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ETMM

Ercoftac Symposium on
Engineering Turbulence
Modelling and Measurements

6th - 8th September 2023
Barcelona, Spain

# 14th International ERCOFTAC Symposium on Engineering Turbulence Modelling and Measurements

## PROCEEDINGS

# Spatial averaging effects in adverse pressure gradient turbulent boundary layers

***Fermin Mallor*[1] , *Ramis Örlü*[1,2], *Philipp Schlatter*[1,3]**

[1] ***SimEx, Engineering Mechanics, KTH Royal Institute of Technology, Stockholm, Sweden***
[2] ***Department of Mechanical, Electronic and Chemical Engineering***
***OsloMet – Oslo Metropolitan University, 0166 Oslo, Norway***
[3] ***Institute of Fluid Mechanics (LSTM)***
***Friedrich-Alexander-Universität (FAU) Erlangen-Nürnberg, Germany***

*mallor@kth.se*

## Abstract

Thermal anemometry sensors for time-resolved velocity measurement average the measured signal over the length of their sensor, thereby attenuating fluctuations stemming from scales smaller than the wire length. Several compensation methods have emerged, the most prominent ones in wall turbulence rely on the small-scale universality in canonical flows or on the reconstruction based on two attenuated variance profiles. To extend these methods to non-canonical flows, the present work considers various adverse-pressure gradient (APG) turbulent boundary layer (TBL) flows in order to explore how the small-scale energy is affected in the inner and outer layer and how the two prominent correction methods perform as function of wall-distance, wire length and flow condition. Our findings show that the increased levels of small scale energy associated with APG TBLs reduces the applicability of empirical methods based on the universality of the small-scale energy. On the other hand, a correction based on the relationship between the spanwise Taylor microscale and the two-point streamwise velocity correlation function, is able to correct the attenuated profiles of non-canonical cases. We therefore explore the development of a composite profile for the Taylor microscale, which could then be used for the correction of probe-length attenuation effects across a multitude of flow conditions.

## 1 Introduction

In the pursuit of accurate velocity measurements in high Reynolds ($Re$) number turbulent boundary layer (TBL) flows, spatial averaging effects rising from the use of finite-length sensing probes (most prominently hot-wires), have been a widely acknowledged and discussed problem. As pointed out by Hutchins et al. (2009), hot-wire spatial averaging effects have been the root cause of major disagreements in high-$Re$ streamwise velocity fluctuation profiles and trends in the literature, e.g., with respect to the scaling of the inner-peak (Örlü & Alfredsson, 2013) or the existence of an outer peak in the streamwise variance profile (Alfredsson et al., 2011). Due to the importance of this phenomena, a number of correction schemes for the missing fluctuations have been proposed during the past 20 years. Miller et al. (2014) assessed the most relevant ones in the literature, including the one by Smits et al. (2011) based on the attached eddy hypothesis, and the one by Segalini et al. (2011) based on the relation of the spanwise Taylor microscale ($\lambda_g$) and the two-point streamwise velocity correlation function, finding good results for pipe, channel and zero-pressure gradient (ZPG) TBL flows.

However, most of the aforementioned hot-wire spatial resolution corrections were developed for canonical flows under the premise of universality of the small-scale energy scaled with viscous wall units ($L^+$) (Mathis et al. (2011)). The one by Segalini et al. (2011), however, might be more generally applicable, although it relies on a correlation between the attenuation and $\lambda_g$, taking only ZPG TBL data as reference. As discussed in Sanmiguel Vila et al. (2020), this could be a potentially erroneous assumption in non-canonical flows, such as TBLs subjected to adverse pressure gradients (APGs) or flows along roughness walls (Gatti et al. (2022)). When the flow is decelerated due to the presence of an APG, the overall structure and dynamics of the TBL are greatly affected: internal shear layers appear, with large scale structures leaving a larger imprint at the wall, and small scale energy becoming more relevant in the outer region of the TBL as discussed in Sanmiguel Vila et al. (2020) and Pozuelo et al. (2022). The introduction of the pressure gradient, summed with the already-known high Reynolds number effects, leads to an increase in the fluctuations in the outer region of the TBL, which depending on the APG strength, may show a dominant peak in the streamwise component of the Reynolds stresses (Sanmiguel Vila et al., 2020). Nevertheless, the conclusions drawn from spatial attenuation studies in canonical flows have been directly transferred over to the study of APG TBLs. For example, it is

Table 1: Numerical datasets used in the present work.

