

# Progress in Simulations of Turbulent Boundary Layers



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## Philipp Schlatter

Ramis Örlü, Qiang Li, Geert Brethouwer,  
Henrik Alfredsson, Arne Johansson, Dan Henningson

**Linné FLOW Centre**, KTH Mechanics,  
Stockholm, Sweden

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# Outline: Turbulent Boundary Layers



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- Comparison of DNS
  - Re-evaluation of data
- New KTH simulations and experiments
  - Something about codes
  - Establishment of fully-developed turbulence
  - Detailed comparison to experiments
- Some findings and detours
  - Wall shear stress, negative velocities and high flatness
  - Modulation of near-wall turbulence
  - Three-dimensional effects
  - Suction boundary layer
  - Coherent structures (Eduction, Visualisations)
  - Passive scalars and free-stream turbulence
  - Sublayer scaling, finding the wall and correcting hotwires
  - Ongoing new simulation
- Conclusions

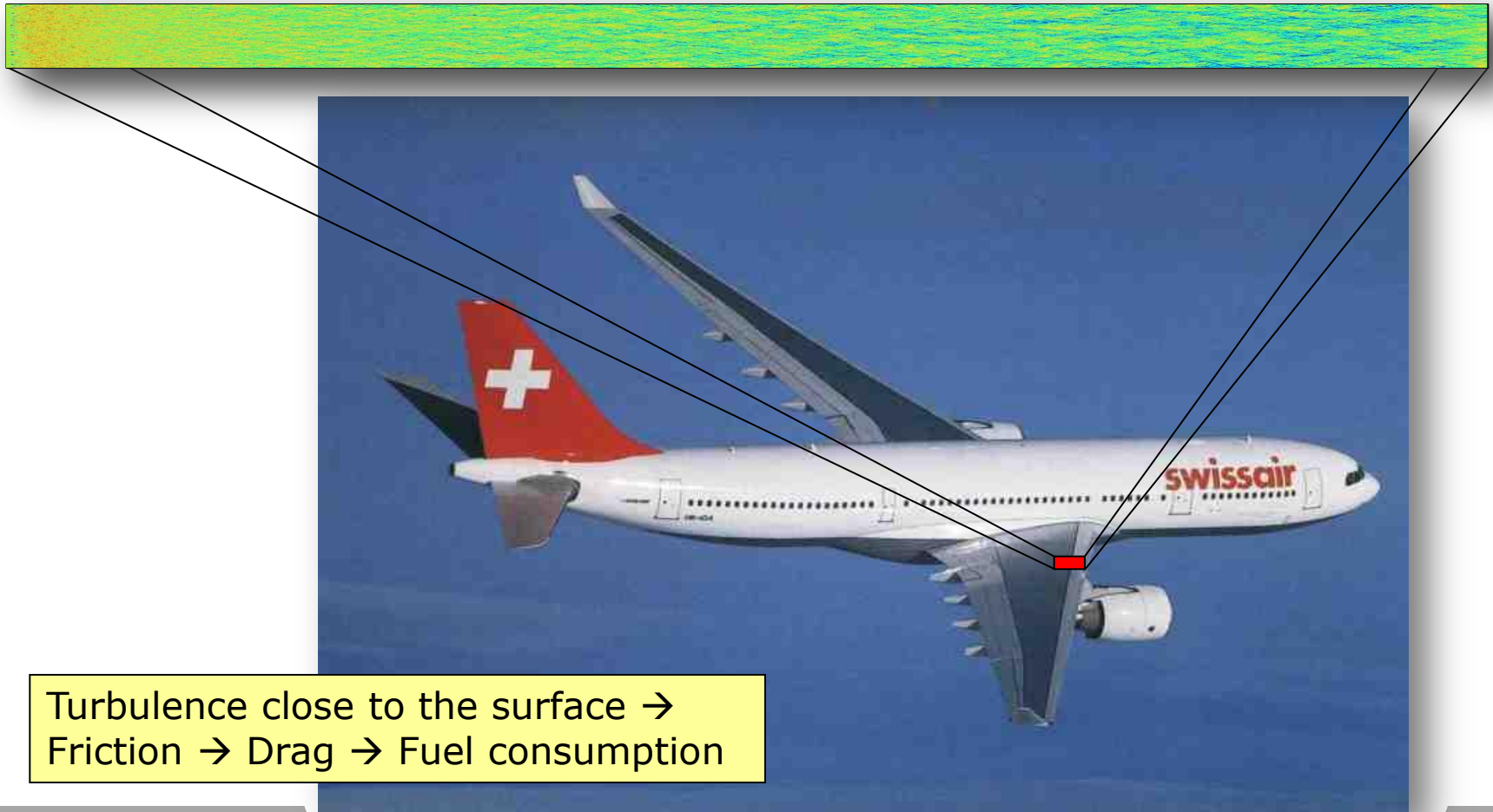
# Outline: Turbulent Boundary Layers



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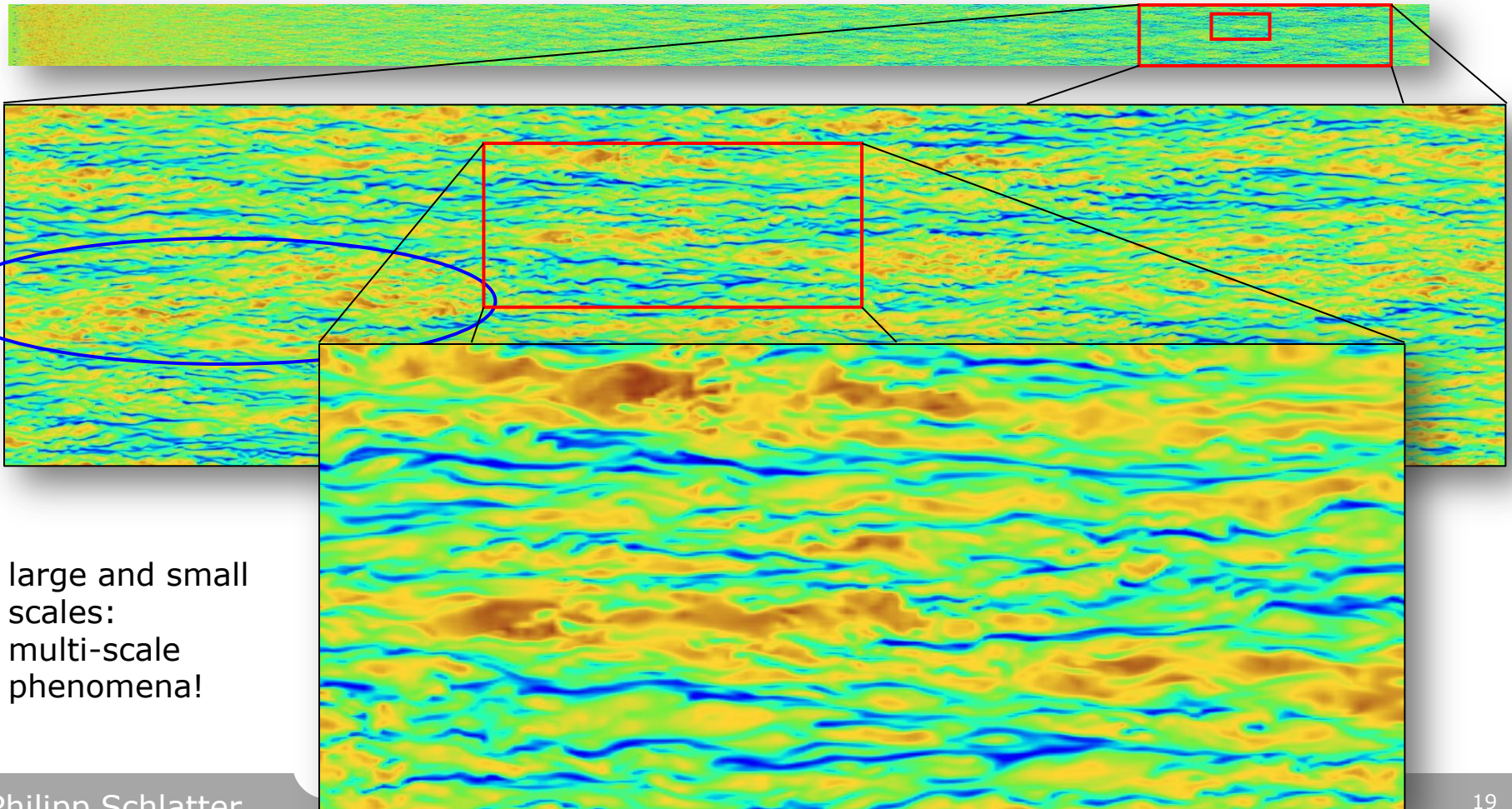
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# Turbulent flow close to solid walls...

