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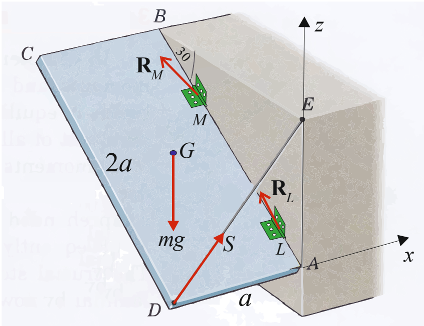
Lösningar



1) Bestäm $\mathbf{S}$:s riktning

$D(-a,0,0)$, $E(0,0,a)$, $B\left(0,\sqrt{3}a,a\right)$,

$G\left(-\frac{a}{2},\frac{\sqrt{3}a}{2},\frac{a}{2}\right)$, $\mathbf{e}_{AB}=\frac{\left(0,\sqrt{3},1\right)}{2}$

$$\Rightarrow \quad \mathbf{S}=S\mathbf{e}_{DE}=\frac{S}{\sqrt{2}}(1,0,1).$$

2) Formulera momentekvationen med avs. på *A*,

$$\mathbf{M}_A=\mathbf{r}_{AD}\times\mathbf{S}+\mathbf{r}_{AG}\times m\mathbf{g}+\mathbf{r}_{AL}\times\mathbf{R}_L+\mathbf{r}_{AM}\times\mathbf{R}_M=\mathbf{0}$$

Formulera momentekvationen med avs. på axeln genom *AB*:

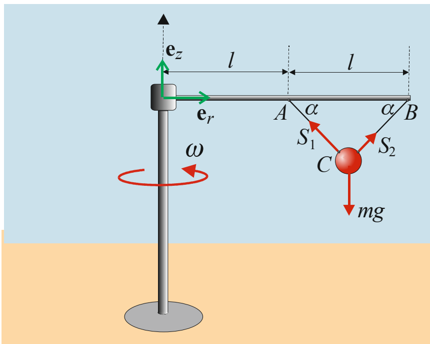
$$M_{AB}=\mathbf{M}_A\cdot\mathbf{e}_{AB}=\mathbf{r}_{AD}\times\mathbf{S}\cdot\mathbf{e}_{AB}+\mathbf{r}_{AG}\times m\mathbf{g}\cdot\mathbf{e}_{AB}+\underbrace{\mathbf{r}_{AL}\times\mathbf{R}_L\cdot\mathbf{e}_{AB}}_{0}+\underbrace{\mathbf{r}_{AM}\times\mathbf{R}_M\cdot\mathbf{e}_{AB}}_{0}=\mathbf{0}$$

$$\frac{S}{\sqrt{2}}\begin{vmatrix}\mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z\\ -a & 0 & 0\\ 1 & 0 & 1\end{vmatrix}\cdot\frac{\left(0,\sqrt{3},1\right)}{2}+\begin{vmatrix}\mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z\\ -\frac{a}{2} & \frac{\sqrt{3}a}{2} & \frac{a}{2}\\ 0 & 0 & -mg\end{vmatrix}\cdot\frac{\left(0,\sqrt{3},1\right)}{2}=0,$$

$$\frac{S}{\sqrt{2}}(0,a,0)\cdot\frac{\left(0,\sqrt{3},1\right)}{2}+\left(-\frac{\sqrt{3}}{2}mga,-\frac{mga}{2},0\right)\cdot\frac{\left(0,\sqrt{3},1\right)}{2}\quad\Rightarrow\frac{\sqrt{3}}{2\sqrt{2}}Sa-\frac{\sqrt{3}}{4}mga=0\Rightarrow$$

$$\underline{\underline{S=\frac{mg}{\sqrt{2}}}}$$

2)



1) Formulera kraftekvationen i cylinderkoordinater

$$\mathbf{e}_r:\quad m\left(\ddot{r}-r\dot{\theta}^2\right)=\left(-S_1+S_2\right)\cos\alpha$$

$$\mathbf{e}_z:\quad 0=\left(S_1+S_2\right)\sin\alpha-mg$$

Med $r=\frac{3}{2}l$ fås:

$$S_1-S_2=\frac{3}{2}\frac{ml\omega^2}{\cos\alpha}\quad(1)$$

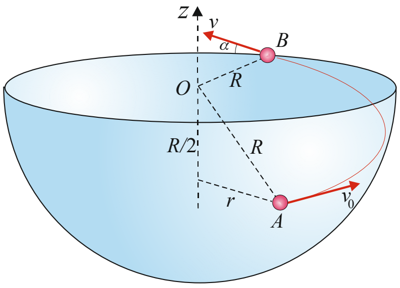
och $S_1+S_2=\frac{mg}{\sin\alpha}$ (2). $(1)+(2)\quad\Rightarrow\quad \underline{\underline{S_1=\frac{mg}{2\sin\alpha}+\frac{3}{4}\frac{ml\omega^2}{\cos\alpha}}}$

$$(2)-(1)\quad\Rightarrow\quad \underline{\underline{S_2=\frac{mg}{2\sin\alpha}-\frac{3}{4}\frac{ml\omega^2}{\cos\alpha}}}$$

$$S_2=0\quad\Rightarrow\quad\frac{3}{4}\frac{l\omega^2}{\cos\alpha}=\frac{g}{2\sin\alpha}\quad\Rightarrow\quad\underline{\underline{\omega=\sqrt{\frac{2g}{3l\tan\alpha}}}}$$

Obs! Det var fel i första varianten av facit!