| Dataset | $Re_\theta$ | $Re_\tau$ | $H_{12}$ | $\beta$ |
|---|---|---|---|---|
| ZPG — | 3116 – 6634 | 1002 – 2007 | 1.41 – 1.37 | 0 |
| APG Pozuelo — | 4894 – 8940 | 1031 – 1967 | 1.61 – 1.50 | 1.56 – 1.12 |
| APG Bobke — | 3079 | 748 | 1.56 | 0.85 |
| Wing — | 3158 | 671 | 1.65 | 2.7 |

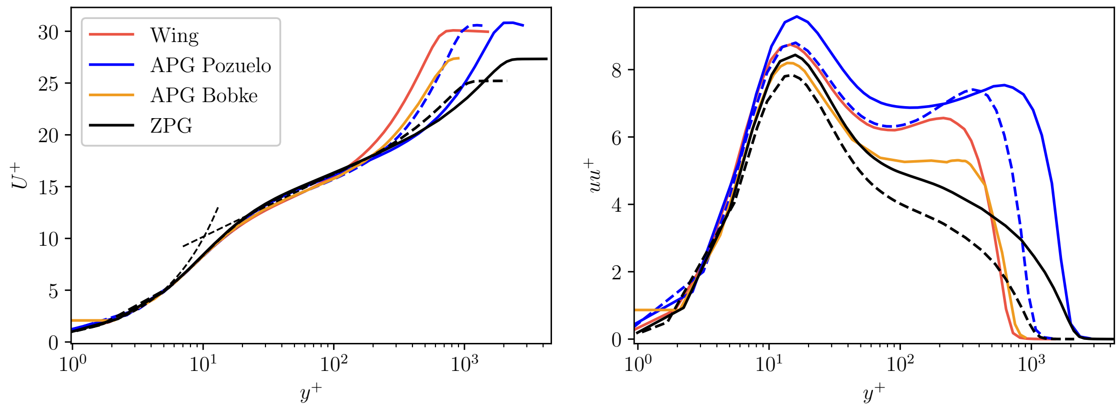


Figure 1: Inner-scaled streamwise velocity mean (left) and fluctuations (right) profiles of the numerical datasets used in this work. In the APG Pozuelo and ZPG datasets, the high and low $Re_\tau$ are represented by solid and dashed lines respectively. Black, dashed lines show the linear ($U^+ = y^+$) and logarithmic ($U^+ = \frac{1}{\kappa} \ln y^+ + B$, with $\kappa = 0.38$ and $B = 4.1$) velocity profiles.

common practice to use the same hot-wire length (in viscous units) for flows with different pressure gradients as e.g. done in Mathis et al. (2011) and Harun et al. (2013), thereby seemingly assuming that pressure-gradient effects may not be biased by spatial resolution effects.

Therefore, despite the plethora of correction techniques available, none of them is directly applicable to APG TBLs, as they were developed with or at least based on data from canonical ZPG TBLs in mind. In this work, we first study the hot-wire resolution effects in APG TBLs and show their differences to the ZPG cases. Furthermore, we assess the applicability and effectiveness of the aforementioned correction methods, as well as propose new pathways in the correction of hot-wire anemometry (HWA) measurements of APG TBLs that can also shed light on ways to include other non-canonical effects.

## 2 Data Sets

In the following, we use data from previous high-fidelity numerical simulations of ZPG by Eitel-Amor et al. (2014) and near-equilibrium APG by Pozuelo et al. (2022) and by Bobke et al. (2017). Moreover, new wall-resolved LES simulations of a NACA 4412 wing profile at an angle of attack of 5 degrees, and chord-based Reynolds number of $10^6$ are used. The rapidly evolving pressure gradient over the suction side (in the present work, the profile analyzed is located at $x/c = 0.75$) makes this new dataset ideal for the study of non-equilibrium APGs. These simulations were possible thanks to the introduction of adaptive mesh refinement into the spectral-element method code Nek5000, described and validated by Tanarro et al. (2020). These data sets, summarized in Table 1, are exploited to study the effect of probe spatial averaging effects on the streamwise-velocity fluctuations and in conjunction with it the effect on the spanwise Taylor microscale in APG TBLs, which according to Segalini et al. (2011) is associated with the attenuation effect. As shown in Figure 1, the datasets studied are quite diverse, with the matching $Re_\tau$ cases for different $\beta$ conditions showing clear differences in both their mean and fluctuation profiles.