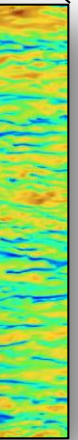
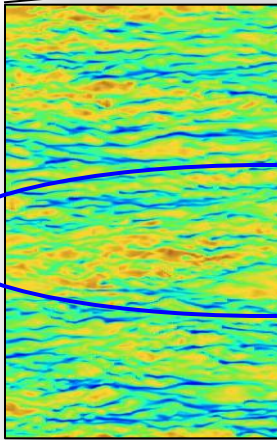
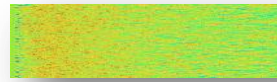
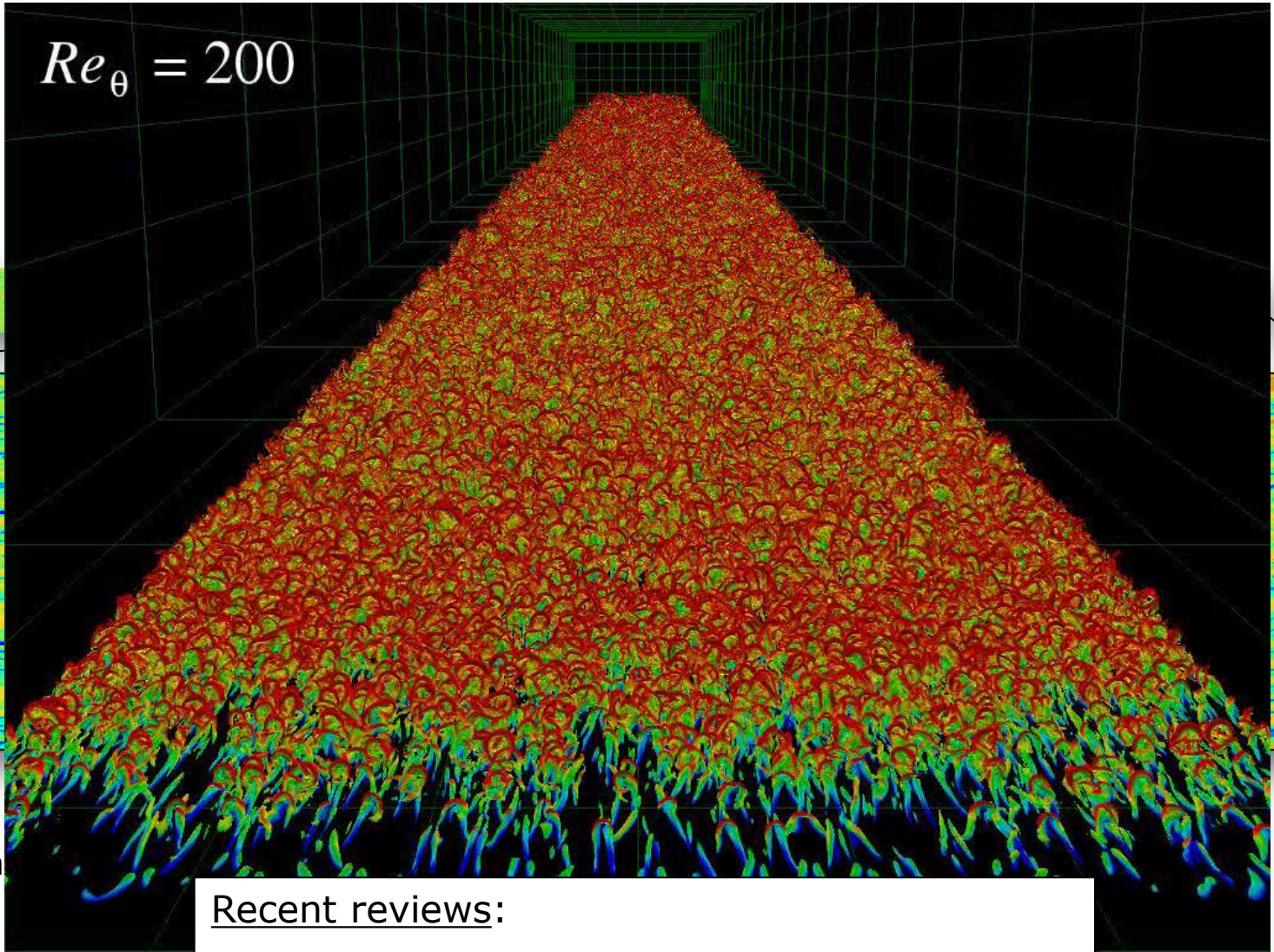


# Turbulent flow close to solid walls...

(simulation result)



$$Re_{\theta} = 200$$



large and small scales:  
multi-scale phenomena!

Recent reviews:

Marusic *et al.*, *Phys. Fluids*, 2010

Klewicky, *J. Fluids Eng.*, 2010

Smits *et al.*, *Annu. Rev. Fluid Mech.*, 2011



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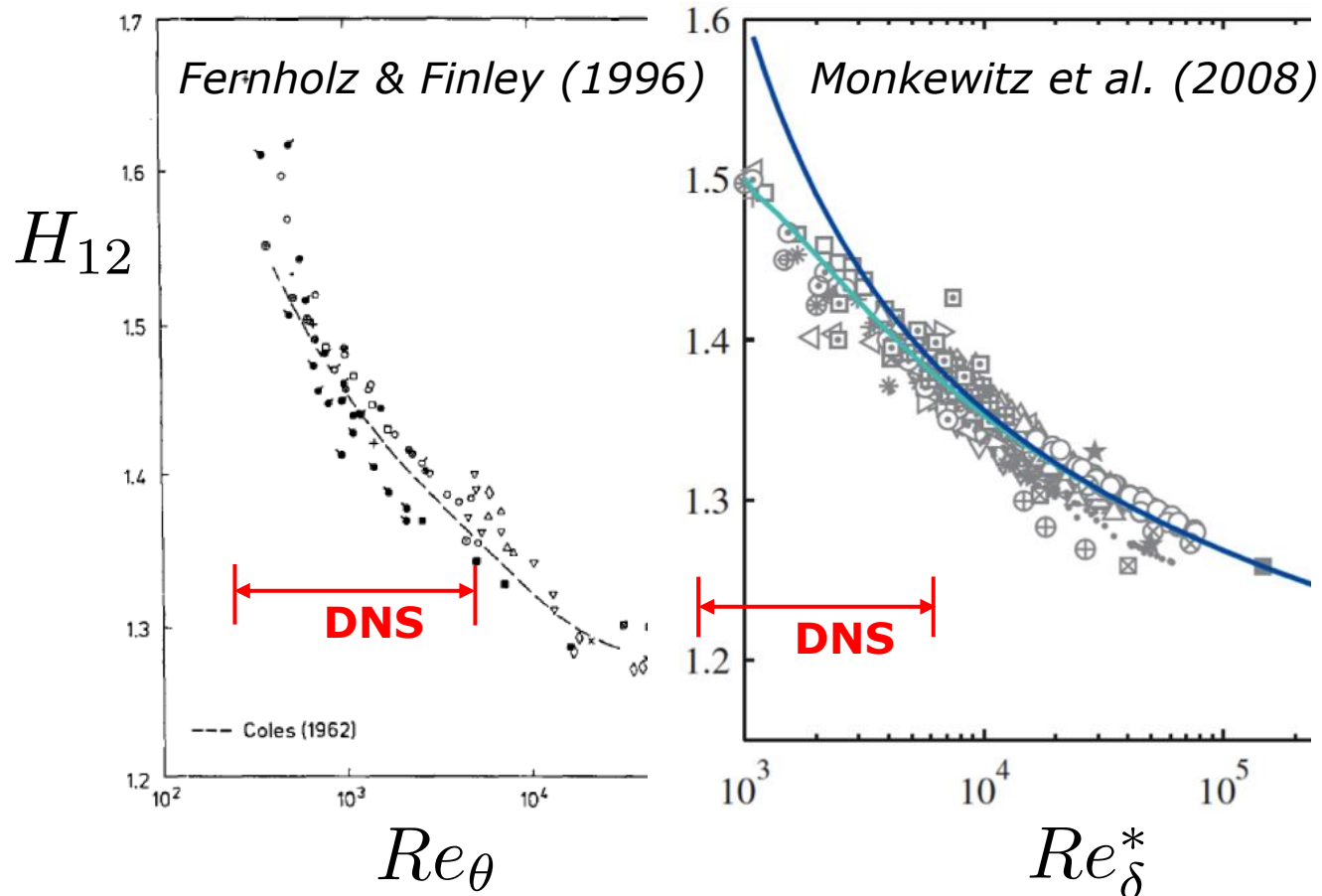
# DNS of Turbulent Boundary Layers (TBL)

# What we are used/expect to see ...



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Compilation/  
Assessment of  
experimental data  
from ZPG TBL flows



**Physical experiments** are commonly scrutinised before they are employed to calibrate, test, or validate other experiments, scaling laws or theories

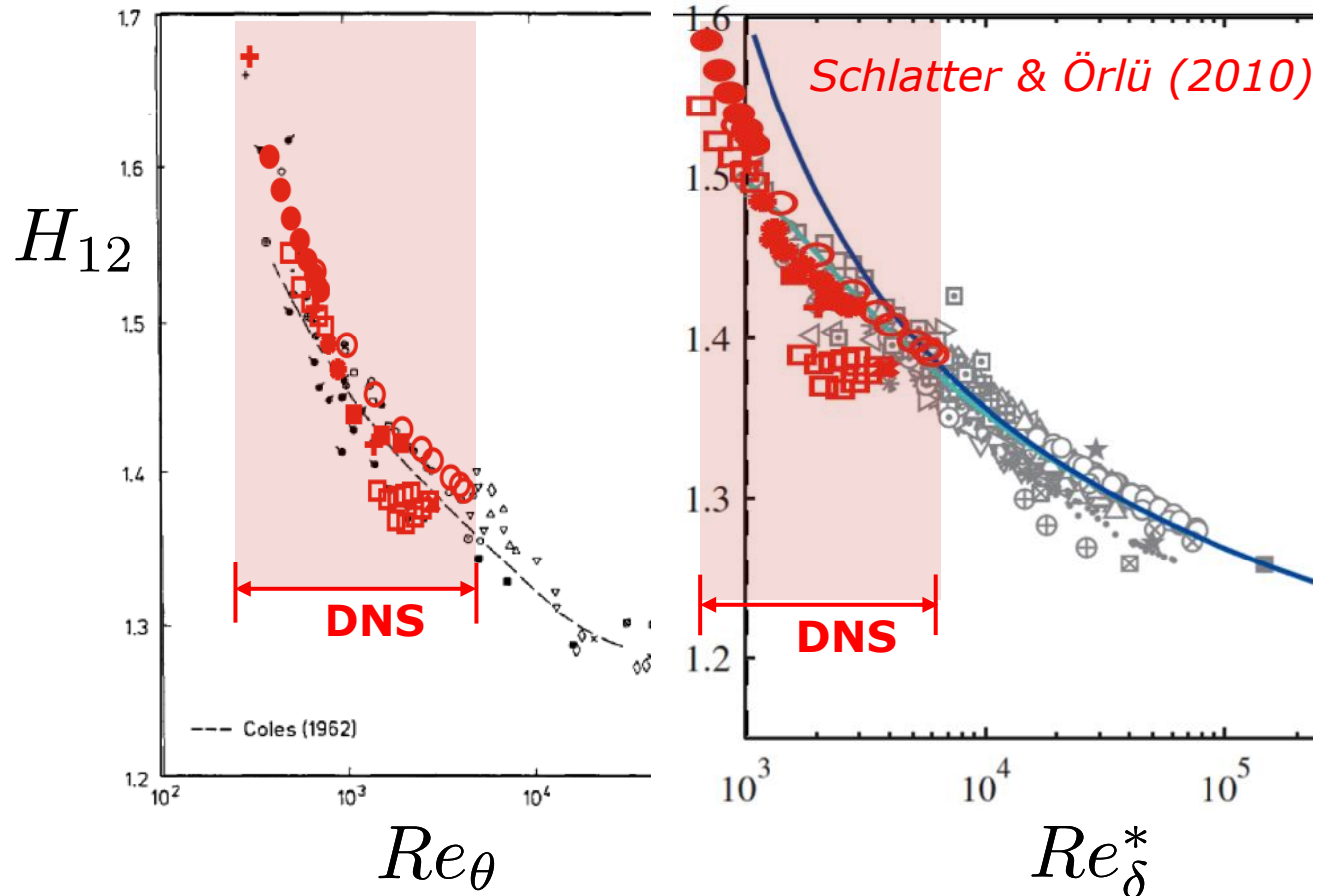


# ... and what "we" are not so used to see



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Red symbols are  
data from 7  
independent DNS  
from ZPG TBL flows



