0023 | - | 0.0025 |

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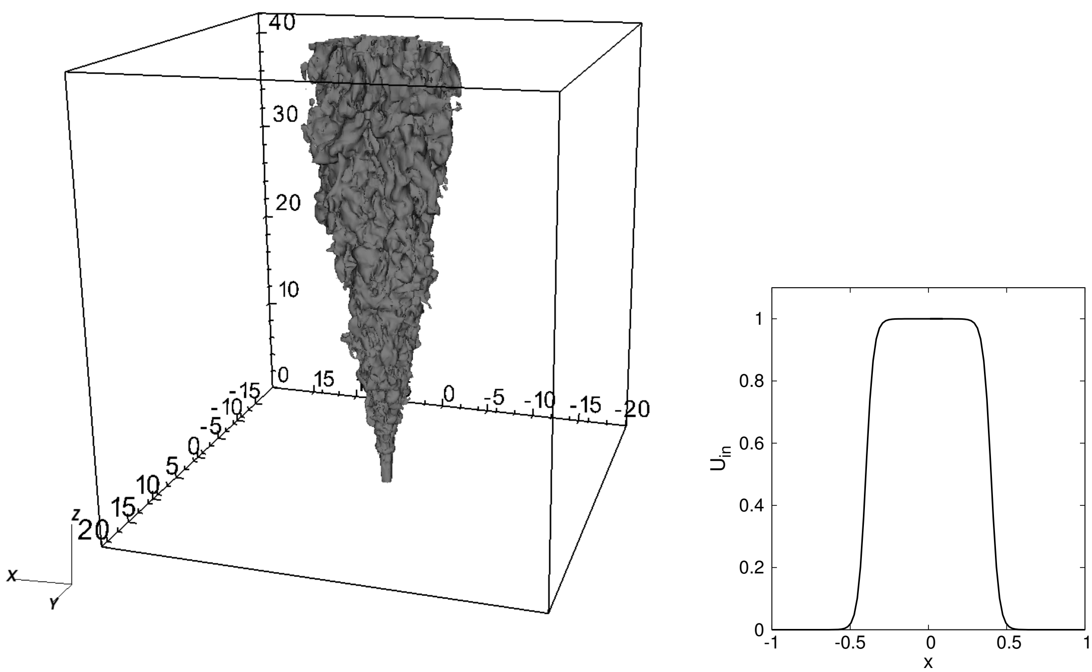



FIG. 1. Left panel: domain configuration for the turbulent jet in a homogeneous fluid. The jet is shown by the contour surface of vertical velocity $u_z = 0.03$. Right panel: an inflow velocity profile.