## 3 Results

To simulate the effect of spatial resolution in hot-wire anemometry measurements, the approach described in Örlü and Schlatter (2013) and Philip et al. (2013) is followed. In short: a box filter (in physical space) with length equal to that of the hot-wire sensor is applied to the velocity signals. This filter is a good surrogate for wires with high length-to-diameter (aspect) ratios, for which end-conduction effects can be neglected, as discussed by Philip et al. (2013).

The effect of insufficient spatial resolution for ZPG TBLs is well-known and reproduced in the top row of Fig. 2. It is apparent that the small-scale dominated inner layer is strongly attenuated, while the outer layer does not exhibit any reduced amplitude. For the APG

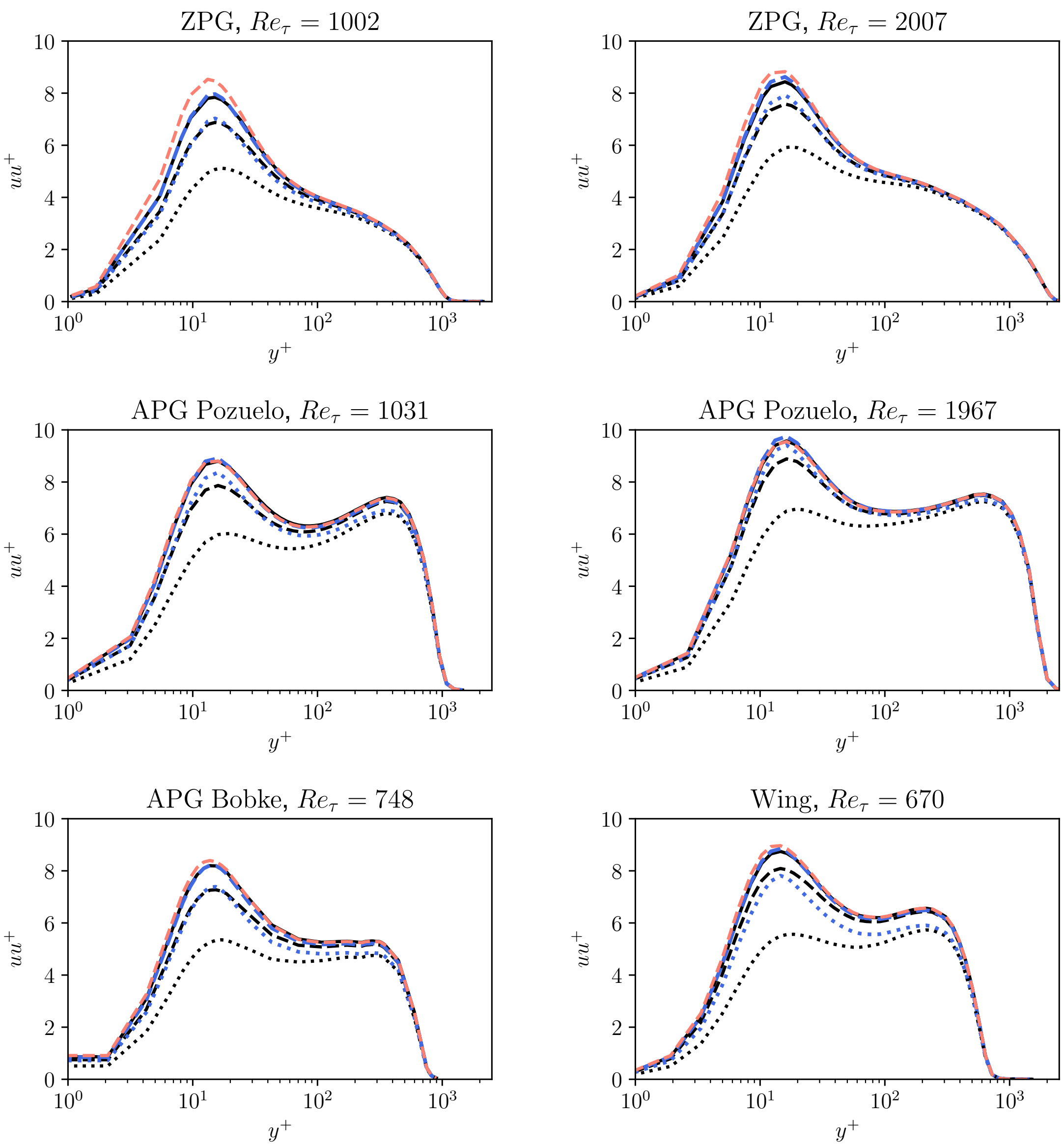


Figure 2: Streamwise velocity fluctuations in a ZPG (top row), mild, quasi-equilibrium APG (middle row and bottom left) and strong, non-equilibrium APG (bottom right). Full black lines are used for the original fluctuation profile, while dashed and dotted lines represent the attenuated profiles obtained by using a probe of 20 and 50 viscous units length. Profiles reconstructed using the correction by Smits et al. (2011) are shown in blue (with line-style depending on the probe-length from which the profile was reconstructed), whereas profiles reconstructed using the correction by Segalini et al. (2011) (which uses both attenuated profiles) are shown in red.