**Simulation data** are hardly scrutinised, when it comes to basic (integral) quantities

# Employed ZPG TBL DNS data

Reference	$Re_\theta$	Method	Symbol
Spalart (1988)	300, 600, 1410	spectral spatio-temporal	+
Komminaho & Skote (2002)	383 – 716	spectral tripping at $Re_\theta \approx 200$ domain up to $Re_\theta \approx 750$	●
Khujadze & Oberlack (2004; 2007)	489 – 2807	spectral tripping close to resp. inflow domain length $\Delta Re_\theta \approx 500$	□
Ferrante & Elghobashi (2005)	2900	finite differences rescaling and recycling domain $Re_\theta = 2340 - 2900$	×△
Simens <i>et al.</i> (2009)	1000, 1550, 1968	finite-difference/spectral rescaling and recycling domain $Re_\theta = 620 - 2140$	■
Schlatter <i>et al.</i> (2009a; 2009b)	677 – 4271	spectral tripping at $Re_\theta = 180$ , single domain up to $Re_\theta \approx 4300$	○
Wu & Moin (2009)	800, 900	finite difference free-stream disturbances	*
Wu & Moin (2010)	900 – 1840	laminar inflow at $Re_\theta = 80$	*
Lee & Sung (2011)	2560	finite differences, recycling	◇

Ref.: Schlatter & Örlü, *J. Fluid Mech.* 2010

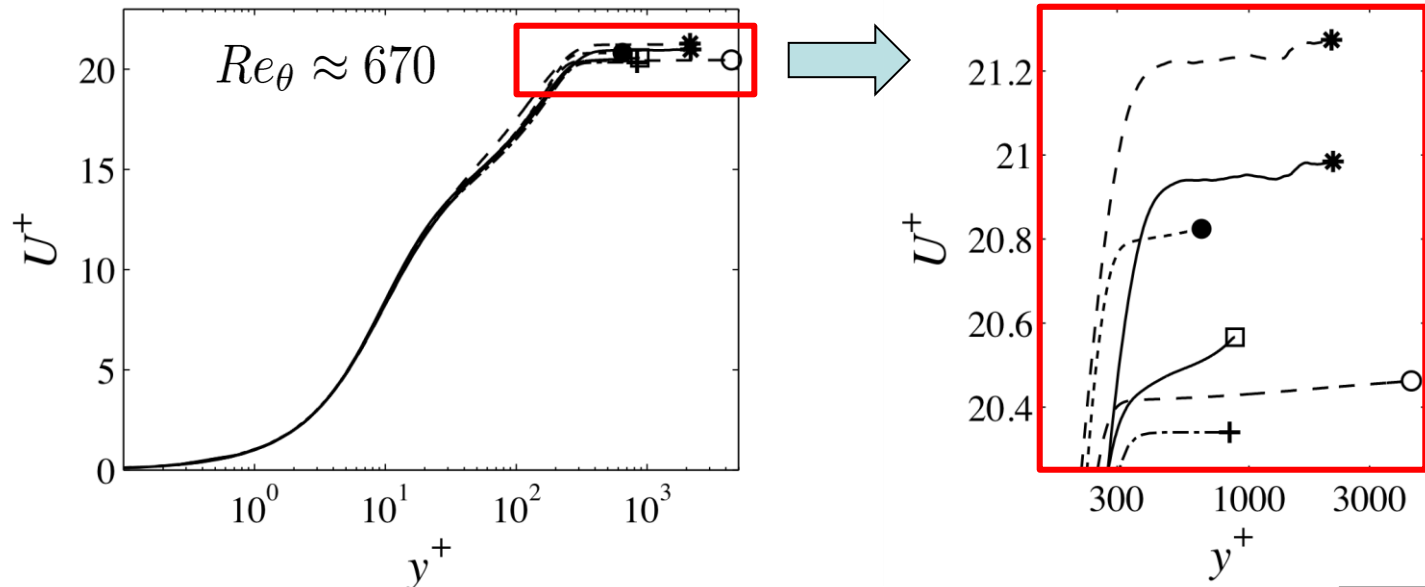
# Justification for re-evaluation

- Integral quantities are often given as function of  $Re$ , however, **how** these were computed is often not given in detail
- Varying free-stream velocities for  $y^+ > \delta^+$  implies (in conjunction with quite varying box height) unambiguous upper integral bound and free-stream velocity

→ Need for consistent re-evaluation!



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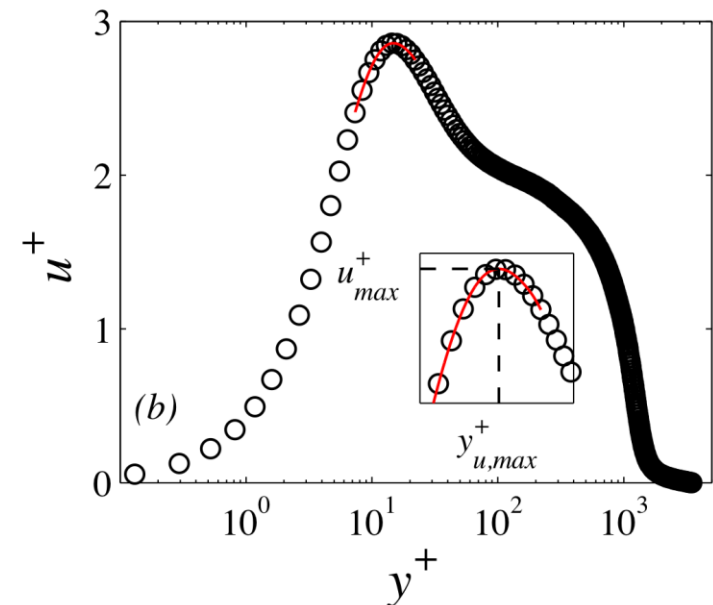
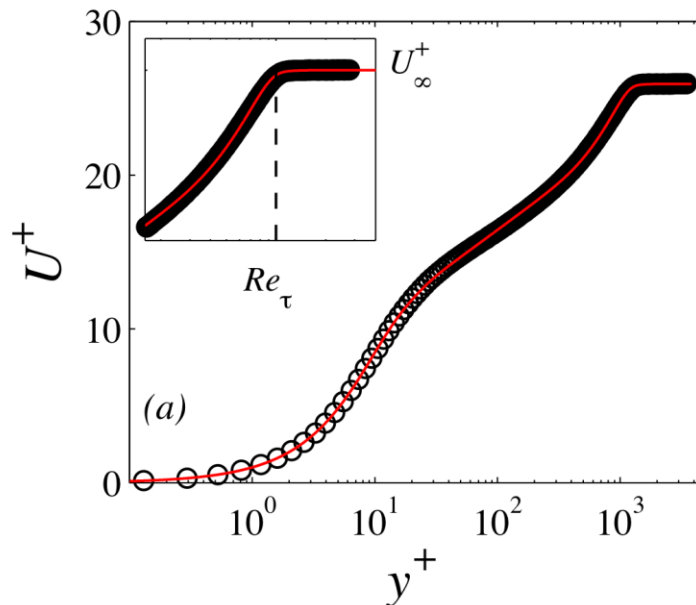


# Consistent way to re-evaluate

- For the following re-evaluation we make use of the **Nickels (2004)** composite profile to determine free-stream velocity and the 99% boundary-layer thickness
- **Chauhan, Monkewitz & Nagib (2009)** composite profile for near-wall comparisons
- 4th order polynomial fit around maxima of Reynolds stresses to determine peak value and location



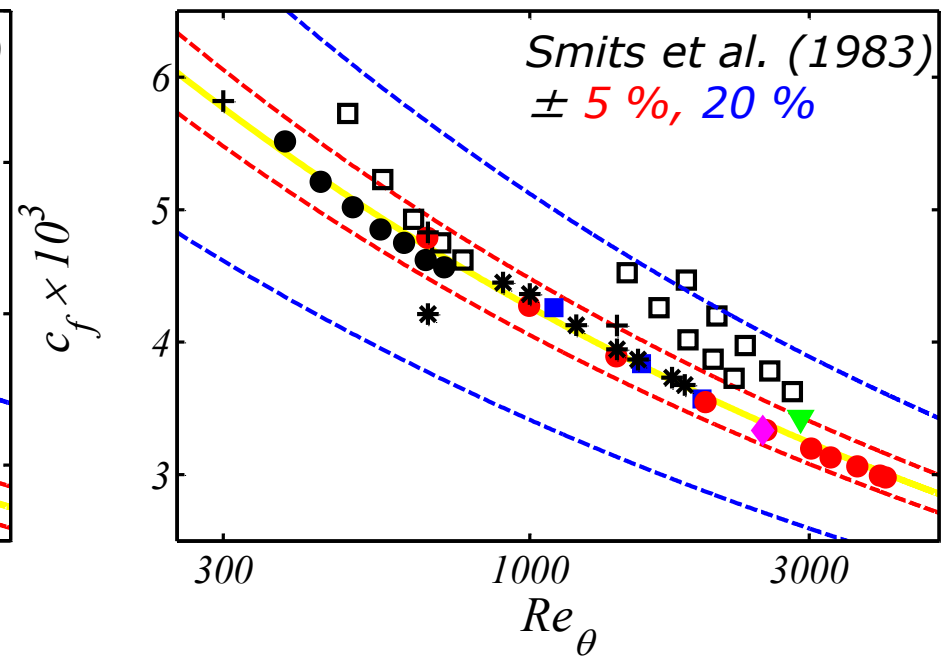
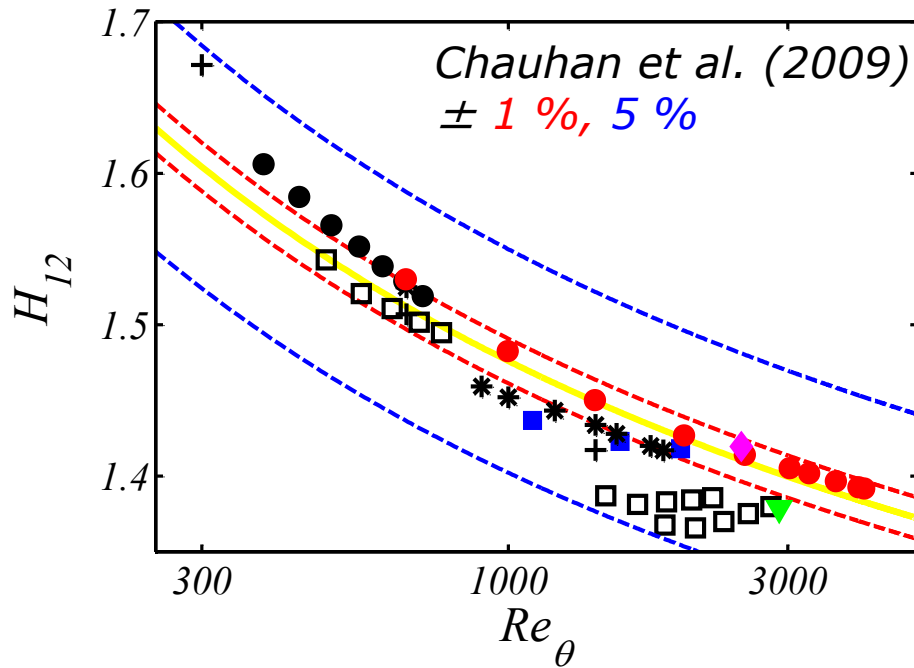
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# A closer look at DNS from ZPG TBL flows

## *Shape factor & Skin friction*

- “we” are usually very confident about DNS data, at least when it comes to **basic integral quantities**, but ...