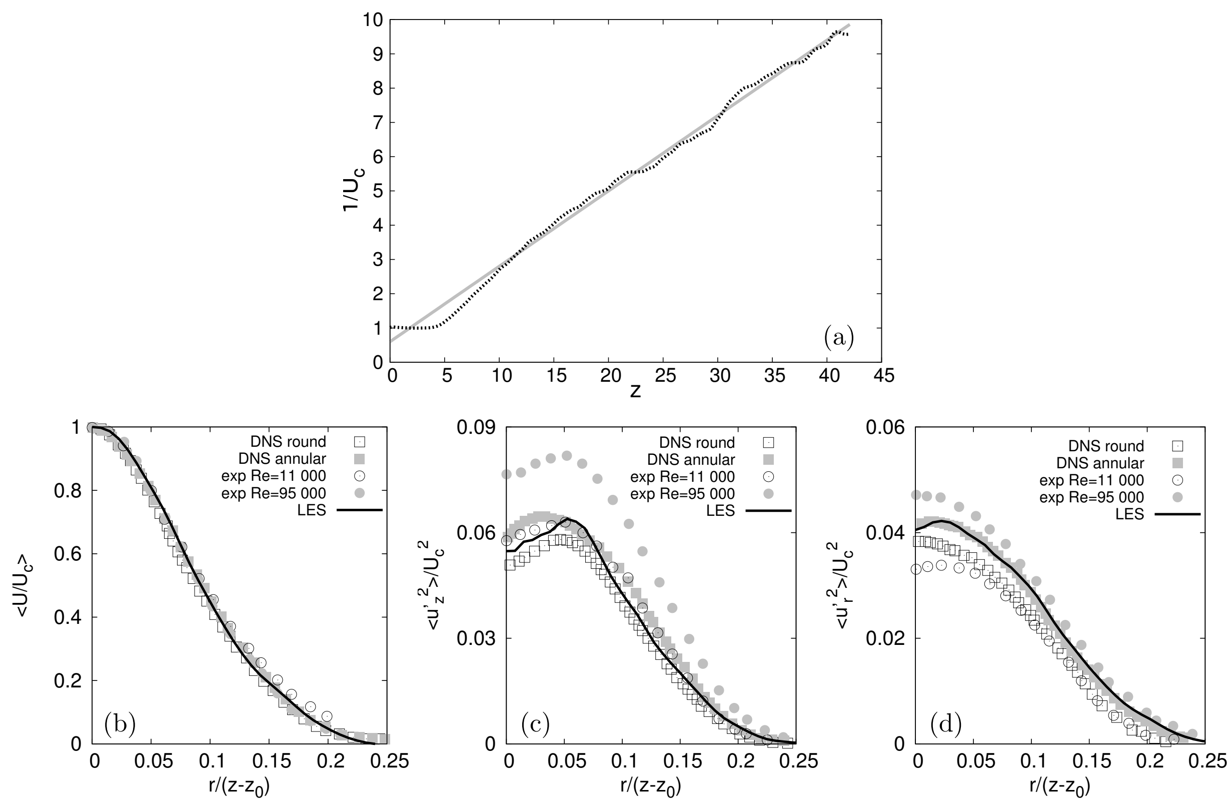



FIG. 2. (a) Inverse mean centerline velocity as a function of the distance from the nozzle (black dots - LES data, gray - theory). (b) The far-field $z$-velocity profile, (c) turbulent stresses $< u_z'^2 > /U_c^2$ and (d) $< u_r'^2 > /U_c^2$. The data for comparison are available from the following papers: 'DNS round' - Picano and Casciola (2007), 'DNS annular' - Picano and Hanjalic (2012), 'exp Re=11 000' - Panchapakesan and Lumley (1993), 'exp Re=95 000' - Hussein et al. (1994).

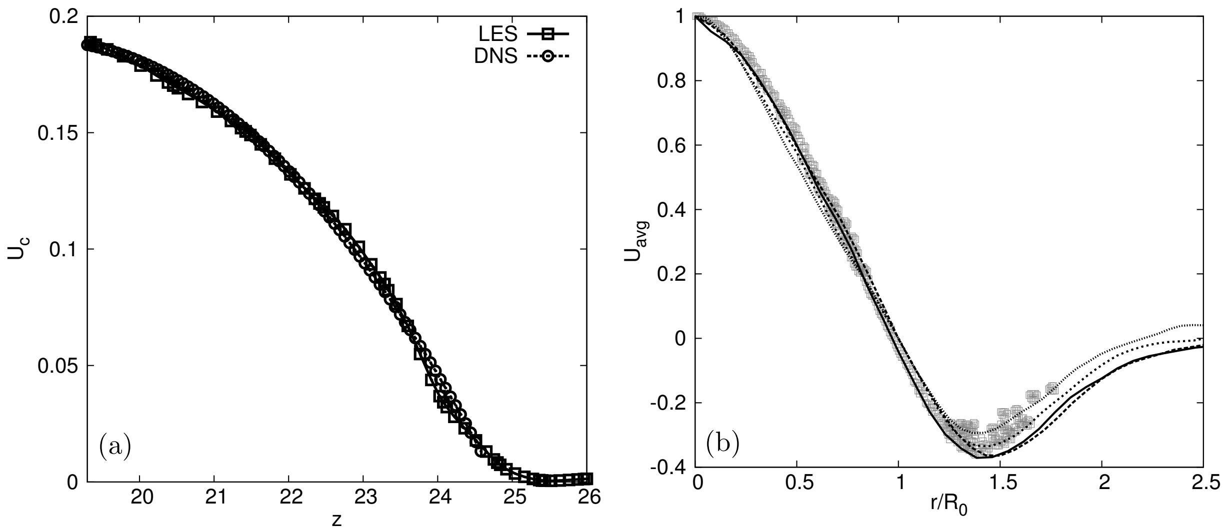



FIG. 3. Comparison of present LES data with direct numerical simulations (DNS) and experiments. (a) Mean axial vertical velocity versus $z$ against the DNS data for weak fountains by Lin and Armfield (2000). (b) Mean vertical velocity profiles in the cross-sections along the jet axis in the thermocline (curves) against the experimental data on turbulent jets in a stratified fluid by Ezhova and Troitskaya (2012) (symbols).

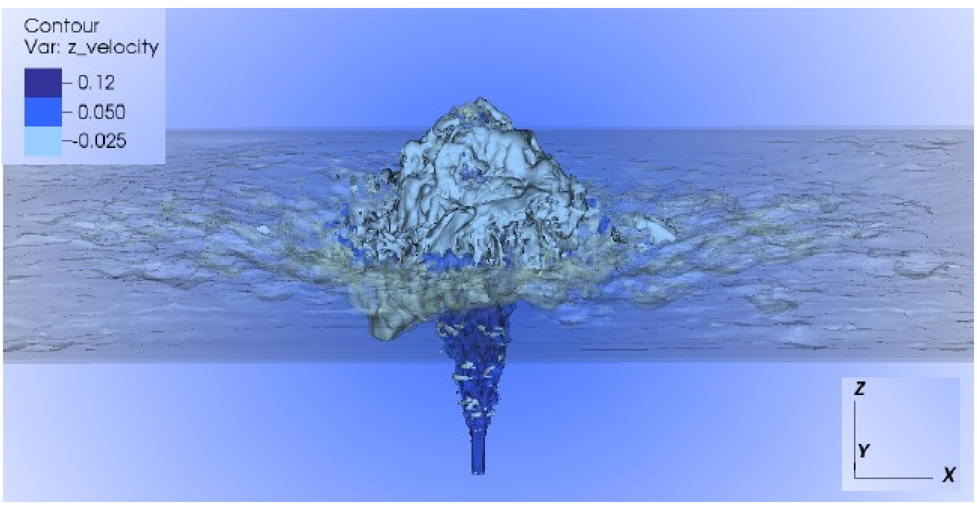



FIG. 4. Illustration of the jet in a stratified fluid by surfaces of constant vertical velocity and temperature for the thick thermocline, $Fr = 22$ ($Fr_t = 1.89$). Waves are visualized by surfaces of constant temperature $T = 0.03$ and $T = 0.97$.