TBLs, on the other hand, the attenuation is also present in the outer layer. In particular it is apparent that attenuation is higher for lower $Re_\tau$ and stronger $\beta$: the strongest attenuation being present for the "wing" and weakest for "APG Pozuelo". It can hence be anticipated that correction methods based on the assumption of small-scale universality limited to the inner layer will not be able to correctly account for the missing small-scale energy in the outer layer.

To assess the performance of the correction schemes by Smits et al. (2011) and Segalini et al. (2011), they have been applied on two attenuated variance profiles, *i.e* representing measurements with probe lengths of $L^+ = 20$ and 50, see Fig. 2. For the smaller probe length, the effects of the APG on the lost energy is minimal, as only the inner region is affected. Nonetheless, as the probe length increases, the effect of the APG becomes apparent: not only the near-wall peak of the variance profile is attenuated, but also the outer peak. It is in this more extreme case in which the limitations of the widely-used correction by Smits et al. (2011) or related correction schemes based on the small-scale universality can be appreciated: Although the effect of the missing small-scale energy on the inner peak is reasonably well corrected, the outer region of the TBL remains problematic. This stems from the

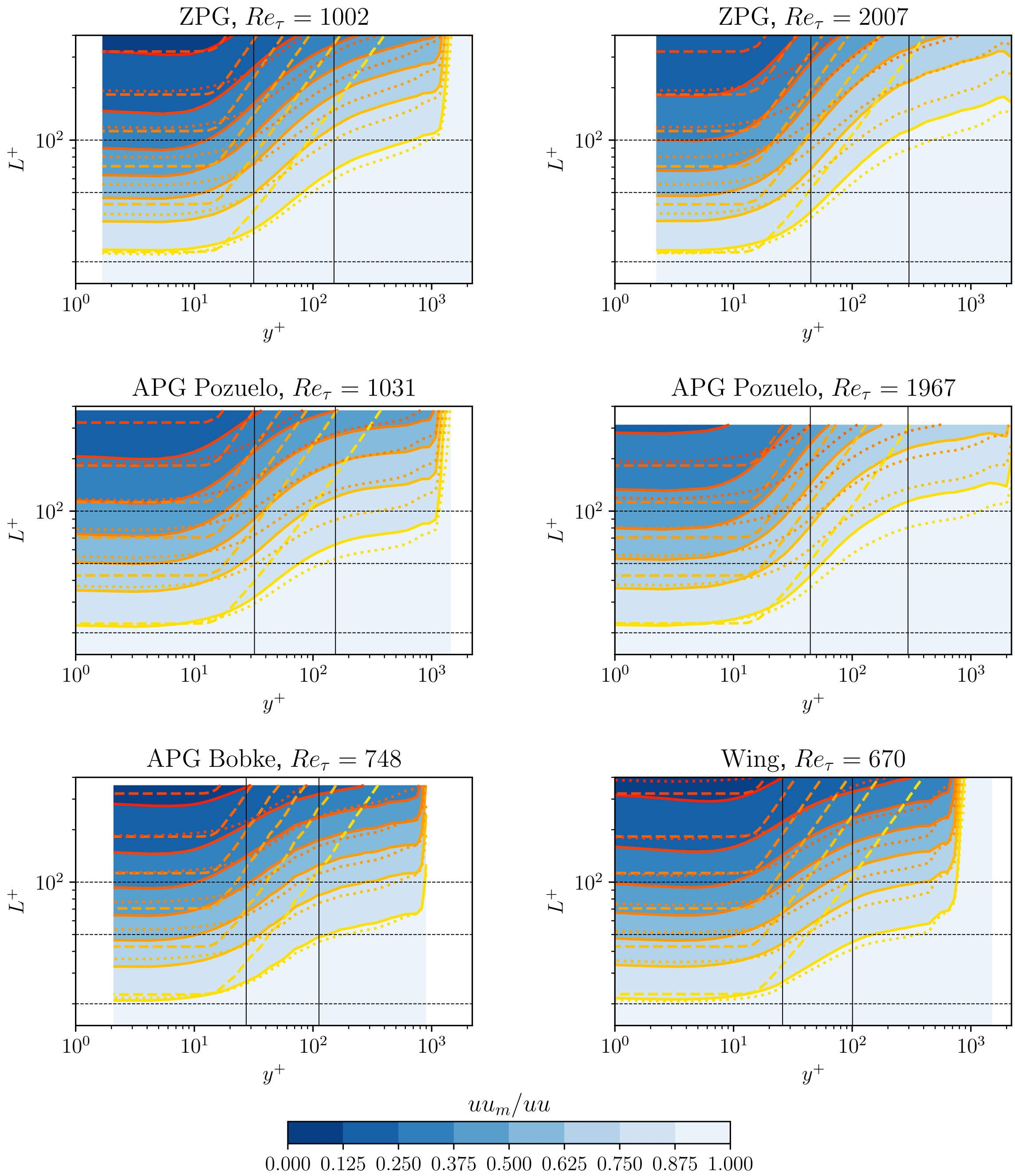