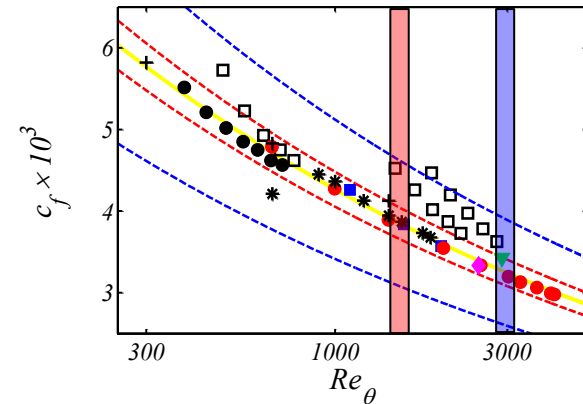
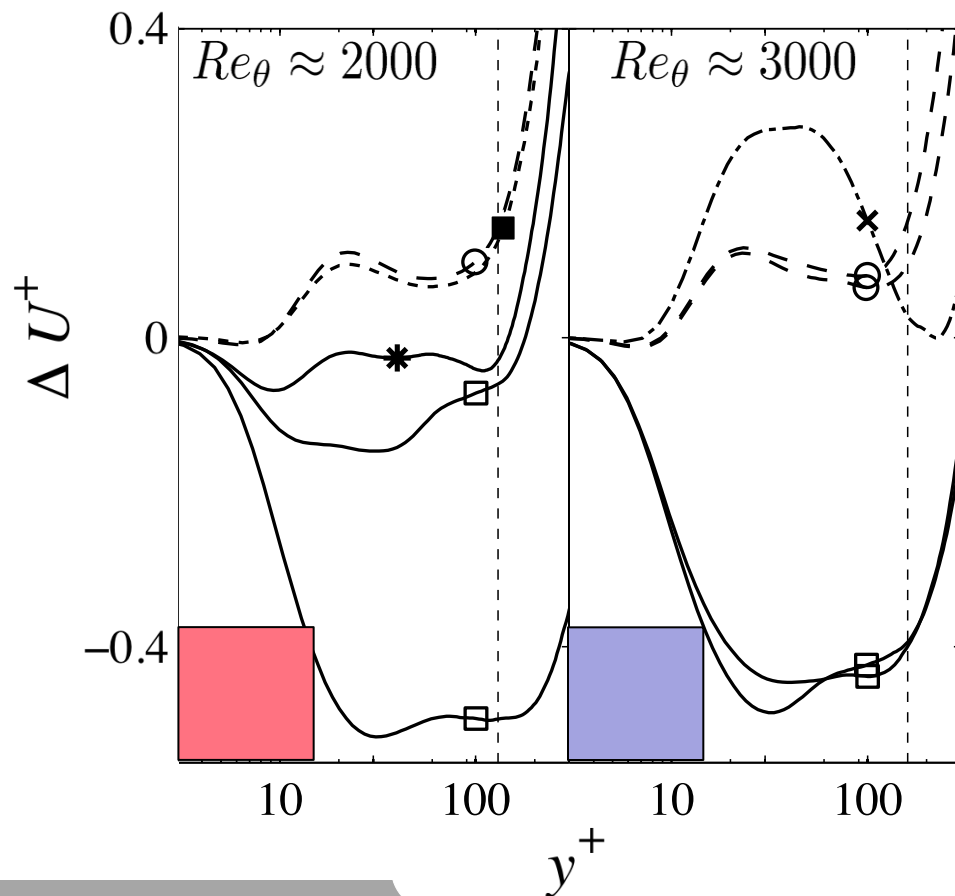


Data from 8 independent DNS (Schlatter & Örlü, *JFM*, 2010)

# A closer look...

## Inner layer

$$\Delta U^+ = U_{\text{DNS}}^+ - U_{\text{Chauhan et al. (2009)}}^+$$



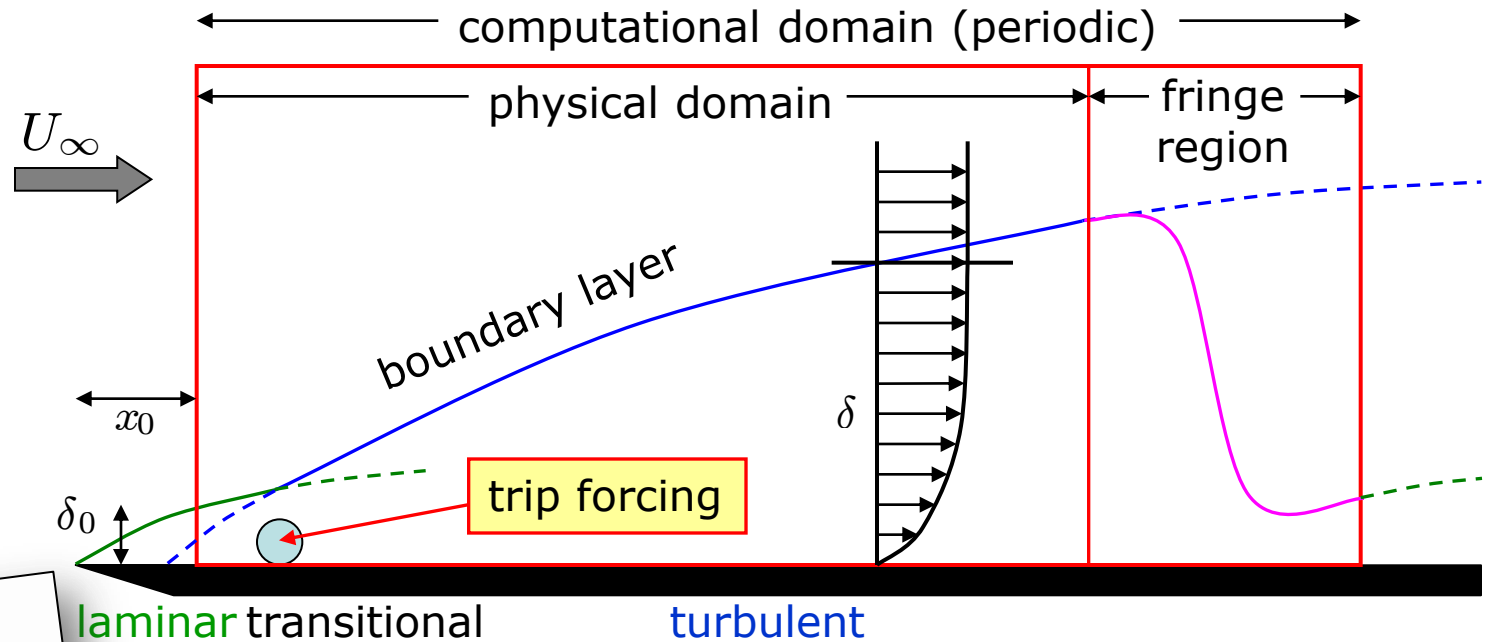
- “we” are usually very confident about DNS data, when it comes to **mean velocity profiles**, but ...
- **Note of caution**: profiles have been utilised in the past to develop corrections for total-head probes, wall position, friction velocity, etc...  
(see Örlü *et al.*, JPAS, 2010)



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Need for new simulations  
close to experiments...