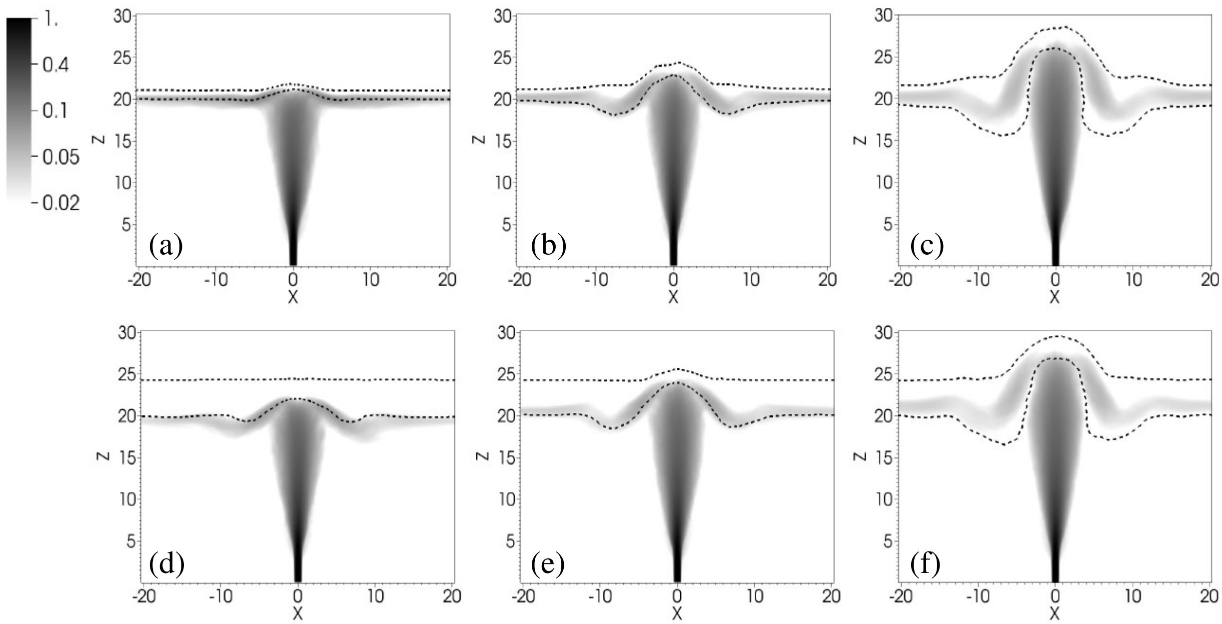



FIG. 5. Magnitude of the mean flow velocity for $Fr$=7 ($Fr_t = 0.6$) (a,d), 13 (1.11) (b,e), 22 (1.89) (c,f) (upper panel - thin thermocline, lower panel - thick thermocline). Dashed curves correspond to the contour lines of temperature $T = 0.1$ and $T = 0.9$.

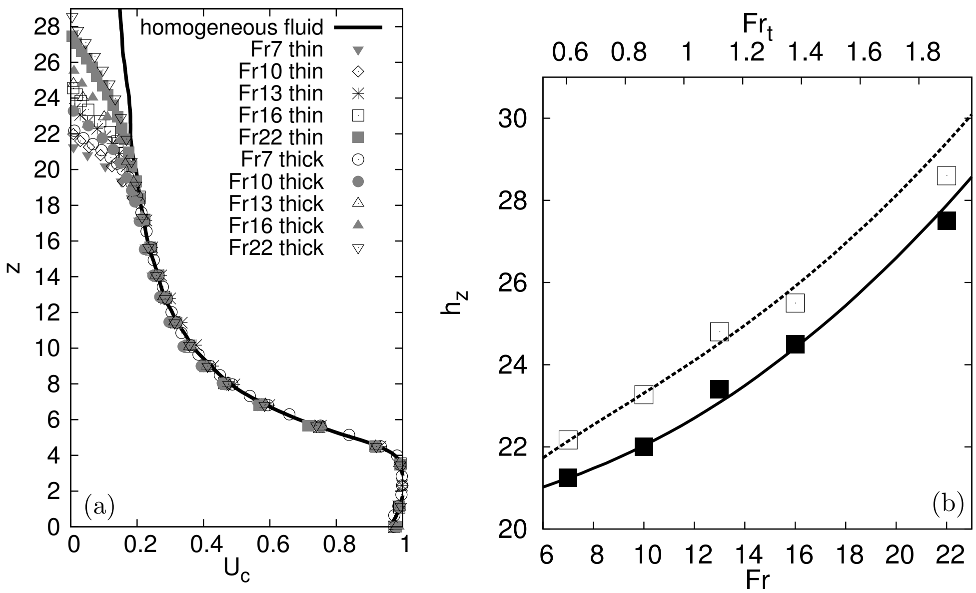



FIG. 6. (a) Mean centerline velocities for the jet in a homogeneous fluid and for all the simulations in the stratified media indicating mean jet penetration. (b) Theoretical prediction of the mean penetration height versus the Froude number: $\gamma = 2$ (solid), $\gamma = 0.5$ (dashed). Symbols ■ and □ denote the mean heights obtained in the simulations for $\gamma = 2$ and $\gamma = 0.5$ respectively.

