ON THE PASSIVE CONTROL OF THE NEAR-FIELD OF COAXIAL JETS BY MEANS OF VORTEX SHEDDING

Ramis Örlü*[1], Antonio Segalini[2], P. Henrik Alfredsson[1] and Alessandro Talamelli[2]

[1]Linné Flow Centre, KTH Mechanics, SE-100 44 Stockholm, Sweden
ramis@mech.kth.se

[2]Dept. of Mechanical and Aerospace Engineering (DIEM), University of Bologna, I-47100 Forlí, Italy

ABSTRACT

The present paper confirms experimentally the theoretical result by Talamelli and Gavarini (Flow, Turbul. & Combust., 2006), who proposed that the wake behind a separation wall between two streams of a coaxial jet creates the condition for an absolute instability. This instability, by means of the induced vortex shedding, provides a continuous forcing mechanism for the control of the flow field. The potential of this passive mechanism as an easy, effective and practical way to control the near-field of interacting shear layers has been demonstrated and its effect towards increased turbulence activity has been shown.

1 INTRODUCTION

Flow control in transitional and turbulent flows has become more practical since the recognition of organised motions attributed to coherent structures embedded in the incoherent turbulent background [1, 2]. Through these coherent structures, which comprise a considerable fraction of the total turbulent energy, ways have been opened to manipulate the dynamics of the flow [3]. In order to highlight the mechanism governing coherent structures and to enhance our understanding of how these can be utilised to control the transport and mixing characteristics, a variety of passive and active flow control mechanisms have been tested and applied within the last decades, principally in single jets [4, 5, 6] and other canonical flows [7].

In the case of coaxial jets, however, flow control studies have been primarily investigated regarding the receptivity to active flow control strategies [8, 9]. Passive strategies, on the other hand, have mainly been disregarded, despite their promising results in other flow cases [4, 5, 6, 7]. One reason for this imbalance might be the fact that a full characterisation of the apparently simple geometry of coaxial jets, depicted in figure 1, is governed by a multitude of parameters [10].

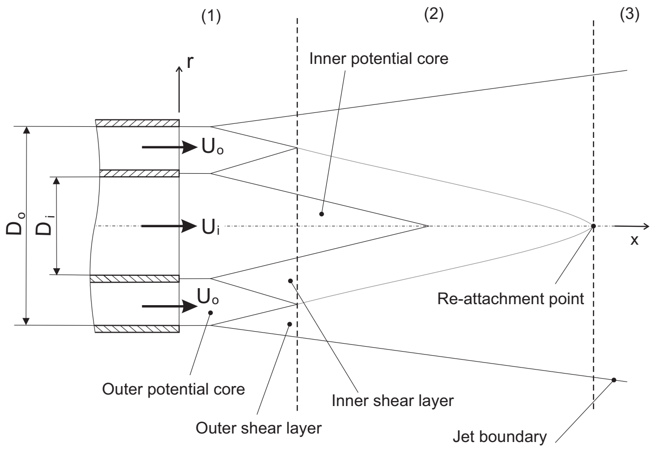

Figure 1: Sketch of the flow field of a coaxial jet configuration and its main parameters. (1) Initial merging zone, (2) Intermediate zone, (3) Fully merged zone.

The prevailing model for more than a decade, put forward by Ko and Kwan [11], was that coaxial jets could be considered as a simple combination of single jets, where the two shear layers originating from the nozzles develop independently from each other. This view was modified when Dahm et al. [12] in their flow visualisation study showed the existence of different topological flow regimes for different velocity ratios ($r_u = U_o/U_i$, here U_i and U_o denote the maximum absolute velocity of the inner and outer streams at the nozzle exits, respectively), as well as absolute velocities. Furthermore the same authors, as well as Wicker and Eaton [13], found that the vortical motion for $r_u > 1$ is dominated by the vortices emerging in the outer shear layer. They showed that the evolution of the vortices of the inner shear layer is dictated by the outer vortices. They are hence trapped in the spaces left free between two consecutive outer shear layers vortices; this scenario became known as the so-called 'locking phenomenon'. Consequently the vortex passage frequency of the inner shear layer will differ from the one predicted by stability analysis for a single axisymmetric shear layer as shown by Dahm et al. [12] and da Silva et al. [14] by means of flow visualisations and direct numerical simulations, respectively. This finding has led to an increased focus on the control of the outer shear layers' vortices [9, 15].