# Spatial Boundary Layer



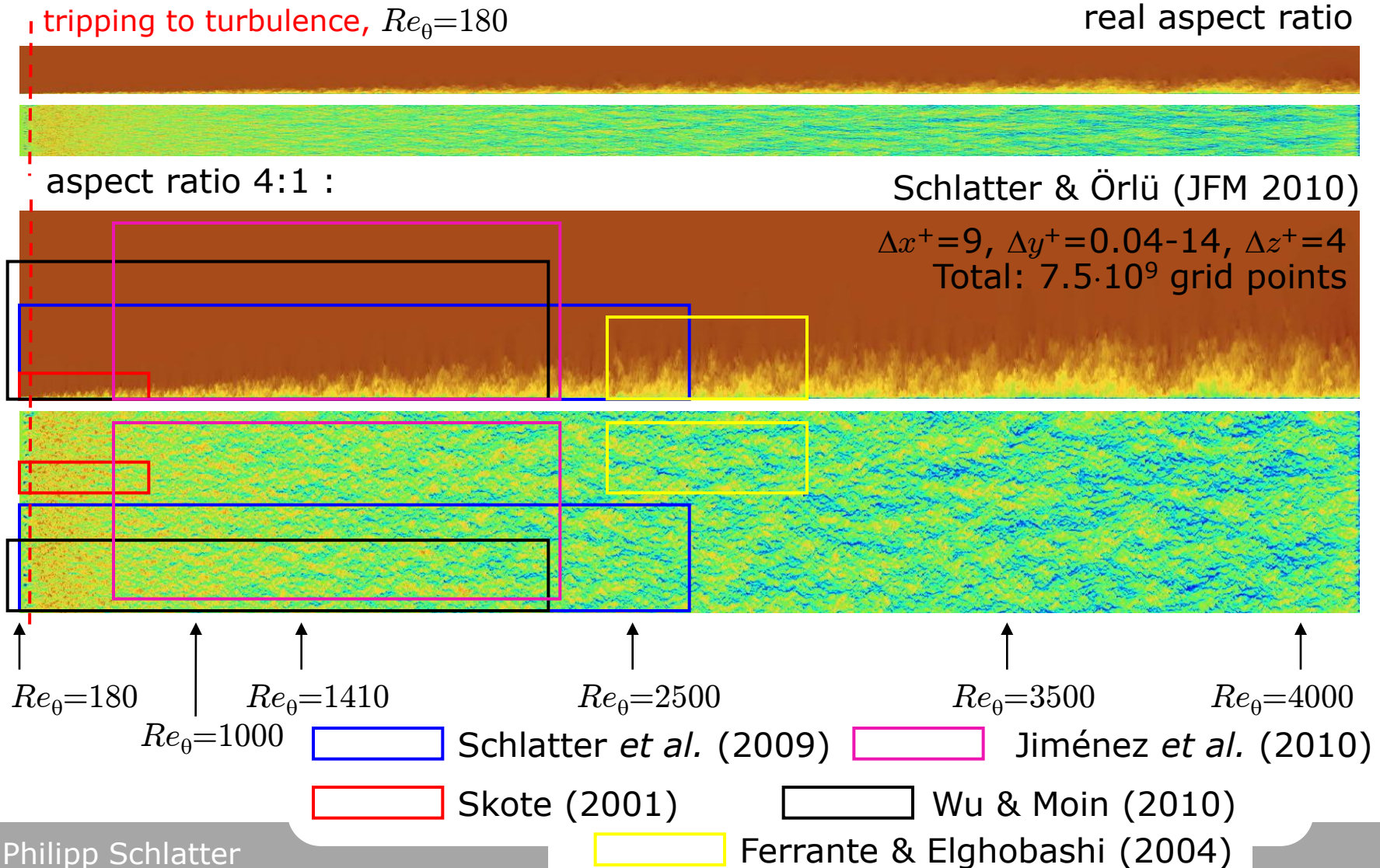
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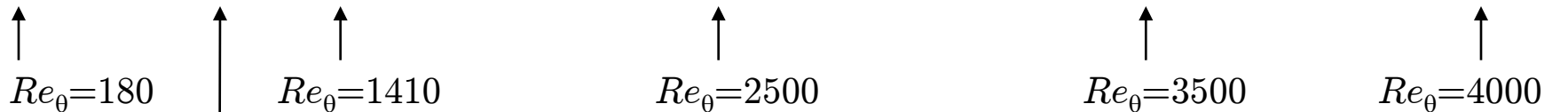
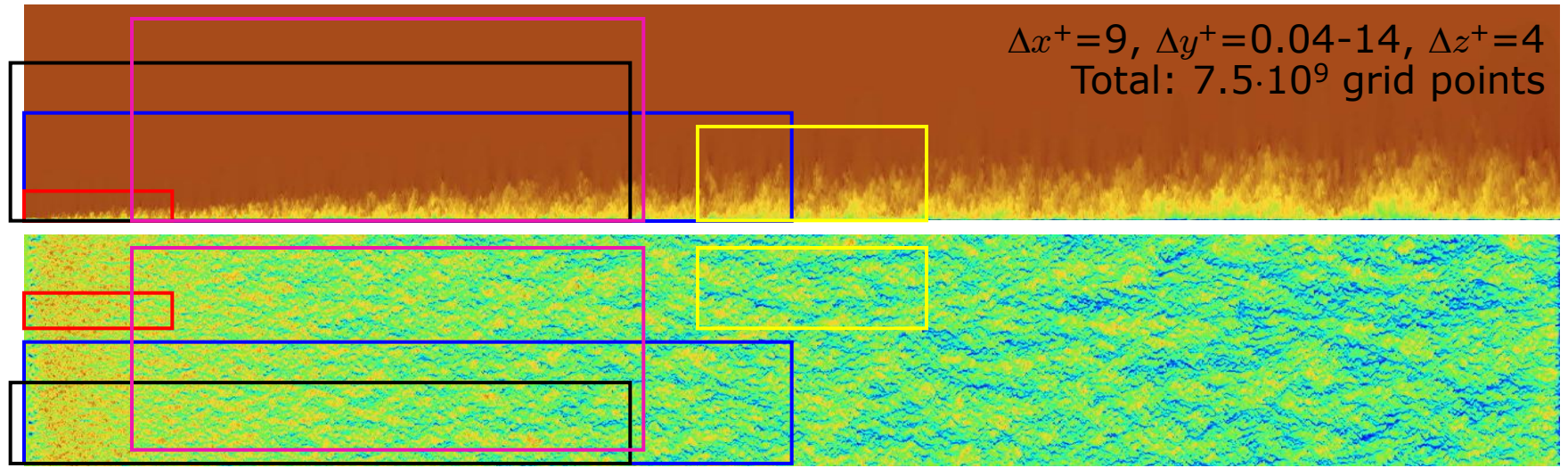
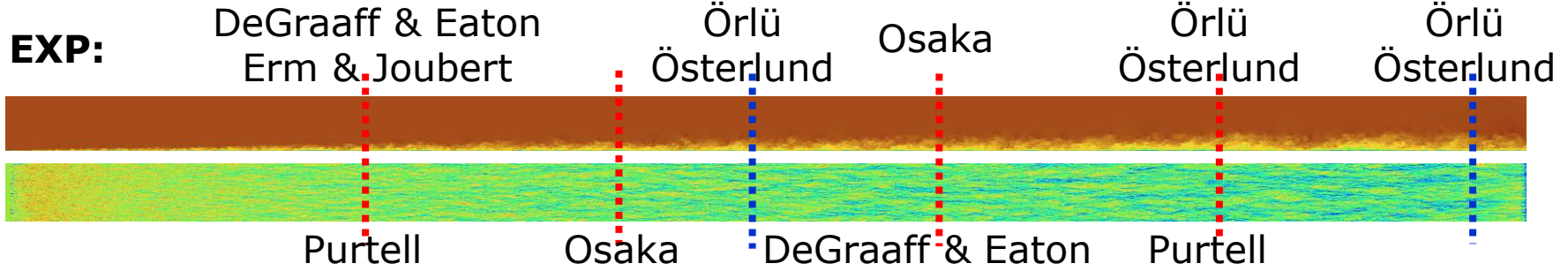
- Fully spectral method: Fourier/Chebyshev tau method
- Periodic boundary condition in the wall-parallel directions, no-slip at lower wall, Neumann conditions at upper boundary.
- Fringe region (volume force) to enforce laminar Blasius inflow
- Trip forcing to induce "natural" laminar-turbulent transition
- Code **SIMSON** (Chevalier *et al.* 2007) on up to 16384 cores BG/P



# TBL up to $Re_\theta = 4300 \dots$



# TBL up to $Re_\theta = 4300 \dots$



- Schlatter *et al.* (2009)
- Jiménez *et al.* (2010)
- Skote (2001)
- Wu & Moin (2010)
- Ferrante & Elghobashi (2004)

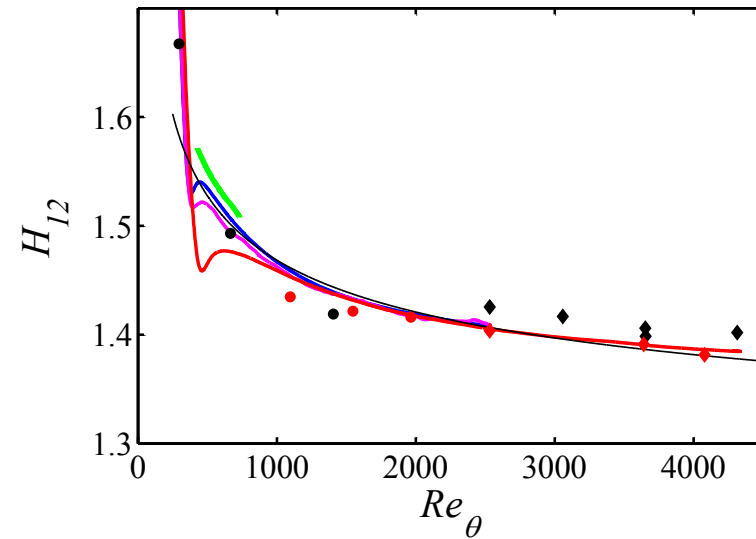
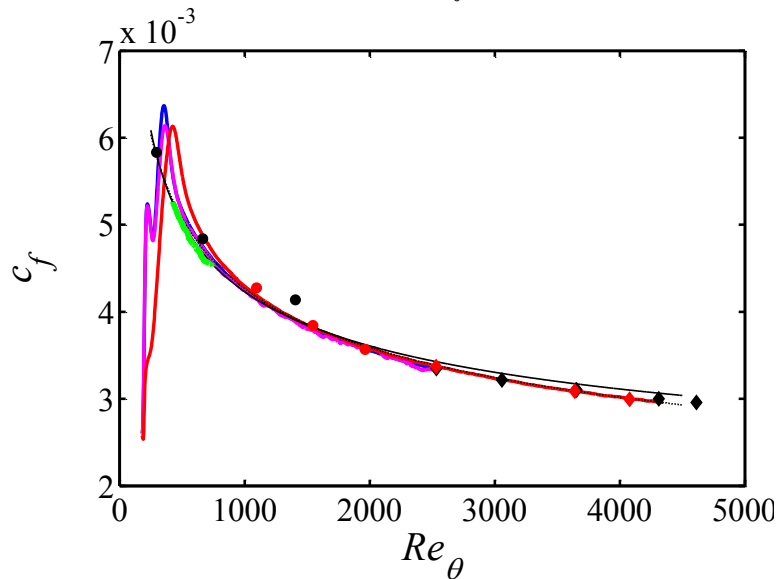


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# Some quick statistics...

