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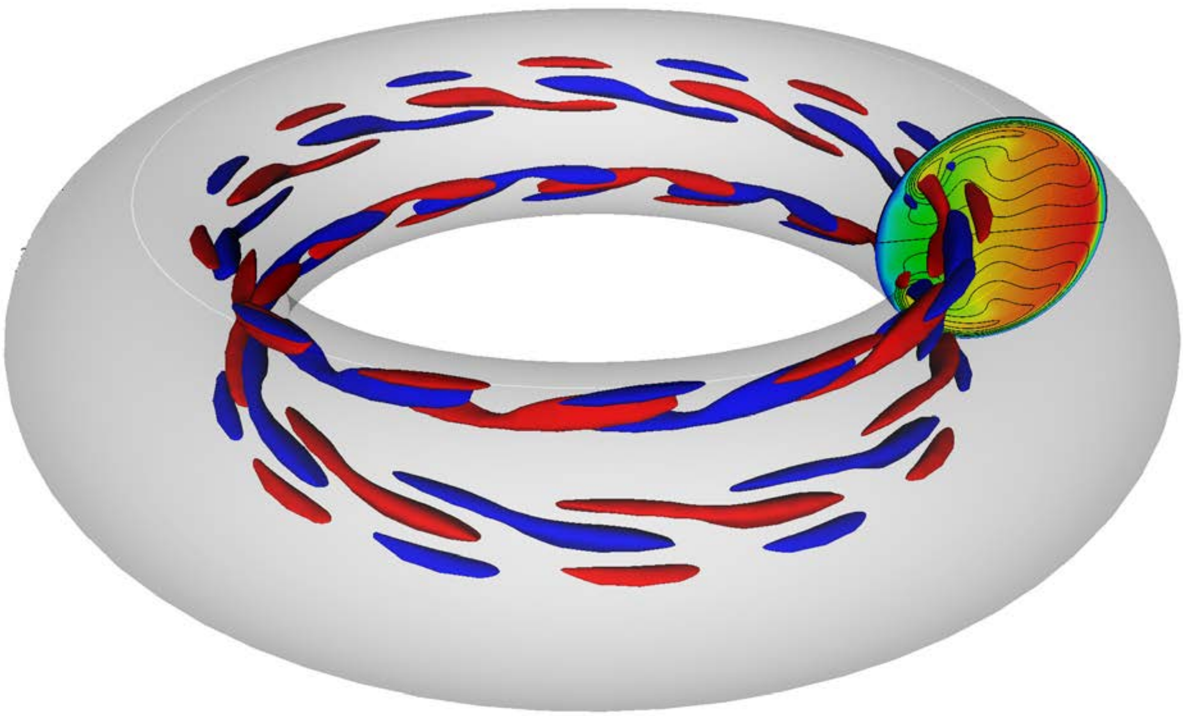

TRACKING THE FIRST BIFURCATION OF THE FLOW INSIDE A TOROIDAL PIPE

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This work is concerned with the numerical investigation of the linear stability properties of the viscous, incompressible flow inside a toroidal pipe. Bent pipes are omnipresent: going from industry where they are employed in devices ranging from heat exchangers, to exhausts, to junctions between pieces of straight pipe (for a review see Ref. [1]), to biology as, for instance, in respiratory and circulatory systems. In this work we start by analysing a toroidal pipe, which differs from a straight pipe by the addition of one single parameter, the curvature. Despite its relatively simple configuration, the flow inside a toroidal pipe cannot be described analytically. This has lead to the use of approximations to derive a closed-form solution to the Navier–Stokes equations in this geometry. The first solution has been proposed by Dean [2] who treated the problem in the limit of vanishing curvature and computed the first steady state solutions. Experimental works mostly consist in the study of the flow inside helical pipes in the low-pitch limit [3, 4, 5]. Only one experiment inside a toroidal pipe is known to the authors [6].

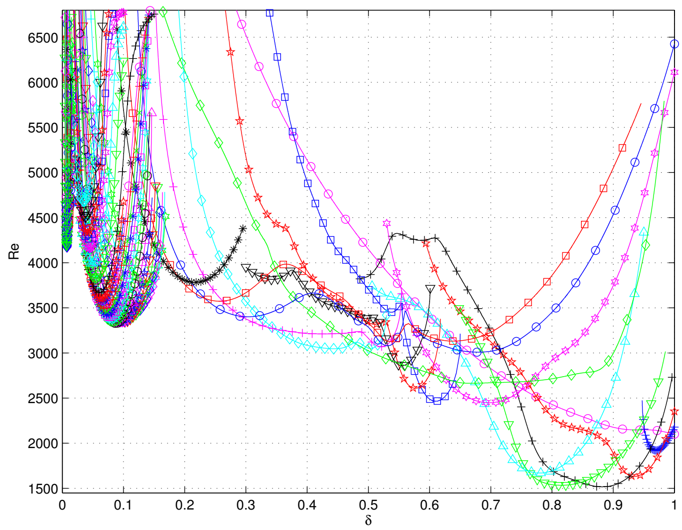

FIGURE 1. *Neutral curves in the $\delta - Re$ plane. Each line corresponds to the neutral curve of one mode; the envelope of the lines constitutes the neutral curve for the flow. It is possible to discern five families, each composed by a handful of modes with common characteristics.*

The flow is found to be modally unstable even at extremely low curvatures and it encounters a Hopf bifurcation as first instability on its route to turbulence. An algorithm for efficiently tracking this bifurcation in phase space is devised and presented. The eigenspectra reveal that the bifurcation is not identified by a single isolated mode: different families of modes and a few isolated eigenvalues take turns as critical modes, as depicted in figure 1 in which every line corresponds to the neutral curve of one mode. The neutral curve for this flow is thus composed by considering the envelope of these curves. The devised algorithm has been employed to trace the curves one by one, but has proven unsuitable for very low curvatures where the wave number characterising the modes increases extremely rapidly.

References

[1] S. Vashisth, V. Kumar, and K. D. P. Nigam. A review on the potential applications of curved geometries in process industry. *Ind. Eng. Chem. Res.*, 47:3291–3337, 2008.
[2] W. R. Dean. Note on the motion of fluid in a curved pipe. *Phil. Mag.*, 7:208–223, 1927.
[3] H. Ito. Friction Factors for Turbulent Flow in Curved Pipes. *J. Basic Eng.*, 81:123–134, 1959.
[4] K. R. Sreenivasan and P. J. Strykowski. Stabilization effects in flow through helically coiled pipes. *Exp. Fluids*, 1:31–36, 1983.
[5] A. Cioncolini and L. Santini. An experimental investigation regarding the laminar to turbulent flow transition in helically coiled pipes. *Exp. Therm. Fluid Sci*, 30:367–380, 2006.
[6] J. Kühnen, M. Holzner, B. Hof, and H. C. Kuhlmann. Experimental investigation of transitional flow in a toroidal pipe. *J. Fluid Mech.*, 738:463–491, 2014.