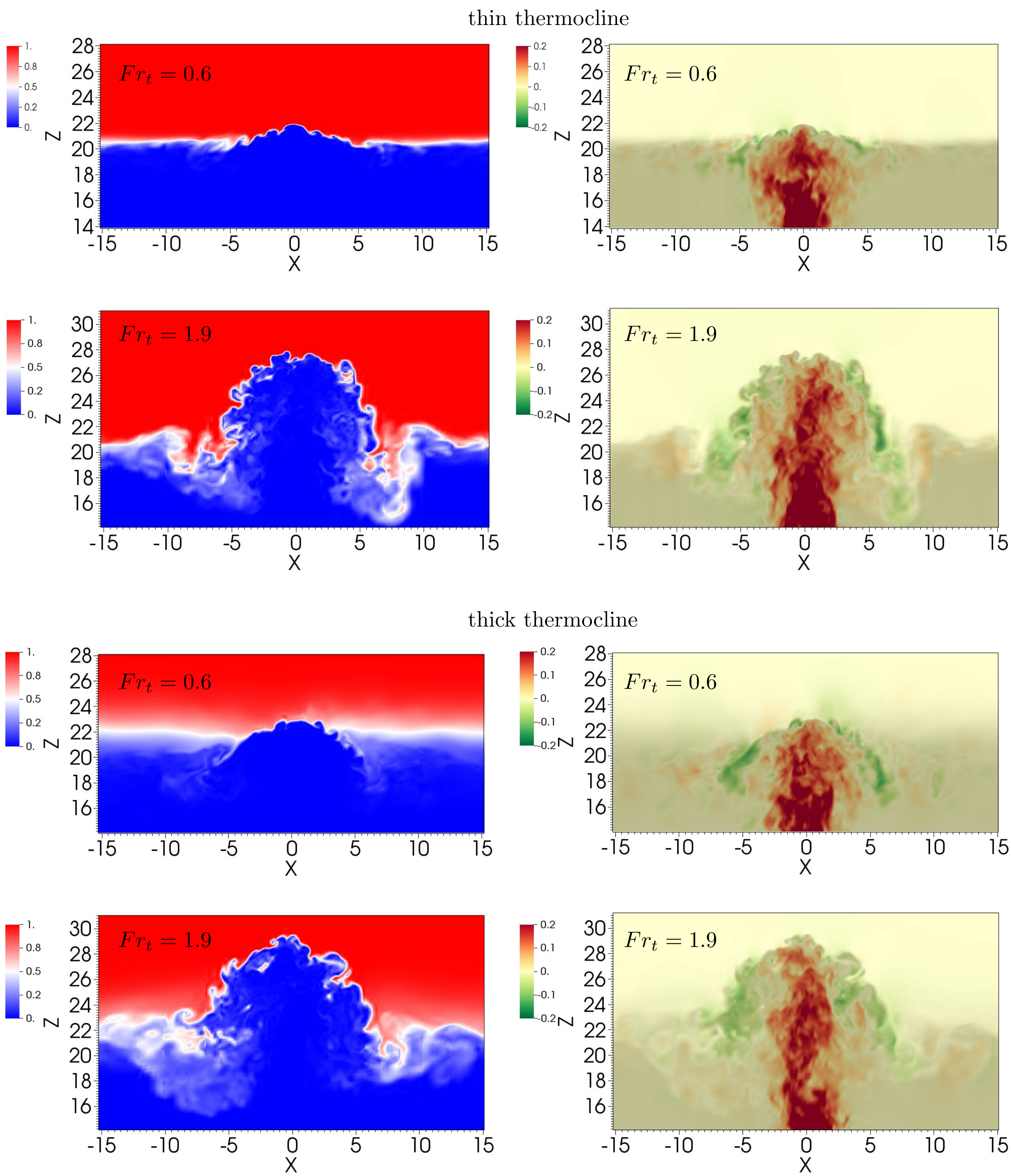



FIG. 7. The instantaneous temperature (left) and vertical velocity (right) fields in the thermocline for both stratifications.

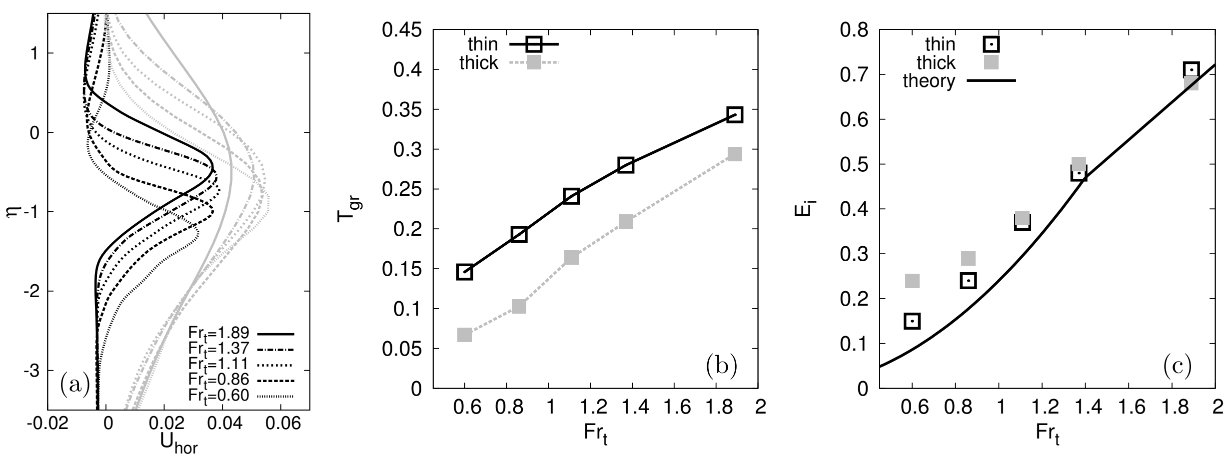



FIG. 8. (a) Velocity profiles of the gravity currents propagating at the level of neutral buoyancy at a distance $r = 20$ from the jet axis (gray curves - thin thermocline, black curves - thick thermocline). (b) Average temperature of the gravity current as a function of the thermocline Froude number. (c) Entrainment flux obtained from (17) as a function of the thermocline Froude number.