# Integral Quantities up to $Re_\theta=4300$

- Skin friction  $c_f$  and shape factor  $H_{12}$



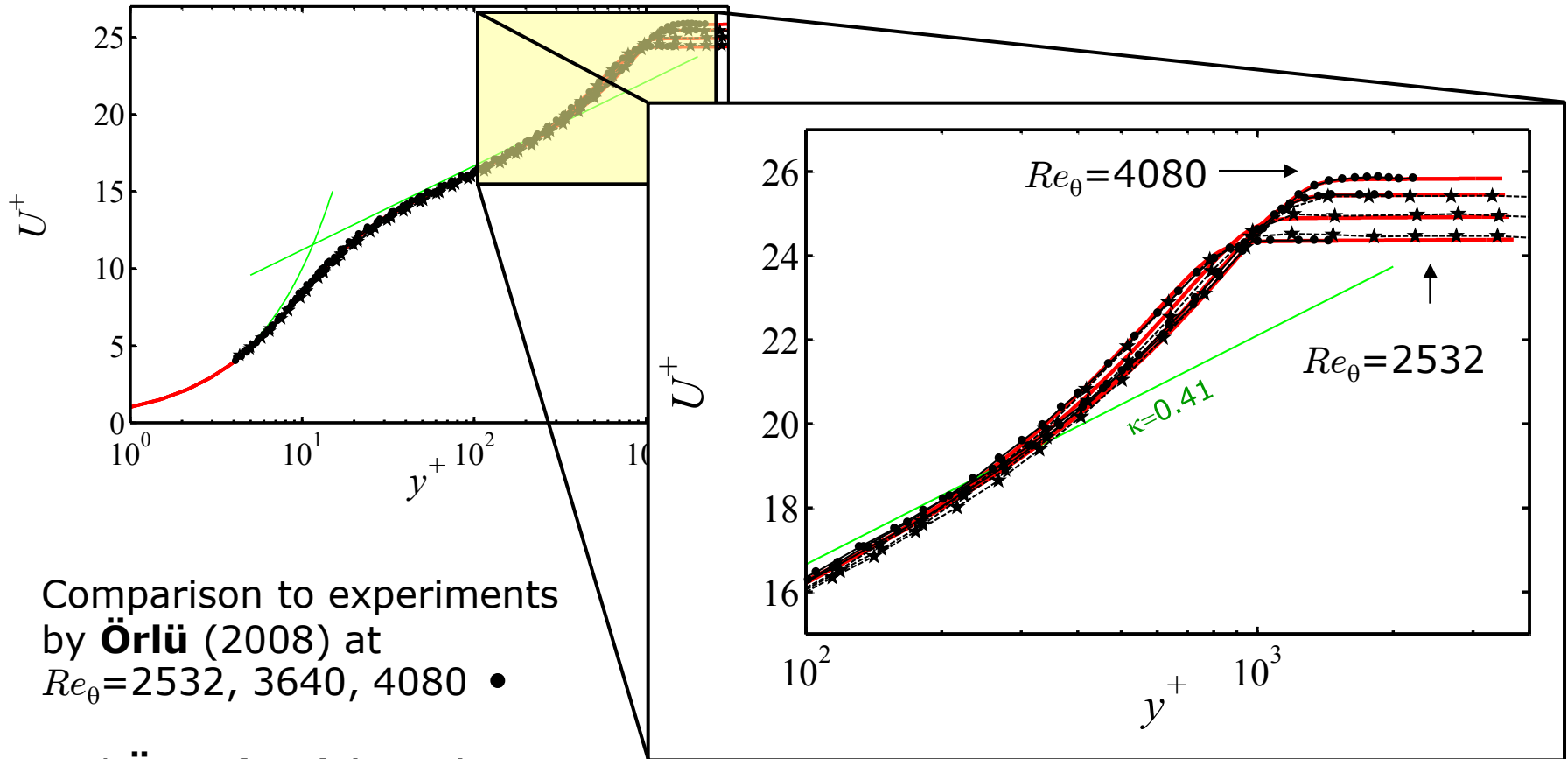
- DNS Spalart (1988)  $Re_\theta=300, 670, 1410$
- DNS Jiménez *et al.* (2009)  $Re_\theta=1100, 1550, 2000$
- EXP Österlund (1999)  $Re_\theta=2500, 3000 \dots$
- EXP Örlü (2008)  $Re_\theta=2500, 3000 \dots$
- Correlations (Monkewitz *et al.* 2007, Österlund (1999))
- DNS Skote (2001)

— Medium DNS — Fine DNS — High DNS



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# DNS – Comparison to EXP



Comparison to experiments by **Örlü** (2008) at  $Re_\theta = 2532, 3640, 4080$  •

and **Österlund** (1999) at  $Re_\theta = 2532, 3060, 3651$  ★

— present DNS at matching  $Re_\theta$

# Von Kármán Integral Equation

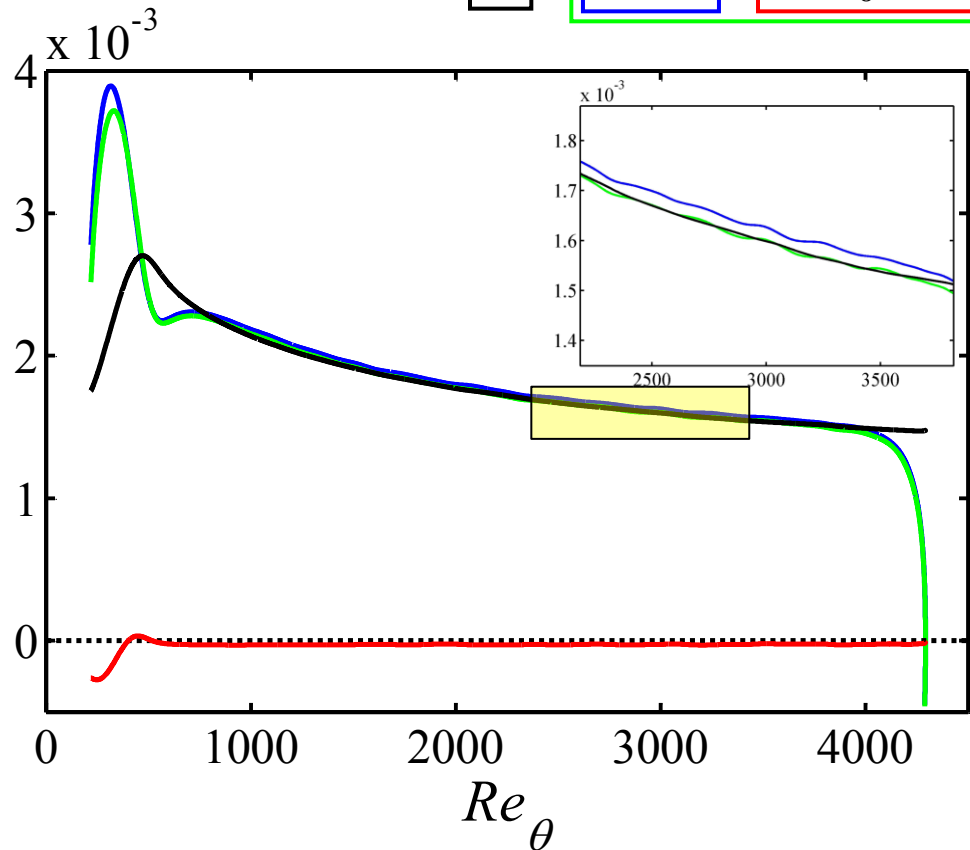
- von Kármán equation  $u_\tau^2 = U_\infty^2 \frac{d\theta}{dx} + \frac{d}{dx} \int_0^\infty (\langle u'^2 \rangle - \langle v'^2 \rangle) dy$

$\approx -3 \cdot 10^{-5} U_\infty^2$



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$$(u_\tau / U_\infty)^2 = (1/2) c_f$$



- Derived based on boundary-layer approximation
- Terms balance up to  $\mathcal{O}(0.1\%)$



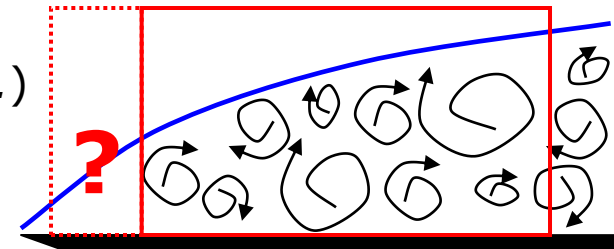
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However, how about  
lower Reynolds numbers?

# Evolution from initial conditions

- Spatial development → turbulence needs to be continuously generated close to / at the inflow:

- artificial turbulence (e.g. Klein *et al.*)
- precursor (periodic) simulation
- recycling/rescaling (Lund *et al.*)
- **tripping/transition to turbulence**



- Immediate questions:
  - Depending on method, what inflow length is necessary?
  - Pressure gradient during adaptation/transition?
  - what is the lowest  $Re$  for "fully developed" turbulence

**Similar issues in experiments**, see e.g. Erm & Joubert (1991), Castillo *et al.* (2004)

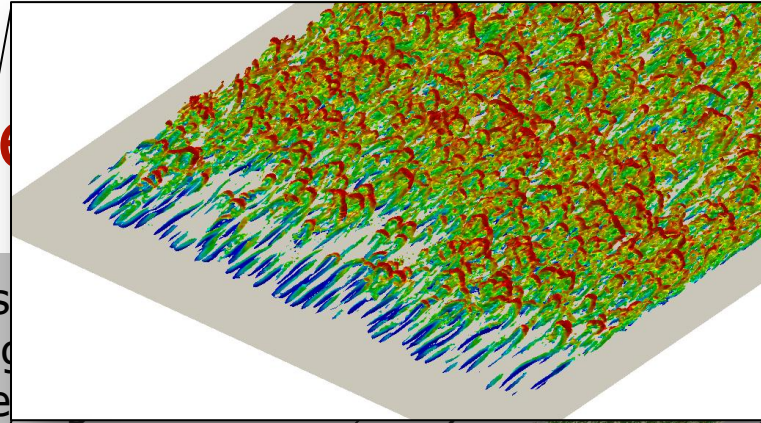


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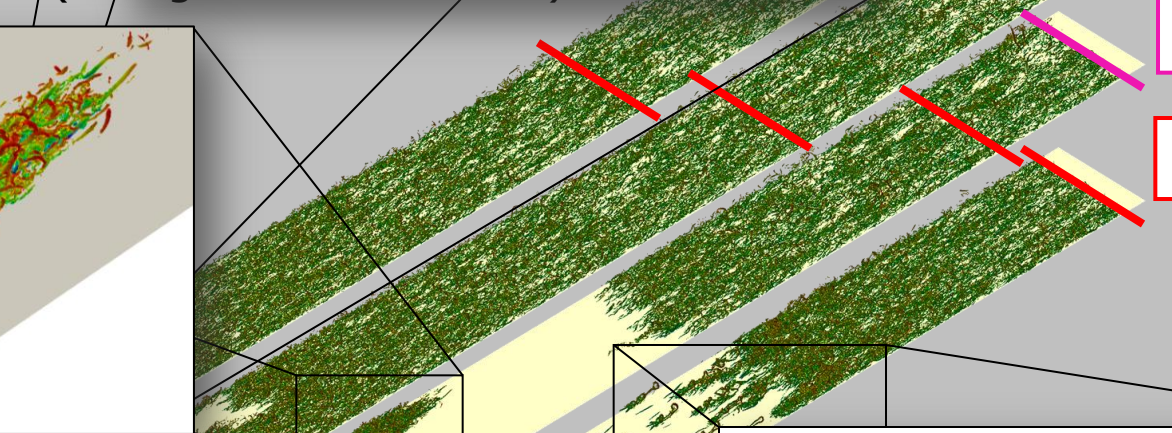
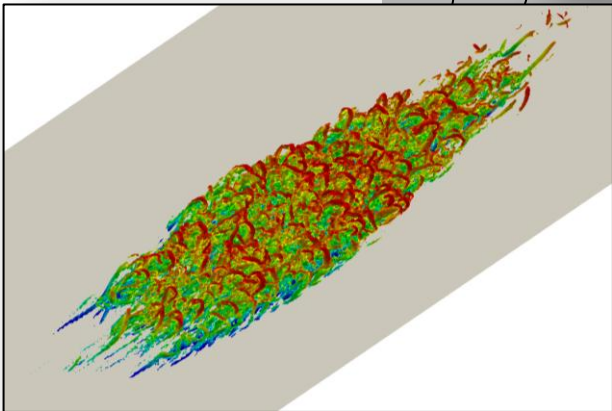