One of the parameters, emphasised by Buresti et al. [10], playing an important role in the evolution of transitional coaxial jets, is the thickness of the (duct) wall, t, separating the two streams from each other. It was shown by the same authors that two trains of alternating vortices are shed from both sides of the inner wall with a well-defined frequency, which scales with the wall thickness and the average velocity of the two streams. In fact both the wall thickness and the velocity ratio were found to be crucial in determining whether the behaviour in the near-field of coaxial jets could be considered as wake-like or shear-layer-like [12, 16].

In a recent study Talamelli and Gavarini [17] formulated a theoretical background for this experimental finding. They showed, by means of linear stability analysis, that the alternate vortex shedding behind the inner wall can be related to the presence of an absolute instability, which exists for a specific range of velocity ratios and for a finite thickness of the wall separating the two streams. The authors proposed that this absolute instability may provide a continuous forcing mechanism for the destabilisation of the whole flow field even if the instability is of limited spatial extent. It is important to point out that this mechanism does not require any external energy input, being considered passive, and therefore is attractive for practical applications.

The present paper aims at verifying the proposed idea of Talamelli and Gavarini [17], namely to test if the absolute instability behind an inner wall of a coaxial jet nozzle with finite thickness can be utilised as a continuous forcing mechanism and hence as a passive flow control mechanism for the near-field of coaxial jet flows. Furthermore, the experimental results will surpass what linear stability analysis can predict and indicate what the effect of the vortex shedding on the near-field turbulence characteristics is. The experiments show that the vortex shedding behind a thick separating wall can be facilitated as an easily applicable and effective passive control mechanism.

In this context it could be shown that the trapping of the inner shear layers vortices into the free spaces of the Kelvin-Helmholtz instability of the external shear layer, known as the 'locking phenomenon', is indeed reversible; namely we observed that the vortex shedding behind the separating wall with finite thickness dictates the passage frequency of the external shear layers vortices and thereby controls the inner and outer shear layers evolution in the whole near-field region. A clear trend towards increased turbulence activity, both within the inner and outer shear layers and thereby the mixing between the two streams of the coaxial jet as well as the outer jet and the ambient surroundings, could be observed. This makes it possible to specifically utilise the geometry of the inner separating wall not only to control the dynamics of the flow, but also to increase the turbulence activity, giving the presented passive control mechanism great importance both from a fundamental and applied point of view.

The remainder of the paper is organised as follows: Section 2 describes the coaxial jet facility as well as the measurement technique used. Experimental results showing the viability of the passive control mechanism as well as its effect on the turbulence are presented and discussed in section 3. The paper finishes with conclusions in section 4.

2 EXPERIMENTAL SET-UP and MEASUREMENT TECHNIQUE

2.1 The coaxial jet facility

The experiments were carried out in the *Coaxial Air Tunnel* (CAT) facility in the laboratory of the *Second Faculty of Engineering* at the *University of Bologna* in Forlí. The facility, schematically shown in figure 2, is composed of two independent centrifugal blowers (**A** and **B**) equipped with three-phase motors. Two pre-settling chambers (**C** and **D**) are placed downstream the blowers to reduce the disturbances from the blowers. Four plastic hoses (**J**) connect the outer jet pre-chamber to the corresponding settling chamber to increase the symmetry of the flow, while a simple diverging pipe (**E**) connects the inner one. Flow conditioning is performed by means of three screens and a honeycomb (**H**) in the inner pipe as well as five screens and a honeycomb on the outer circuit. The inner and outer contraction ratios are 11:1 and 16.5:1, respectively, whereas the inner and outer coaxial nozzles, of exit diameter $D_i = 50$ mm and $D_o = 100$ mm, end with two straight pipes of 100 mm length (**K**). The experimental facility is placed in a large laboratory and the exit of the coaxial jet is far enough from surrounding walls and the floor, ensuring that the experimental results reported here resemble a jet in an infinite environment (cf. appendix B of [18]).

In the present experiment, two different types of separation walls have been used. The first one has a thickness of $t = 5$ mm and ends in a rectangular geometry, schematically shown in figure 2 (**K**), whereas the second one ends with a sharp trailing edge making the wall thickness negligible ($t \approx 0$ mm) with respect to the sum of the side boundary layers thicknesses. These two separating walls will in the following be denoted as thick and sharp, respectively. The sharp and thick wall cases represent the flow cases in the absence and presence of the vortex shedding phenomenon, respectively, and enable therefore a selective investigation regarding the effect of the absolute instability.