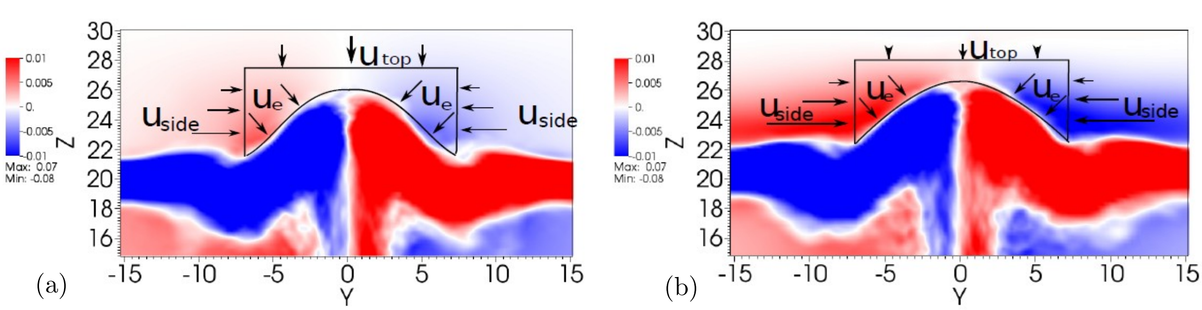



FIG. 9. Mean horizontal velocity fields in the jet impinging on the thermocline for $Fr = 16$ ($Fr_t = 1.37$): (a) - thin thermocline, (b) - thick thermocline.

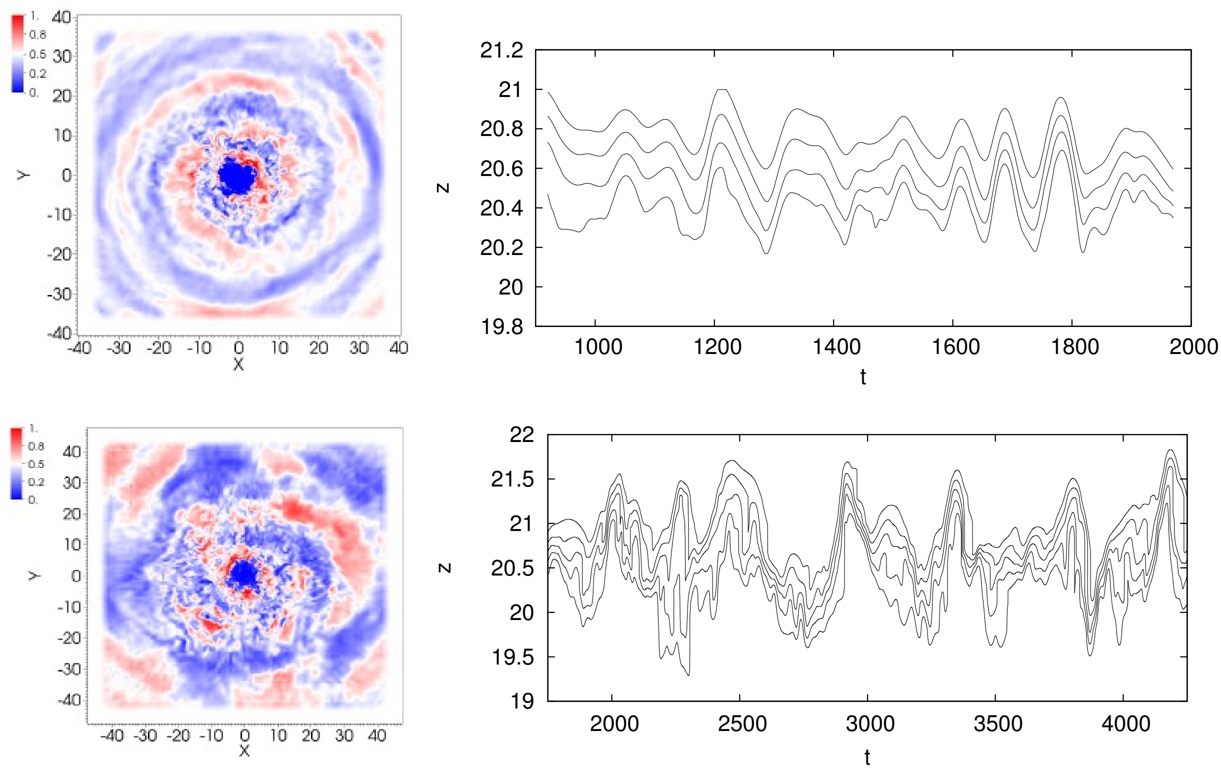



FIG. 10. Instantaneous temperature field in the horizontal plane at the center of the thermocline and time history of the isotherms at distance $r = 20$ from the jet center. The data in the upper panel pertain the simulation of the thin thermocline with $Fr = 10$ ($Fr_t = 0.86$) (isotherms corresponding to temperatures from $T = 0.4$ to $T = 0.7$) and at the lower panel - the simulation with the $Fr = 22$ ($Fr_t = 1.89$) (isotherms from $T = 0.3$ to $T = 0.7$).