$Re_\theta = 1700$

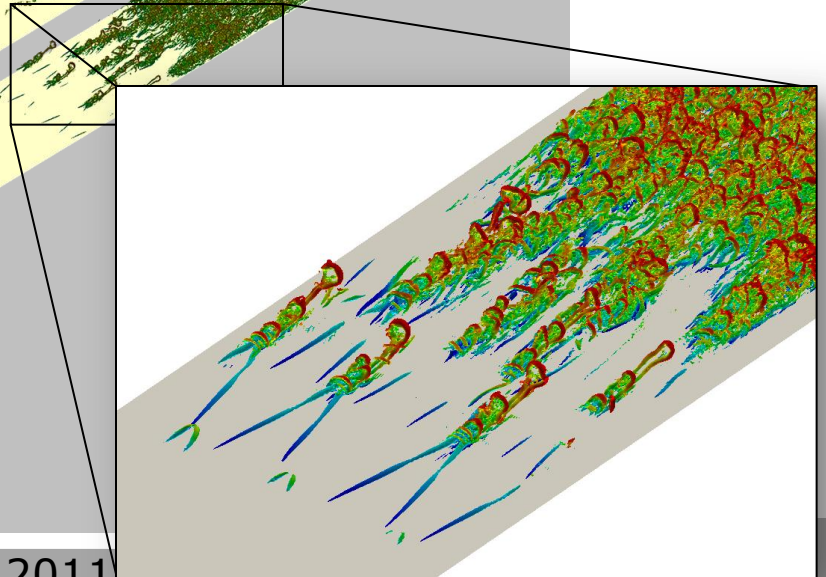
$Re_\theta = 1600$

$Re_\theta = 1250$

$Re_\theta = 1100$

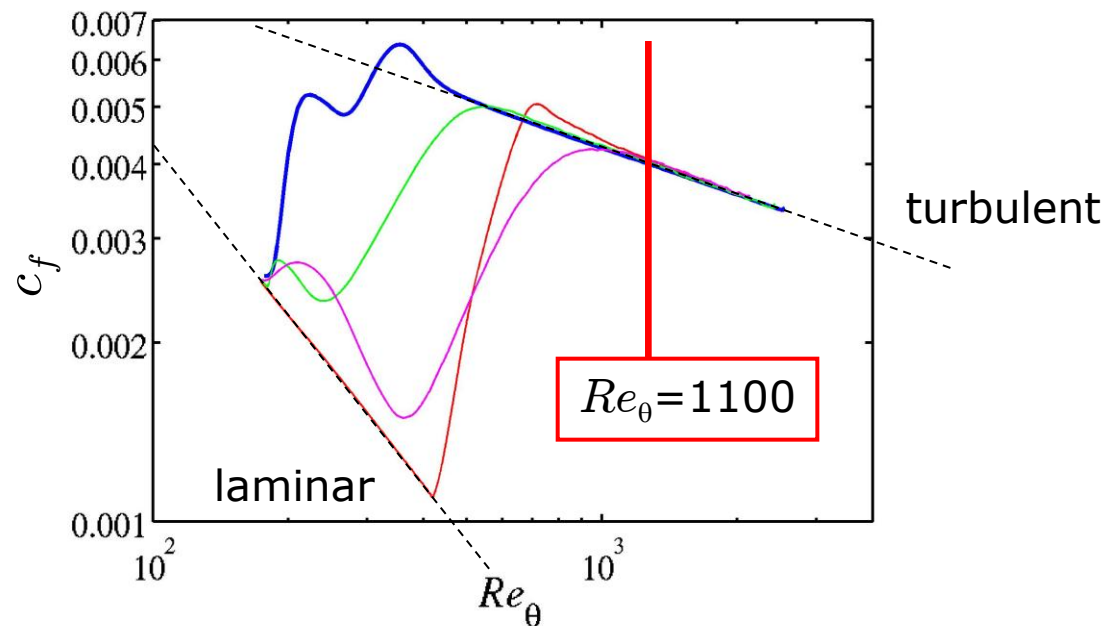


region  $Re_x = 70,000 - 750,000$   
(half of the computational domain)



# Inflow Length: Tripping

- We consider 4 different tripping mechanisms:
  - baseline — blue line
  - low amplitude — green line
  - low frequency — magenta line
  - Tollmien-Schlichting (TS) waves — red line



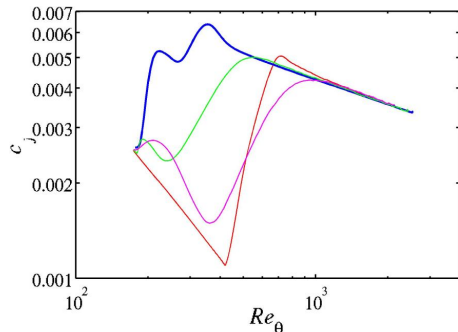
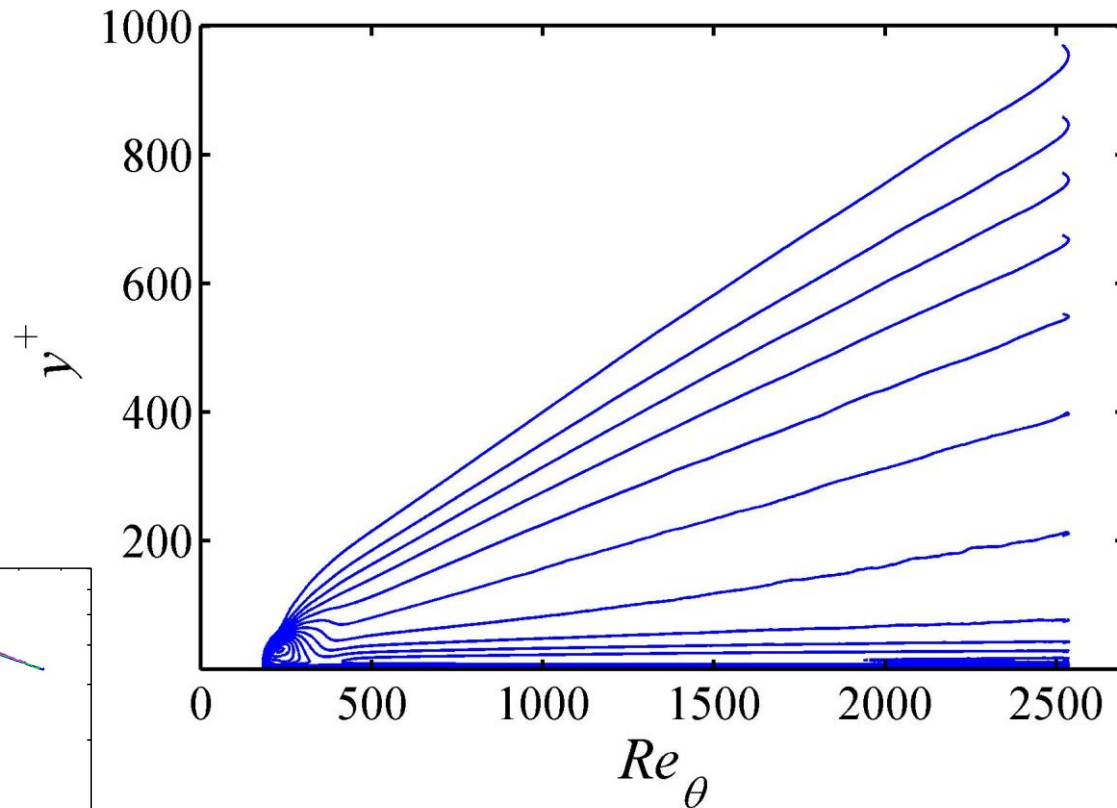
- ✓ All simulations reach a common friction curve at some  $Re_\theta$ .
- ✓ Skin friction  $c_f$  is a measure for inner-layer convergence...?



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# Inflow Length: Tripping

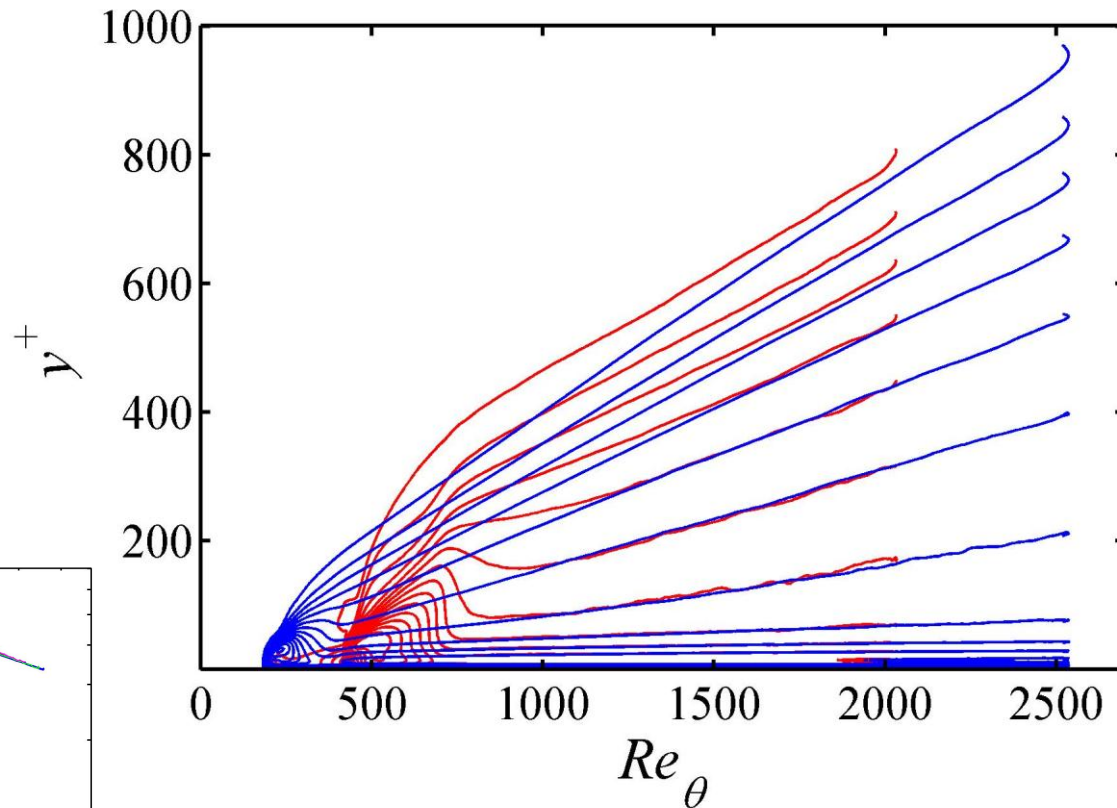
- Contours of  $u^+_{rms}$  (steps 0.25)



- a) baseline — blue — b) low amplitude — green —  
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# Inflow Length: Tripping