The probe is finally positioned in the flow by means of a motorised traversing system capable to move the probe in the axial (**L**) and radial (**N**) direction. The

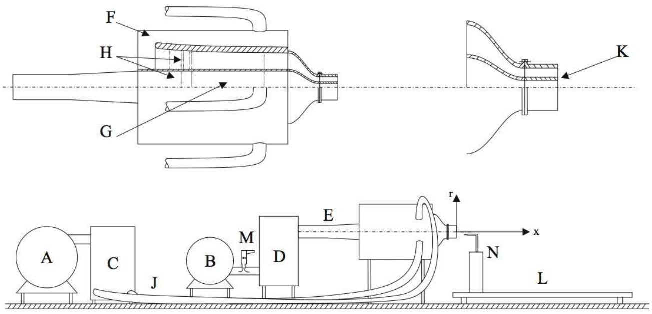

Figure 2: Schematic of the coaxial jet facility. **A** outer jet blower, **B** inner jet blower, **C** outer jet pre-settling chamber, **D** inner jet pre-settling chamber, **E** inner jet diffuser, **F** outer jet settling chamber, **G** inner jet settling chamber, **H** screens and honeycombs, **J** outer jet hoses, **K** close-up of the jet exit with the thick separating wall, **L** axial traversing, **M** heat gun, **N** radial traversing.

traversing system and the data acquisition are controlled by a single PC using a *National Instruments* 16-bit PCI-6035E acquisition board.

2.2 Measurement technique

For the present investigation a variety of hot-wire probes were used. The characterisation of the boundary layers, upstream the orifice exit and in the vicinity of it, were performed by in-house made long and slender boundary layer probes consisting of 2.5 micron Platinum wires with a length-to-diameter ratio of 400 in order to avoid any blockage effects within the nozzle and reduce near wall effects. All other measurements were performed with a *DANTEC* 55P61 X-wire probe. The hot-wires were operated in the constant temperature (CTA) mode with resistance overheat ratios in the range of 70–80 % either by means of an *DANTEC StreamLine* or an *AA-lab AN-1003* hot-wire anemometry system. The hot-wires were calibrated slightly upstream the inner nozzle against a Prandtl-tube. A modified King's law [19] was used, where the yaw response was computed using a sum and difference method [20] with experimentally determined calibration coefficients for the wires.

Most of the measurements have been obtained using one point statistics. The measurement set related to the triggering investigation, on the other hand, has been conducted in the cross stream direction and simultaneously behind the inner and outer wall in order to obtain conditional measurements able to highlight the dynamics of the interacting shear layers. The signals from the CTA channels were offset and amplified through the circuits to match the fluctuating signal components to the voltage range of the 16-bit A/D converter used, and then digitised on a PC at a sampling frequency of 5 kHz (10 kHz for spectral and conditional measurements) and a sampling duration between 10 and 40 seconds depending on the downstream position of the probe.

Additionally a series of flow visualisation photographs were taken for a range of absolute velocities and velocity ratios for both the sharp and thick wall cases. The particles used for the visualisation are small droplets of condensed smoke of polyethylenglycen and were injected at the inlet of the blower for the outer jet. A laser sheet from a 6 W Argon/Krypton Ion laser source was used to illuminate the flow. Images were recorded with a *NanoSense MkI* camera at a sampling frequency of 1 kHz.

3 RESULTS and DISCUSSION

In the following three sections results from the sharp and thick wall cases are presented to highlight the effect of the presence of the vortex shedding mechanism on interacting shear layers, section 3.1, its utilisation to control the flow dynamics, section 3.2, as well as its effect on the turbulence activity, section 3.3. The results shown here are all for the case of equal maximum velocities in the inner and annular jet, i.e. $r_u = 1$. The Reynolds numbers investigated range from around 12000 to 100000, based on the inner nozzle diameter, corresponding to absolute velocities from 4 to 32 m/s, respectively. The velocity ratio of unity was selected, because it lies within the range for which an absolute instability behind a thick wall has theoretically been shown to exist (cf. [17]).

3.1 Vortex shedding effect on interacting shear layers

In the case of the sharp wall, the power-spectral density of the fluctuating streamwise velocity component in the inner and outer shear layer as well as a snapshot of a smoke visualisation are shown in figure 3 and 4, respectively. The Kelvin-Helmholtz instability emerging between the annular stream and the ambient is clearly highlighted in the visualisation as well as in the spectral content of the flow (~ 240 Hz corresponding to a Strouhal number of 0.012 based on the momentum thickness of the external boundary layer, which is related to the 'shear layer mode'; cf. [21]). As the snapshot suggest, the flow behind the outer separating wall resembles a picture of a classical Kelvin-Helmholtz instability, whereas the sharp inner separating wall produces a smooth interface between the coflowing jets, being free of any apparent trace of a wake.