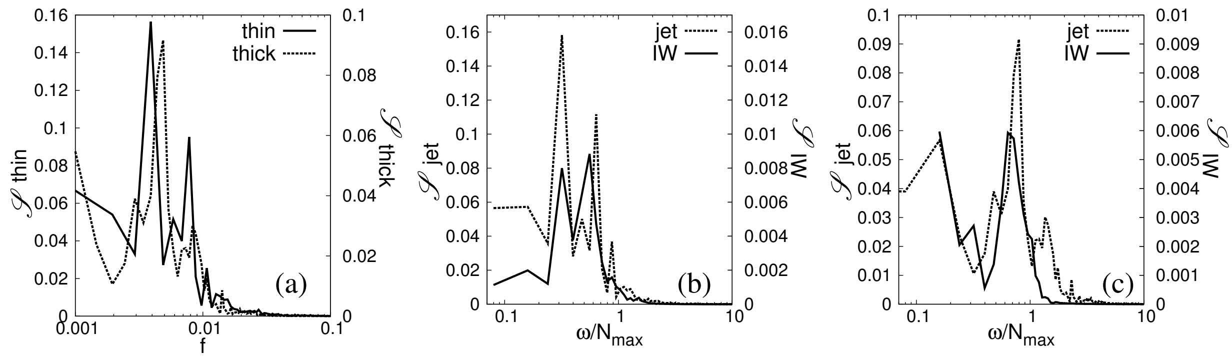
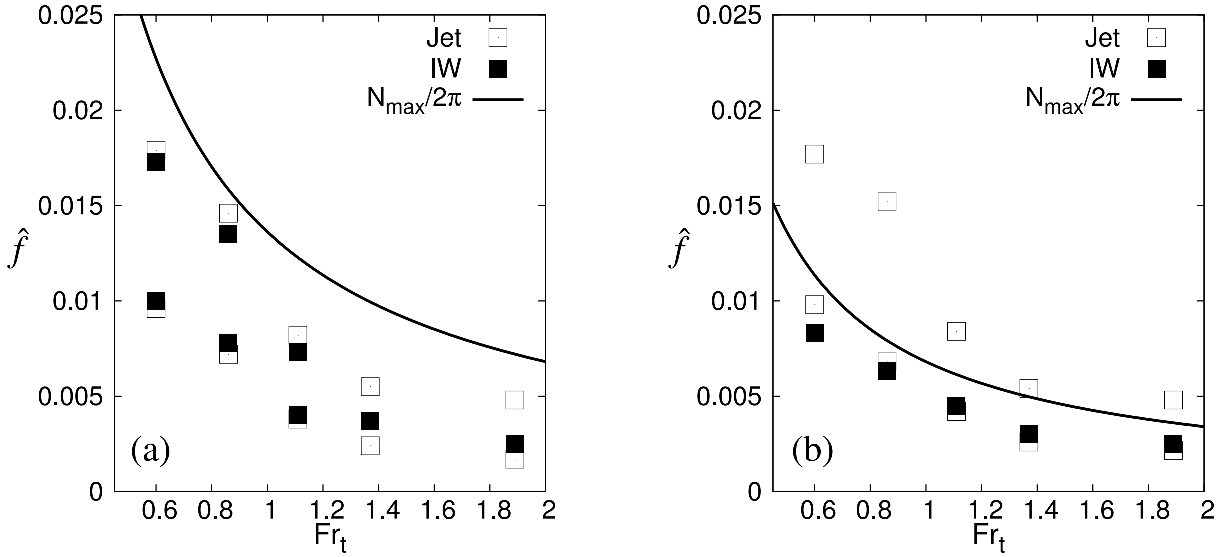



FIG. 11. (a) The spectra of jet oscillations in the thin (solid) and thick thermocline (dashed) for $Fr = 13$ ($Fr_t = 1.11$). (b) The spectra of jet oscillations (dashed) and internal waves (solid), thin thermocline. (c) The spectra of jet oscillations (dashed) and internal waves (solid), thick thermocline.




935 FIG. 12. The frequencies of the jet oscillations and internal waves as the functions of the Froude number: (a) thin thermocline, (b) thick thermocline. The solid curves correspond to the maximal buoyancy frequency.

