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MODAL STABILITY ANALYSIS OF TOROIDAL PIPE FLOW APPROACHING ZERO CURVATURE

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The flow in a straight pipe was found to be linearly stable up to $Re_b = 10^7$ by Meseguer & Trefethen [1], with Re_b being the Reynolds number based on bulk velocity, diameter and kinematic viscosity. The authors also derived an asymptotic expression of the growth rate to show that the linear stability threshold for the straight pipe flow is infinite. However, contrary to plane Couette flow [2], there is no analytical proof for this result in the literature. Indeed, the expression provided by [1] is an extrapolation based on their numerical data. The purpose of the present work is to study the linear stability of the flow in a torus for very low curvatures approaching zero, i.e. the straight pipe, and to provide further evidence of the result in [1]. We perform classical linear stability analysis. Both the base flow computation and the modal stability analysis are carried out using an in-house developed code, which employs a spectral collocation method based on Chebychev nodes in the radial direction and Fourier nodes in the azimuthal one. Proper treatment of the pressure boundary conditions is performed to avoid spurious pressure modes in the base flow computation. The analysis is similar to the BiGlobal stability approach, with the difference of employing a three-dimensional base flow instead of a two-dimensional one. A sensitivity analysis of the stability properties with respect to the underlying base flow is carried out by linearizing around both the computed base flow and the Hagen–Poiseuille velocity profile. In the latter case, the perturbations are allowed to have non-zero curvature. The results show that a proper account of the secondary motion in the base flow is needed to correctly capture the spectrum of the linearized Navier–Stokes operator. The analysis cannot be repeated using the Dean's approximation [4] as base flow, since this solution ceases to be valid for values of curvature and Reynolds number at criticality. Curvatures below 10^{-2} are investigated, and a continuation method is employed to track the most unstable eigenmodes. The tracking is performed by Rayleigh quotient iteration. A wide range of streamwise wavenumbers needs to be taken into account, in order not to miss unstable branches. The neutral curve obtained with preliminary results is shown in the figure below. It indicates that the critical Reynolds number increases with curvature, suggesting that it might go asymptotically to infinity as $\delta \to 0$, i.e. approaching the straight pipe.

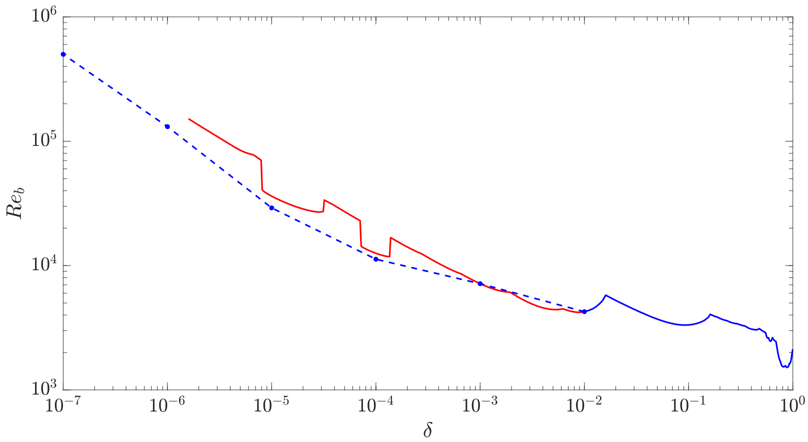

FIGURE 1. *Neutral curve in the $\delta - Re_b$ plane for curvatures $\delta \in [10^{-7}, 1]$. The blue solid line is taken from [5], the red solid line represents data from the present study, the blue dashed line is an exploratory curve obtained performing full spectrum computation at different curvatures.*

References

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