For the thick wall, on the other hand, both the smoke visualisation as well as spectral content of the flow, have strong imprints of the vortex shedding behind the inner wall. This changes the flow scenario drastically, as evident from figures 5 and 6. In contrast to the sharp wall results the spectral energy is highly concentrated in its fundamental peak and harmonics as well as in the incoherent background fluctuations anticipating an increased turbulence activity as well as the presence of organised structures.

The vortex shedding, starting right behind the inner wall and hence upstream the first emergence of the

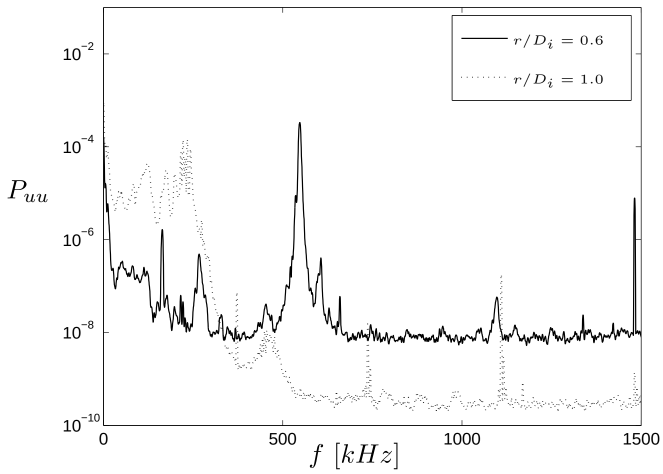

Figure 3: Power-spectral density function of the streamwise velocity component measured behind the inner ($r/D_i = 0.6$) and outer ($r/D_i = 1.0$) wall at $U_o = 8$ m/s, $r_u = 1$ and $x/D_i = 0.5$ for the sharp wall.

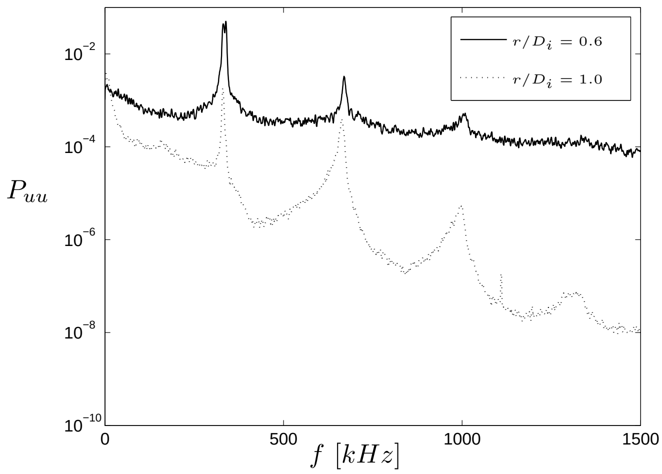

Figure 5: Power-spectral density function of the streamwise velocity component measured behind the inner ($r/D_i = 0.6$) and outer ($r/D_i = 1.0$) wall at $U_o = 8$ m/s, $r_u = 1$ and $x/D_i = 0.5$ for the thick wall.

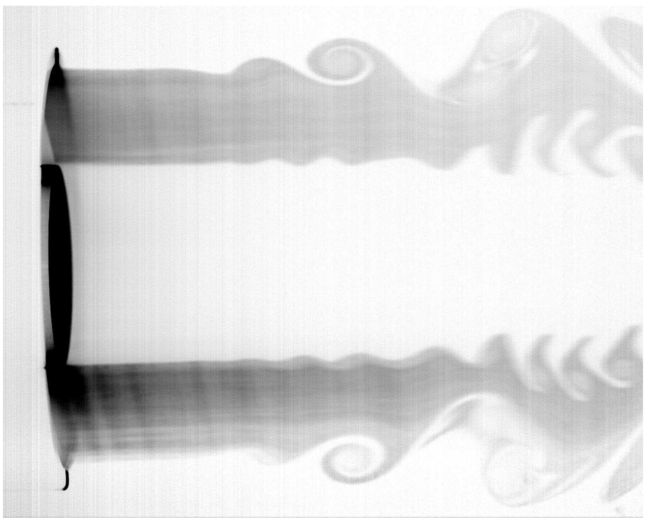

Figure 4: Smoke flow visualisation at $U_o = 4$ m/s and $r_u = 1$ for the sharp wall.