Figure 3: Contours of the attenuation of the streamwise velocity fluctuations ($F_2 = uu_m/uu$) in a ZPG (top row), mild, quasi-equilibrium APG (middle row and bottom left) and strong, non-equilibrium APG (bottom right) as a function of the wall-normal position ($y^+$) and probe length ($L^+$). The full contour lines indicate the actual attenuation, while the dashed ones correspond to the predicted one by the correction introduced in Smits et al. (2011), and the dotted lines to the correction proposed by Segalini et al. (2011) (using the $\lambda_g$ computed from the timeseries). Horizontal dashed lines indicate probe lengths of 20, 50 and 100 viscous units, and the vertical lines delimit the TBL overlap region ($\sqrt{Re_\tau} < y^+ < 0.3Re_\tau$).

fundamental assumption of the correction scheme: the attached eddy hypothesis, which translates into a $y^{-1}$ factor in the correction.

A better reconstruction of the original (non-averaged) data is obtained with the correction scheme proposed by Segalini et al. (2011). In fact, both the inner and outer layer lie on top of the original data after the correction is applied. The method relies on an approximation of the velocity-fluctuation correlation function using the spanwise Taylor microscale (valid in the limit of small probe lengths). The method estimates both the corrected variance of the streamwise velocity fluctuations as well as its spanwise Taylor microscale. The outperformance of the method proposed

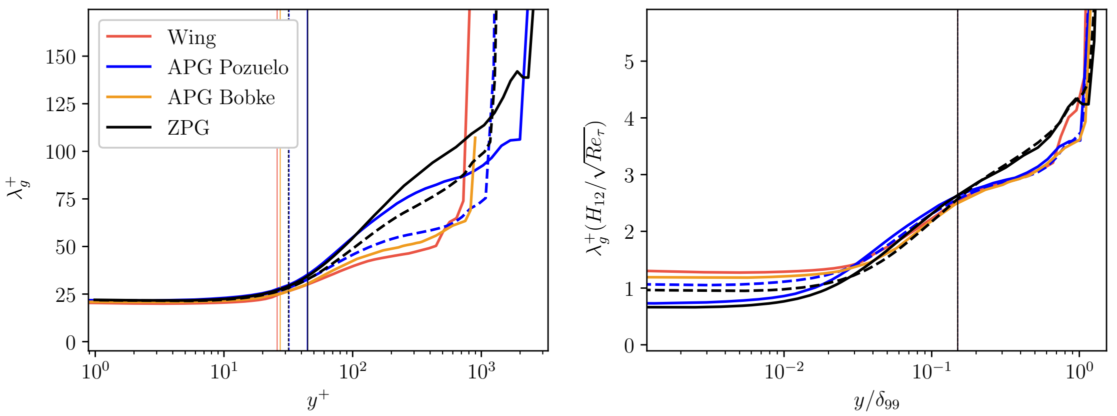


Figure 4: Inner (left) and outer (right) scaled profiles of the Taylor microscale ($\lambda_g$). The vertical lines on the left plot correspond to the position of $y^+ = \sqrt{Re_\tau}$, and the one on the right panel to $y/\delta_{99} = 0.15$.