3)



1) Momentekvationen ger

$$\dot{H}_z = 0 \quad \Rightarrow \quad H_z^{(A)} = H_z^{(B)}$$

$$m v_0 r = m v R \cos\alpha \quad \Rightarrow \quad \cos\alpha = \frac{r}{R}\frac{v_0}{v} \quad (1)$$

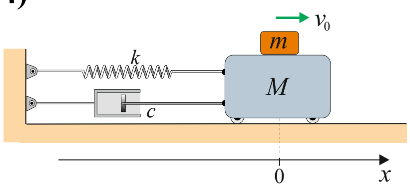
2) Mekaniska energilagen ger

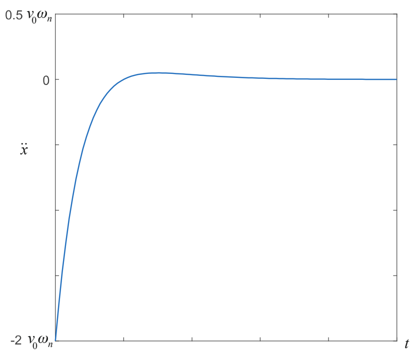
$$T_A + V_A = T_B + V_B$$

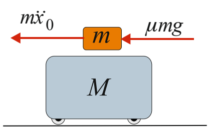
$\frac{1}{2} m v_0^2 + 0 = \frac{1}{2} m v^2 + mg\frac{R}{2} \quad \Rightarrow \quad v = \sqrt{v_0^2 - gR}$. Dessutom $r = \sqrt{R^2 - \frac{R^2}{4}} = \frac{\sqrt{3}}{2} R$

3) Dessa samband insatta i (1) ger $\cos\alpha = \frac{\sqrt{3}}{2}\frac{1}{\sqrt{1-\frac{gR}{v_0^2}}} \xrightarrow[v_0 \to \infty]{} \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \underline{\underline{\alpha_{\max} = 30°}}$

4)







1) Kraftekvationen för hela systemet ger

$$\mathbf{e}_x: \quad (M+m)\ddot{x} = -c\dot{x} - kx \quad \Rightarrow \quad \ddot{x} + \underbrace{\frac{c}{M+m}}_{2\varsigma\omega_n}\dot{x} + \underbrace{\frac{k}{M+m}}_{\omega_n^2}x = 0$$

Svängningsekvationen $\ddot{x} + 2\varsigma\omega_n\dot{x} + \omega_n^2 x = 0$ har vid kritisk dämpning $\varsigma = 1$ den allmänna lösningen på formen

$$x = (At + B)e^{-\omega_n t} \text{ . BV } \begin{cases} t = 0 \\ x = 0 \end{cases} \Rightarrow \quad B = 0 \Rightarrow x = Ate^{-\omega_n t}$$

$$\dot{x} = Ae^{-\omega_n t} - A\omega_n te^{-\omega_n t}$$

$$\begin{cases} t = 0 \\ \dot{x} = v_0 \end{cases} \Rightarrow \quad A = v_0 \quad \Rightarrow \quad x = v_0 te^{-\omega_n t}$$

2) Betrakta nu endast partikeln och formulera kraftekvationen i $x-$ riktningen $\qquad \mathbf{e}_x: \quad m\ddot{x} = \mu mg \quad (1)$

Den maximala friktionskraften fås vid maximal acceleration $\ddot{x}_{\max}$ enligt kraftekvationen. Vi har

$$x = v_0 te^{-\omega_n t} \quad \Rightarrow \quad \dot{x} = v_0 e^{-\omega_n t} - \omega_n v_0 te^{-\omega_n t} = (1-\omega_n t)v_0 e^{-\omega_n t} \quad \Rightarrow \quad \ddot{x} = -\omega_n v_0 e^{-\omega_n t} - \omega_n(1-\omega_n t)v_0 e^{-\omega_n t} =$$

$$= -\omega_n(2-\omega_n t)v_0 e^{-\omega_n t} \quad \Rightarrow \quad \dddot{x} = \omega_n^2 v_0 e^{-\omega_n t} + \omega_n^2(2-\omega_n t)v_0 e^{-\omega_n t} = \omega_n^2(3-\omega_n t)v_0 e^{-\omega_n t} = 0$$

Vilket ger $t_1 = \frac{3}{\omega_n}$ och insättning i accelerationen ger $\ddot{x}_{\max} = \omega_n v_0 e^{-3}$. Men observera att $|\ddot{x}_0| = |\ddot{x}(0)| = 2\omega_n v_0 > \ddot{x}_{\max}$ (se grafen). Friktionstalet μ fås därför från (1) med detta värde,

$$\mu = \frac{|\ddot{x}_0|}{g} = \frac{2\omega_n v_0}{g} = \underline{\underline{\sqrt{\frac{k}{M+m}}\frac{2v_0}{g}}}$$