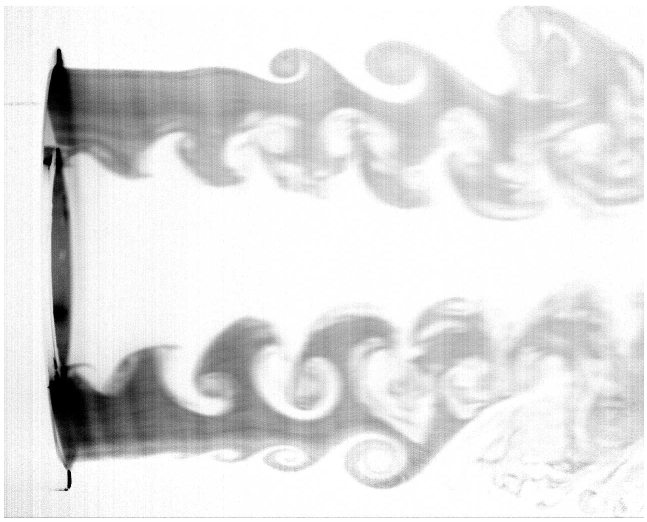

Figure 6: Smoke flow visualisation at $U_o = 4$ m/s and $r_u = 1$ for the thick wall.

Kelvin-Helmholtz instability of the outer shear layer in the sharp wall case, traps the outer shear layers vortices into the free spaces left between the ones of the inner shear layer. This increases the coherency between the two shear layers and can further be evinced through figure 7, where a pseudo-flow visualisation has been plotted through a conditional sampling technique. The vortices in the outer shear layer resemble the famous Kelvin's 'cat's eye' pattern predicted by stability analysis [22], whose organisation is strongly coupled to the vortex shedding in the inner shear layer. This again underlines the strong organisation and mutual interaction of the two shear layers in the whole near-field region as could be anticipated from the flow visualisation. An X-wire probe was thereby triggered by the vortex shedding behind the inner separating wall detected by a single hot-wire probe which was placed at the same downstream location, $x/D_i = 0.26$. Figure 8 depicts the instantaneous axial and radial velocities above their mean in the outer shear layer, $r/D_i = 1$, at $x/D_i = 0.26$ from an X-wire probe. The clear cyclical path in the (u, v)-space is a strong indication for the presence of a dominant peak in the outer region, and may also anticipate a formation of larger vortices [3].

The clear spectral peak of the vortex shedding phenomenon, the pronounced fundamental and its harmonics in the outer shear layer in figure 5, anticipate a clear cyclical path of the streamwise and radial velocity component as found in the conditionally sampled pseudo-flow visualisation. Although the instantaneous smoke visualisation image has been taken at a lower velocity (due to the difficulty to obtain clear images at higher velocities),

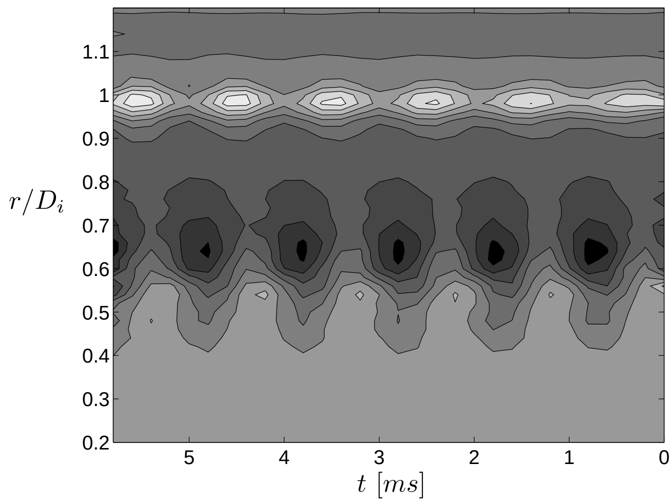

Figure 7: Pseudo-flow visualisation by means of conditional sampling technique of the radial velocity component at $U_o = 20$ m/s and $r_u = 1$ for the thick wall. Colormap from white to black corresponding to inflow and outflow conditions, respectively.