by Segalini et al. (2011) over the one by Smits et al. (2011) is clearly reflected in Figure 3, in which the contours of the fluctuation attenuation predicted by both methods for different probe-lengths at different wall-normal positions are overlayed on top of the actual attenuation. As discussed previously, the assumption of the attached eddy hypothesis leads to a decrease proportional to $y^{-1}$ in the correction scheme by Smits et al. (2011), a faster rate than the actual one. This leads to an under-correction of the attenuation starting in the overlap layer of the TBL. In cases in which the small-scale energy is confined to the inner layer (*i.e.* ZPGs or mild APGs at high $Re_\tau$), this departure from the actual attenuation is not as drastic as long as the probe lengths are small. However, for high $\beta$ and lower $Re_\tau$ cases, larger deviations from the actual attenuation are observed starting from the overlap layer at moderate probe lengths. On the other hand, the attenuation predicted by Segalini et al. (2011) correction scheme is almost on top of the actual attenuation for all the considered cases across the whole TBL. Nevertheless, this method has two major drawbacks: it requires two measurements (although not simultaneous) using different wire lengths, and, as noted by Miller et al. (2014) the optimization process used is prone to amplifying experimental noise. Furthermore, the fit used for estimating the attenuation as function of $\lambda_g$ may need to be adapted for non-canonical flow cases as e.g., for strong pressure gradient cases.

In order to circumvent the issues presented by both the Smits et al. (2011) and Segalini et al. (2011) correction schemes, both approaches should be combined. We propose the development of a correction scheme based on a single expression as in the Smits et al. (2011) correction which should be capable of providing a meaningful correction for the streamwise velocity fluctuations in the outer layer of non-canonical flows (i.e., APGs). This can be achieved through the relationship between the spanwise Taylor microscale and the streamwise velocity two-point correlation function Segalini et al. (2011). As shown in Fig. 4, the spanwise Taylor microscale plateaus in the inner region when scaled in inner units across all cases. In the outer region, the effects of the pressure gradient (and its history effects) become relevant. A better collapse of the data in the overlap region ($\sqrt{Re_\tau} < y^+ < 0.15Re_\tau$) is achieved by scaling the wall-normal distance with the boundary layer thickness, and the spanwise Taylor microscale with the square root of the friction Reynolds number (a mixed scaling as proposed in the original work by Segalini et al. (2011)) with the addition of the inverse of the shape factor, which is a quantity closely related with both the strength and the history of the pressure gradient, as discussed by Dróżdż et al. (2020). Nonetheless, a collapse of the $\lambda_g$ profiles in the outer region ($y/\delta_{99} > 0.15$) is yet to be found.

## 4 Conclusions and outlook

In this work, we analyzed the effect of hot-wire length on the attenuation of the streamwise fluctuation profiles in canonical and non-canonical TBLs. We focused our attention in three kinds of profiles: ZPGs, near-equilibrium APGs, and non-equilibrium APGs (for which a new high-fidelity numerical simulation of the flow around a NACA 4412 wing profile was carried out). Two correction methods based on different approaches are analyzed: One the one hand, Smits et al. (2011) propose a correction based on the attached eddy hypothesis and empirical fittings to ZPG datasets, and features a single expression; and, on the other hand, Segalini et al. (2011) uses the relationship between the Taylor microscale and the two-point correlation function, which is more general but requires two sets of measurments with different HW lengths.

We show that the correction by Smits et al. (2011) fails to capture the presence of small-scale energy in lower $Re_\tau$ and high $\beta$ cases, leading to an under-correction of the outer peak characteristic of APGs. We therefore recommend its use only in canonical TBLs: either in ZPGs or in near-equilibrium APGs with low $\beta$ and high $Re_\tau$. On the contrary, the correction scheme proposed by Segalini et al. (2011) is shown to perform equally well in canonical and non-canonical flows. Nevertheless, it has one major drawback: it necessitates of two independent fluctuation profiles measured with different HW lengths, and involves an optimization process which could result in the amplification of experimental uncertainties.

Lastly, we propose a path towards a more general correction scheme, which involves the development of a composite profile for the Taylor microscale in TBLs. Insofar, a good collapse has been found for both the inner and overlap regions, but a formulation that collapses the different profiles in the outer (wake) is still an open subject.

## Acknowledgments

The simulations were performed on resources provided by both the Swedish National Infrastructure for Computing (SNIC) at the PDC Center for High Performance Computing, in KTH (Stockholm), and by the Partnership for Advanced Computing in Europe (PRACE) project 2021250090 on HAWK (HLRS Stuttgart).

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