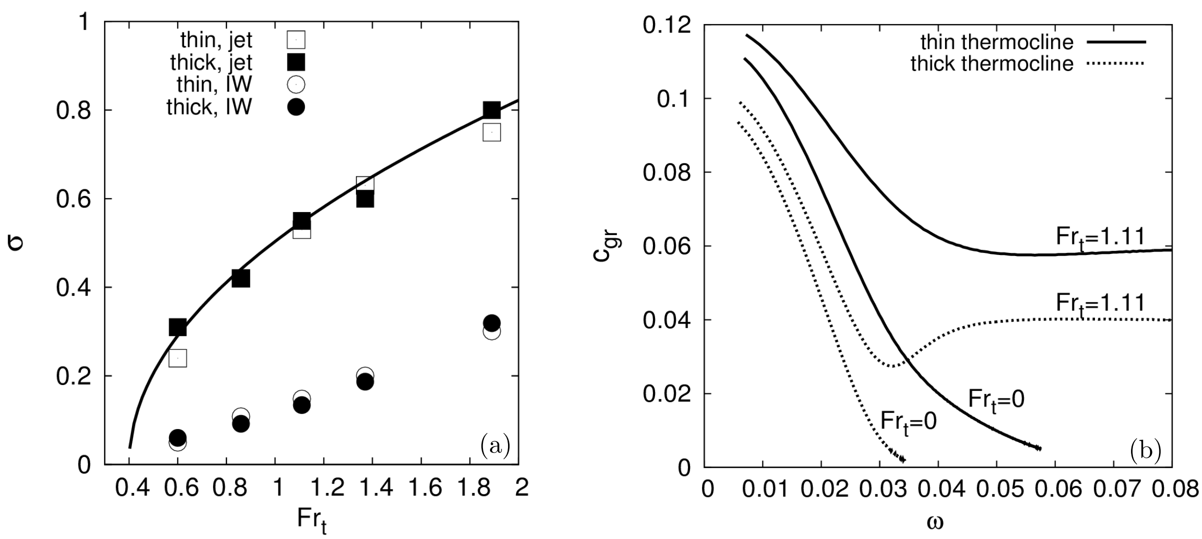



FIG. 13. (a) The amplitudes of the jet oscillations and the internal waves as function of the thermocline Froude number. Solid line represents the stationary solution of Landau equation. (b) The group velocity of the first mode of internal waves as a function of frequency.

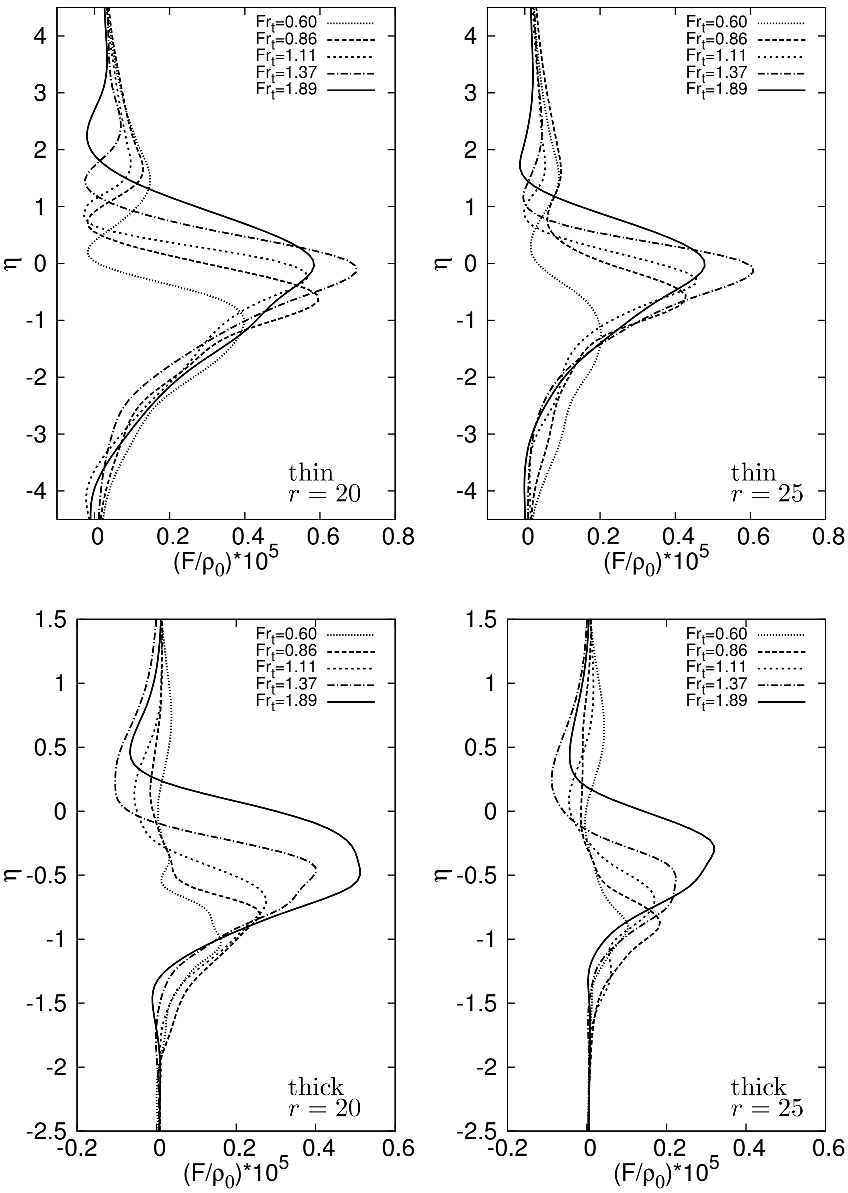



FIG. 14. Vertical profiles of the energy flux of internal waves: upper panel - thin thermocline, lower panel - thick thermocline. Left: $r = 20$, right: $r = 25$.

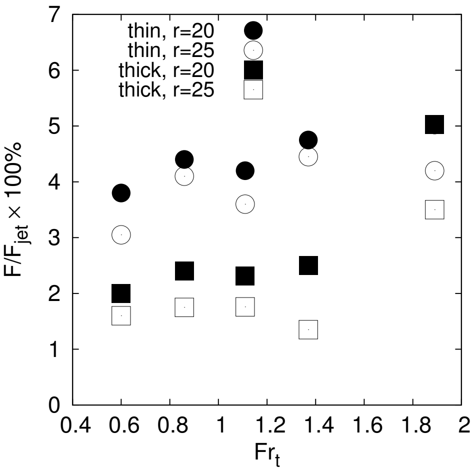



FIG. 15. The surface-integrated wave energy flux normalized with the jet energy flux at the entrance to the thermocline. The values at the largest Froude number for the distance $r = 20$ coincide.