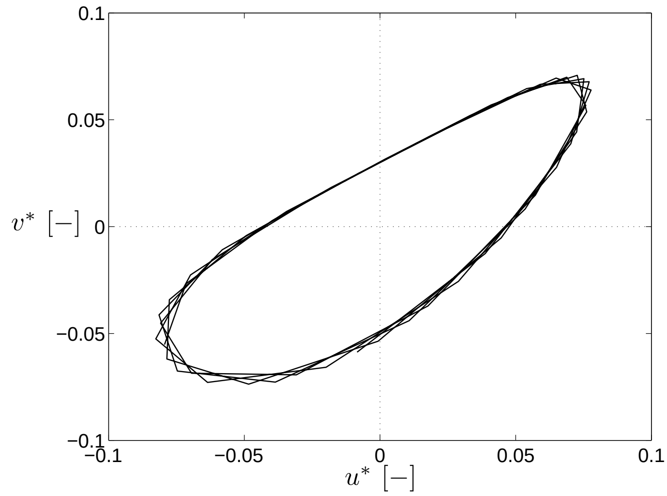

Figure 8: Instantaneous streamwise and radial velocity fluctuations plotted in the (u, v)-space above their mean for the thick wall case for $U_o = 20$ m/s at $x/D_i = 0.26$ in the outer shear layer, $r/D_i = 1$.

compared to the presented spectral measurements and pseudo-flow visualisation, its spatial character complements the quantitative time evolution of the two point hot-wire measurements.

These results demonstrate that the recent finding of Balarac and Métais [23], who stated that (below a critical velocity ratio) the vortices of the outer shear layer develop with a Strouhal number corresponding to the value predicted by linear stability analysis for the Kelvin-Helmholtz instability, is therefore not generally true. This is particularly true for the presence of an absolute instability, which—as shown in figures 5 and 6—dominates the motion of the organised structures.

3.2 Vortex shedding as a passive flow control mechanism

So far we have shown the effect of the vortex shedding on the near-field dynamics of a coaxial jet. The next paragraphs are devoted to the question whether or not the vortex shedding can be used as a viable flow control mechanism and how it affects the turbulence activity. Figure 9 shows the fundamental peak frequency as a function of the absolute velocity for both wall cases. No trend or relationship is observable for the sharp wall case (a) between the vortices in the inner and outer shear layer, whereas the strong controllability of the evolution of the vortices' fundamental frequency in both shear layers is evident from the linear relationship with the absolute velocity in the thick wall (b) case. Hence the absolute instability, by means of the induced vortex shedding behind the inner wall, dictates the evolution of the vortices in both shear layers and hence the whole flow field.

The overlapping at 16 and 20 m/s for the sharp wall in figure 9 could be explained by a whistler tone, which was observed around these absolute velocities (cf. [24]). However, there is no controllability of the fundamental peak, in the sharp wall case, whereas the thick wall presents a means to predetermine the evolution of the vortices in the inner and thereby outer shear layer, just by knowing the separating wall thickness, t, and the ab-

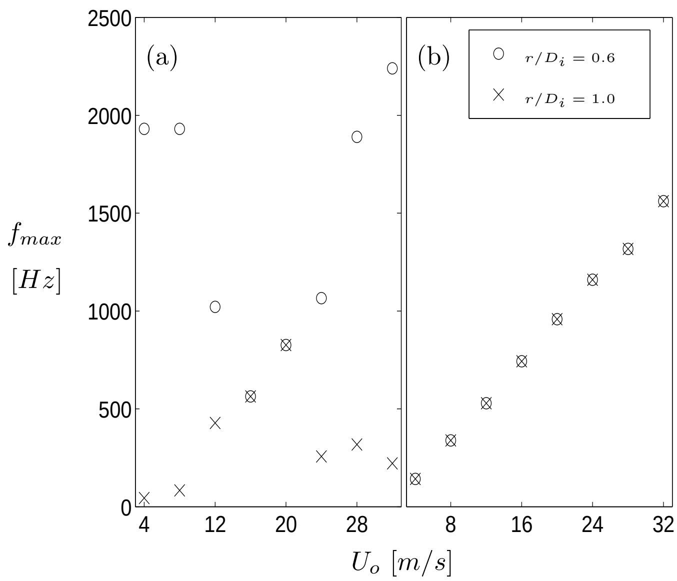

Figure 9: Fundamental frequency as function of the outer jet velocity behind the inner and outer wall at $r_u = 1$ and $x/D_i = 0.26$ for the sharp (a) and thick (b) wall.

