MODAL INSTABILITY OF THE FLOW IN A TOROIDAL PIPE

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Introduction

While hydrodynamic stability and transition to turbulence in straight pipes — as one of the most fundamental problems in fluid mechanics — has been studied extensively, the stability of curved pipes has received less attention. In the present work, the first (linear) instability of the flow inside a toroidal pipe is investigated as a first step in the study of the related laminar-turbulent transition process. The impact of the curvature of the pipe (defined as the ratio between the radius of the pipe and that of the torus) on the stability properties of the flow is studied in the framework of classical linear stability analysis.

We focus on an idealised toroidal geometry which, albeit rarely encountered in industrial applications, is representative of a canonical flow and is relevant in the context of the research on the onset of turbulence. Moreover, the toroidal pipe constitutes the common asymptotic limit of two important flow cases: the curved pipe and the helical pipe. The technical relevance of these flows is apparent from their prevalence in industrial appliances, such as in heat exchangers, exhausts and other devices; for a comprehensive review of the applications, see Vashisth *et al.* [1].

The study of the flow in curved pipes has been the subject of several papers over the last decades: theoretical, experimental and numerical results have been presented [2-4], however, a thorough analysis of the causes and mechanisms of hydrodynamic stability and transition to turbulence in this flow is still missing.

Base flow

In order to determine the linear stability of the toroidal pipe flow, we investigate the growth of infinitesimal disturbances around a basic state. This base flow, *i.e.* the solution to the steady, incompressible Navier–Stokes equations, is invariant with respect to the axial pipe direction and is maintained in motion by a constant volume force. The base flow is characterized, as first discovered by Dean [5], by the presence of two counter-rotating vortices, so-called Dean vortices in his honour. These two primary vortices are present at *every* Re and for *any* value of δ (different from zero), and are located symmetrically with respect to the equatorial plane of the torus. The shape of the vortices and the position of their centres depend on both Re and δ.

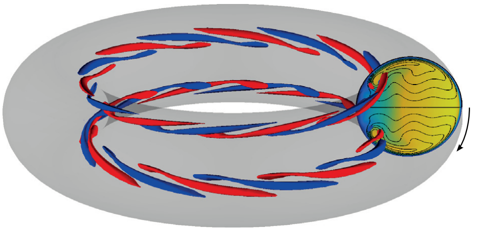

Figure 1: Critical mode for δ =0.3, Re=3379

Stability analysis

Results show that the flow is linearly unstable for all curvatures investigated between 0.002 and unity, and undergoes a Hopf bifurcation at *Re* of about 4000. The bifurcation is followed by the onset of a periodic regime, characterised by travelling waves with wavelength *O*(1) pipe diameters. The neutral curve associated with the instability is traced in parameter space by means of a novel continuation algorithm, which provides a complete description of the modal onset of instability as a function of the two governing parameters. Several different modes are found, with differing properties and eigenfunction shapes. Some eigenmodes belong to groups with a set of common characteristics, deemed 'families', while others appear as 'isolated'. Comparison with nonlinear DNS shows excellent agreement, confirming every aspect of the linear analysis, its accuracy, and proving its significance for the nonlinear flow. Experimental data from the literature are also shown to be in considerable agreement with the present results [6].

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