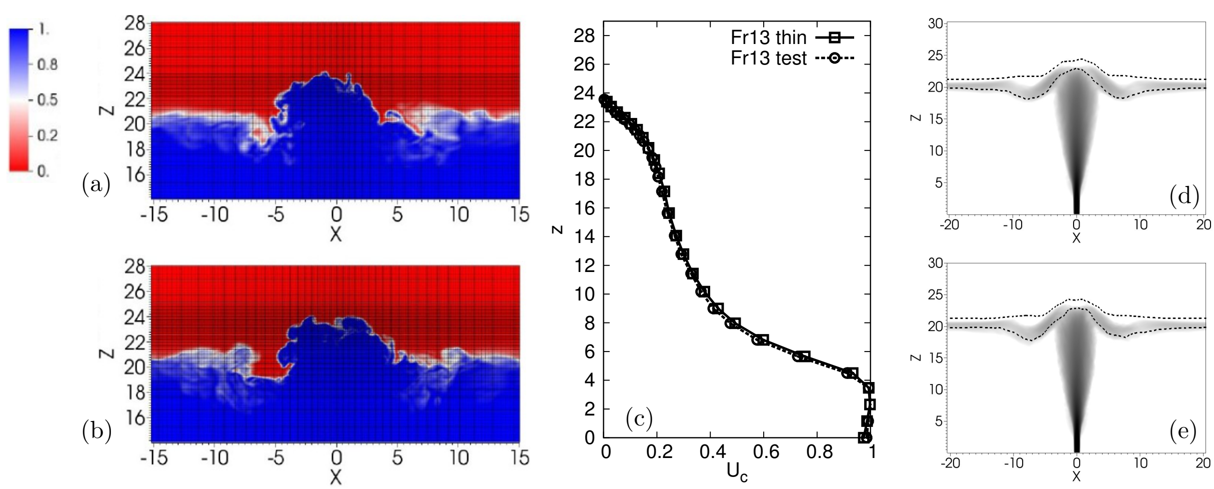



Fig. A1. The mesh and instantaneous temperature fields for $Fr = 13(Fr_t = 1.11)$, thin thermocline, in (a) the regular simulations and (b) in the validation case. (c) Average centerline velocities for the test case and in regular simulations. Average velocity magnitude: (d) in the regular simulations and (e) in the test case. Dashed curves denote the contour lines of average temperature $T = 0.1$ and $T = 0.9$.

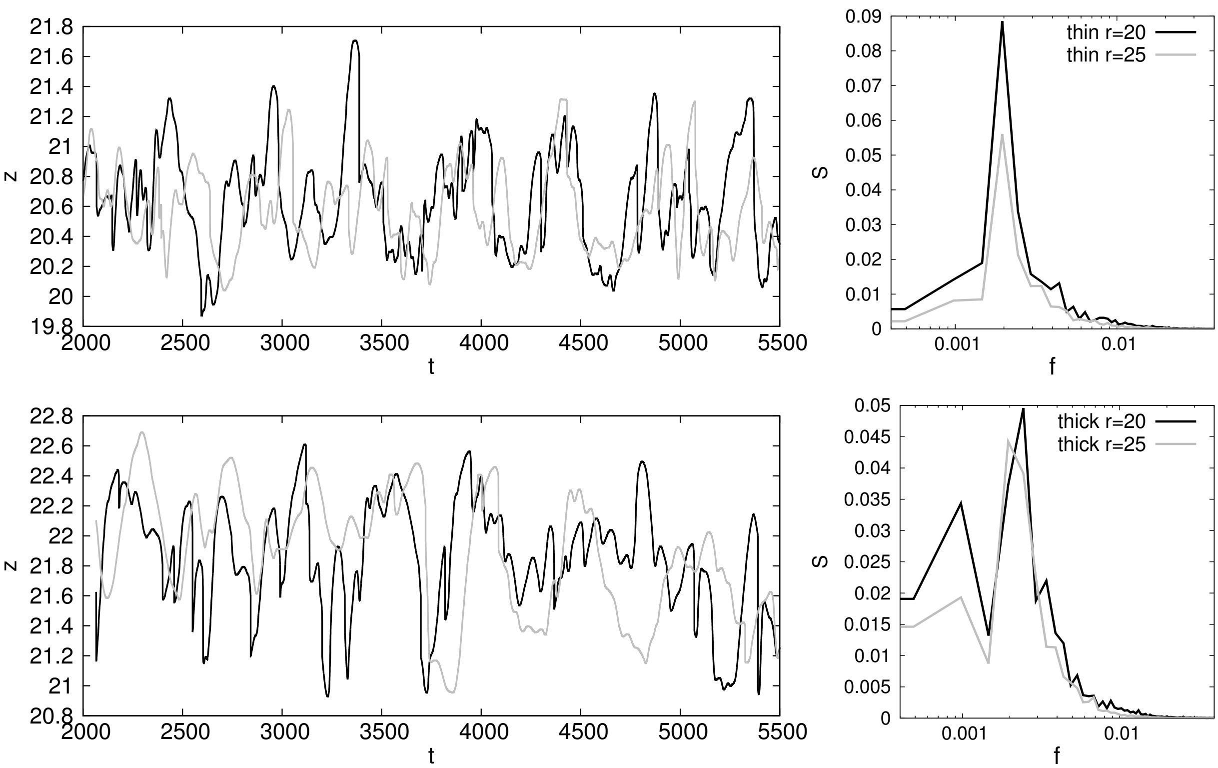



Fig. A2. The examples of the isotherms $T = 0.5$ and average spectra at $r = 20$ and $r = 25$ for the thin thermocline (upper panel) and the thick thermocline (lower panel).