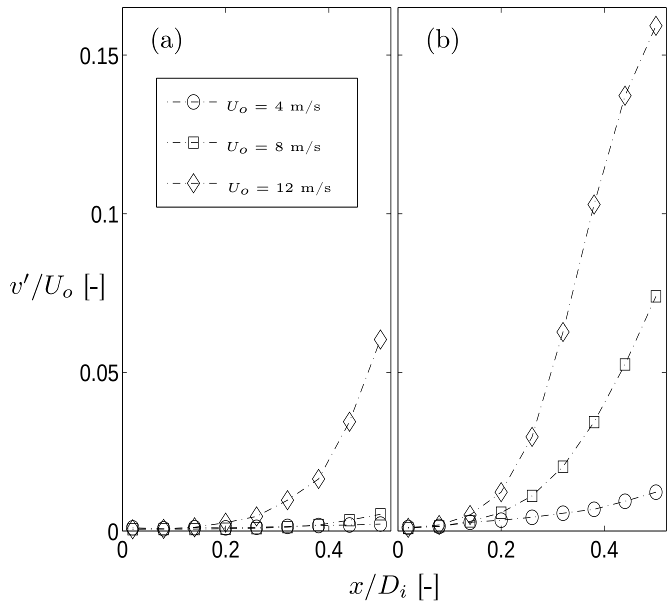

Figure 10: Root mean square value of the radial velocity fluctuations along $r/D_i = 1$ at 4, 8 and 12 m/s for the sharp (a) and thick (b) wall.

solute velocity, U_o, or the average velocity for the case of unequal velocities within the range of the presence of an absolute instability.

3.3 Vortex shedding effect on the turbulence activity

In the presence of active excitation, where the amplitude and phase of the excitation signal can be set arbitrarily, an increase in the organisation and formation of larger vortices can result either in a turbulence suppression [5, 12] or an turbulence enhancement [21, 25]. The power-spectral density functions showed that the energy content of the flow, with the presence of the vortex shedding, is not only brought about by means of the emergence of stronger organised structures, but also by a drastic increase in the incoherent background turbulence. Finally the normalised root mean square values of the radial velocity fluctuations along $r/D_i = 1$ for 4, 8 and 12 m/s are shown in figure 10 for the sharp (a) and thick (b) wall to complete the picture. A clear increase in the rms value of the normalised radial velocity fluctuations can be observed for all downstream positions as well as absolute velocities. Consequently the thick wall can be used not only to trigger and hence control the evolution of the organised structures, but also—and this is of more practical importance—to increase the turbulence activity within the inner and outer shear layers and thereby the mixing between the two coaxial jets streams as well as the annular jet with the ambient fluid.

A probably larger turbulence enhancement can be obtained if both fluid mechanical and geometrical parameters of the coaxial jet, like momentum thicknesses or the thickness of the separating wall, could be more easily varied.

4 CONCLUSIONS

In summary we have presented results from an experimental investigation confirming the theoretical study by Talamelli and Gavarini (2006), where it was suggested that the self-exciting temporally growing wake instability behind an inner thick wall in coaxial jets, may provide a continuous forcing mechanism for the passive control of the whole flow field. The present paper has hence shown, that despite the current trend to utilise mainly active flow control strategies in coaxial jet flows, there are basic flow features, like the vortex shedding behind a thick wall, which—if correctly recognised—can be facilitated to control the flow field without external energy supply.

It was furthermore shown that the 'locking phenomenon', which commonly is restricted to the dominance of the outer shear layers vortices over the inner ones, is reversible and therefore open doors to apply flow control strategies on the inner nozzle wall. This finding also restricts the finding by Balarac and Métais (2005), who showed that (below a critical velocity ratio) the vortices of the outer shear layer develop with a Strouhal number corresponding to the value predicted by linear stability analysis for the Kelvin-Helmholtz instability. It was shown that, particularly in the presence of an absolute instability by means of the vortex shedding mechanism, the vortices of the outer shear layer develop with a Strouhal number corresponding to a value related to the vortex shedding frequency behind the inner separating wall.

The present paper underlines the potential of this passive mechanism as an easy, effective and practical way to control the near-field of interacting shear layers and shows that it can be used to increase the turbulence activity in both shear layers and hence the mixing between the two coaxial jet streams as well as the annular jet with the ambient fluid.

ACKNOWLEDGEMENT

RÖ is supported by *The Swedish Research Council* (VR), and the cooperation between *KTH* and the *University of Bologna* is supported by *The Swedish Foundation for International Cooperation in Research and Higher Education* (STINT), which are both greatly acknowledged. Furthermore Prof. G. Buresti is acknowledged for placing the coaxial jet facility to the disposal of the *Second Faculty of Engineering* of the *University of Bologna* as well as Dr. P. Levoni and Mr. P. Proli for the assistance with the flow visualisations.

References

[1] Hussain, A.K.M.F., *Coherent structures and turbulence*, J. Fluid Mech., Vol. 173, pp. 303-356, 1986

[2] Roshko, A., *Structure of turbulent shear flows: A new look*, AIAA J., Vol. 14, pp. 1349-1357, 1976

[3] Fiedler, H.E., *Coherent structures*, Advances in Turbulence, Springer Verlag, pp. 320-336, 1987.

[4] Bradbury, L.J.S. and Khadem, A.H., *The distortion of a jet by tabs*, J. Fluid Mech., Vol. 70, pp. 801-813, 1975

[5] Zaman, K.B.M.Q. and Hussain, A.K.M.F., *Turbulence suppression in free shear flows by controlled excitation*, J. Fluid Mech., Vol. 103, pp. 133-159, 1981.

[6] Tong, C. and Warhaft, Z., *Turbulence suppression in a jet by means of a fine ring*, Phys. Fluids, Vol. 6, pp. 328-333, 1994

[7] Gad-el-Hak, M., Pollard, A. and Bonnet, J.P., *Flow Control: Fundamentals and Practices*, Springer Verlag, 1998

[8] Kiwata, T., Ishii, T., Kimura, S., Okajima, A. and Miyazaki, K., *Flow Visualization and Characteristics of a Coaxial Jet with a Tabbed Annular Nozzle*, JSME International Journal Series B, Vol. 49, pp. 906-913, 2006

[9] Angele, K.P., Kurimoto, N., Suzuki, Y. and Kasagi, N., *Evolution of the streamwise vortices in a coaxial jet controlled with micro flap-actuators*, J. Turbulence, Vol. 6, pp. 1-19, 2006

[10] Buresti, G., Talamelli, A. and Petagna, P., *Experimental characterization of the velocity field of a coaxial jet configuration*, Exp. Thermal Fluid Sci., Vol. 9, pp. 135-146, 1994

[11] Ko, N.W.M. and Kwan, A.S.H., *The initial region of subsonic coaxial jets*, J. Fluid Mech., Vol. 73, pp. 305-332, 1976

[12] Dahm, W.J.A., Frieler, C.E. and Tryggvason, G., *Vortex structure and dynamics in the near field of a coaxial jet*, J. Fluid Mech., Vol. 241, pp. 371-402, 1992

[13] Wicker, R.B. and Eaton, J.K., *Near field of a coaxial jet with and without axial excitation*, AIAA J., Vol. 32, pp. 542-546, 1994

[14] da Silva, C.B., Balarac, G. and Métais, O., *Transition in high velocity ratio coaxial jets analysed from direct numerical simulations*, J. Turbulence, Vol. 4, pp. 1-18, 2003

[15] Balarac, G., Métais, O. and Lesieur, M., *Mixing enhancement in coaxial jets through inflow forcing: A numerical study*, Phys. Fluids, Vol. 19, 075102, 2007

[16] Braud, C., Heitz, D., Arroyo, G., Perret, L., Delville, J. and Bonnet, J.P., *Low-dimensional analysis, using POD, for two mixing layer-wake interactions*, Int. J. Heat Fluid Flow, Vol. 25, 351-363, 2004

[17] Talamelli, A. and Gavarini, I., *Linear instability characteristics of incompressible coaxial jets*, Flow Turbul. Combust., Vol. 76, pp. 221-240, 2006

[18] Hussein, H.J., Capp, S.P. and George, W.K., *Velocity measurements in a high-Reynolds-number, momentum-conserving, axisymmetric, turbulent jet*, J. Fluid Mech., Vol. 258, pp. 31-75, 1994

[19] Johansson, A.V. and Alfredsson, P.H., *On the structure of turbulent channel flow*, J. Fluid Mech., Vol. 122, pp. 291-314, 1982

[20] Bruun, H.H., *Hot-wire anemometry: Principles and signal analysis*, Oxford University Press Inc., 1995

[21] Zaman, K.B.M.Q. and Hussain, A.K.M.F., *Vortex pairing in a circular jet under controlled excitation. Part 1. General jet response*, J. Fluid Mech., Vol. 101, 449-491, 1980

[22] Drazin, P., *Introduction to Hydrodynamic Stability*, Cambridge University Press, 2002

[23] Balarac, G. and Métais, O. *The near field of coaxial jets: A numerical study*, Phys. Fluids, Vol. 17, 065102, 2005

[24] Örlü, R., Segalini, A., Alfredsson, P.H. and Talamelli, A., *Passive control of mixing in a coaxial jet*, Proc. of 7th Int. ERCOFTAC Symp. on Engineering Turbulence Modelling and Measurements (ETMM7), Vol. 2, pp. 450-455, 2008

[25] Crow, S.C. and Champagne, F.H., *Orderly structure in jet turbulence*, J. Fluid Mech., Vol. 48, 547-591, 1971