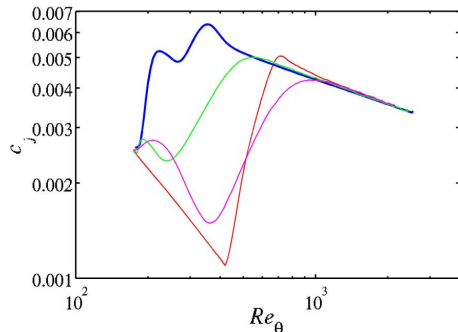
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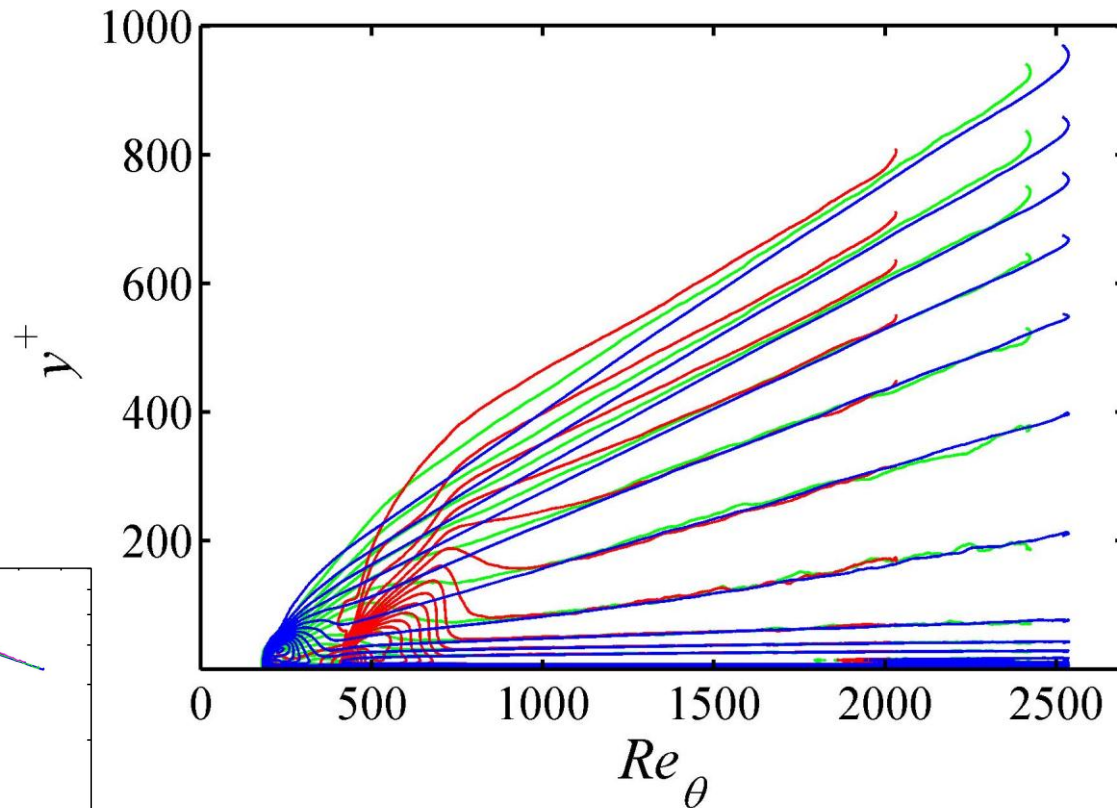


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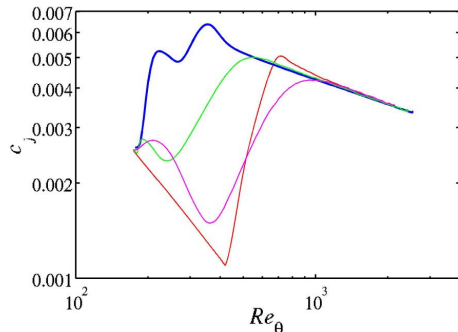
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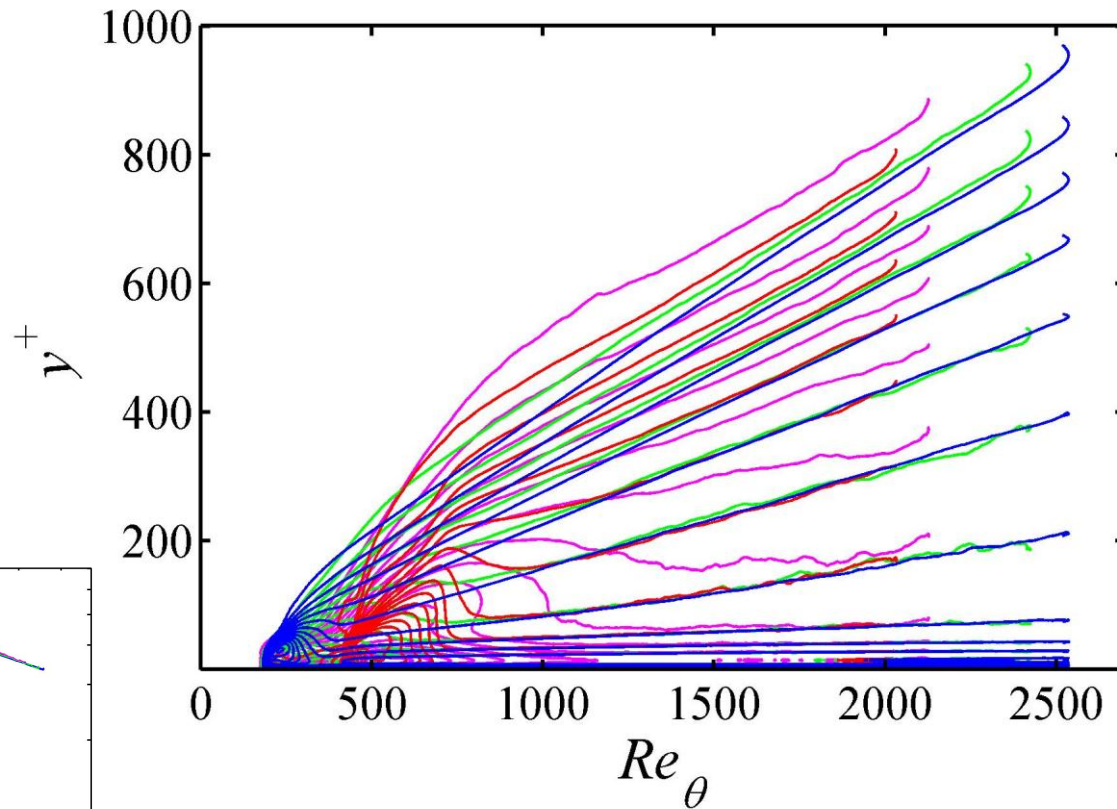


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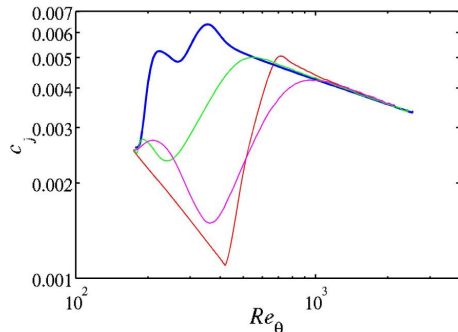
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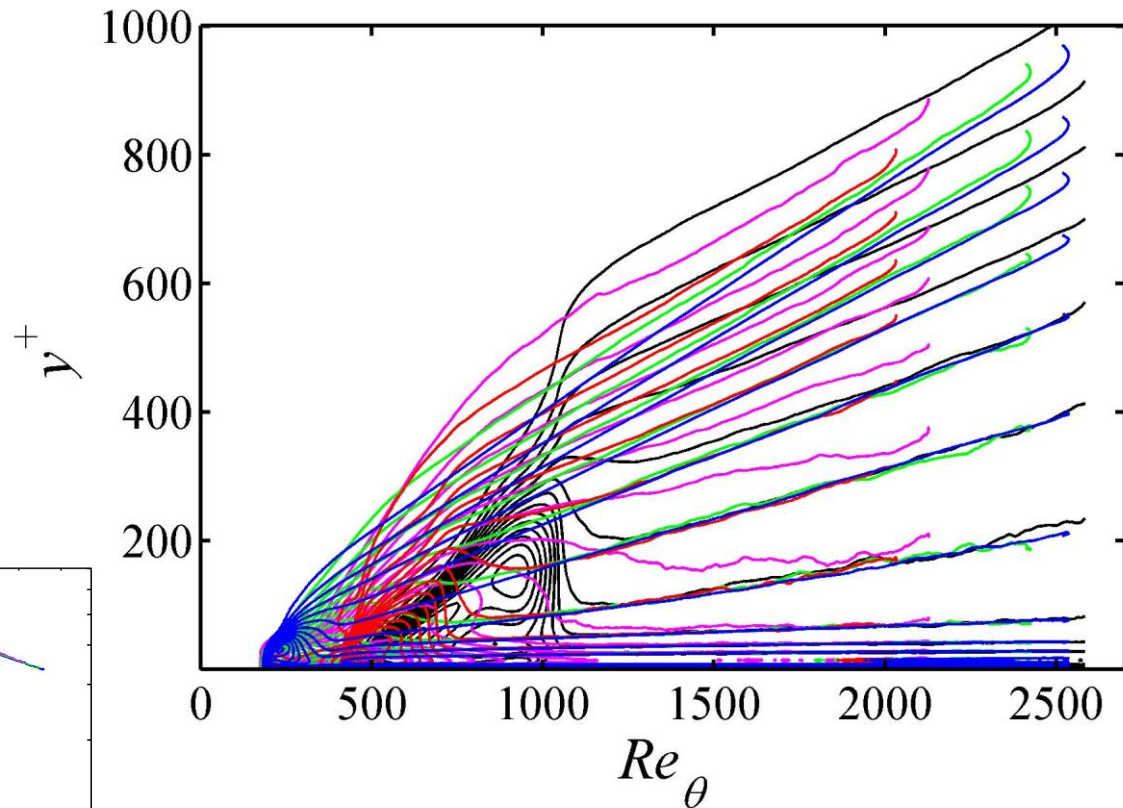


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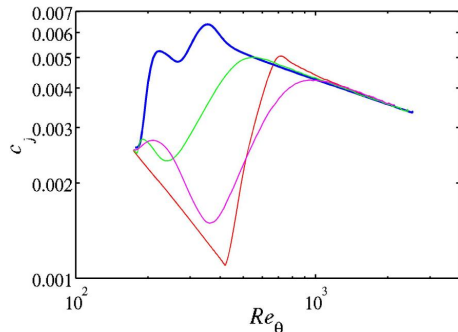
- Contours of  $u^+_{rms}$  (steps 0.25)



- a) baseline — blue — b) low amplitude — green —  
 c) low frequency — magenta — d) Tollmien-Schlichting (TS) waves — red —  
 — black — tripping at higher  $Re$

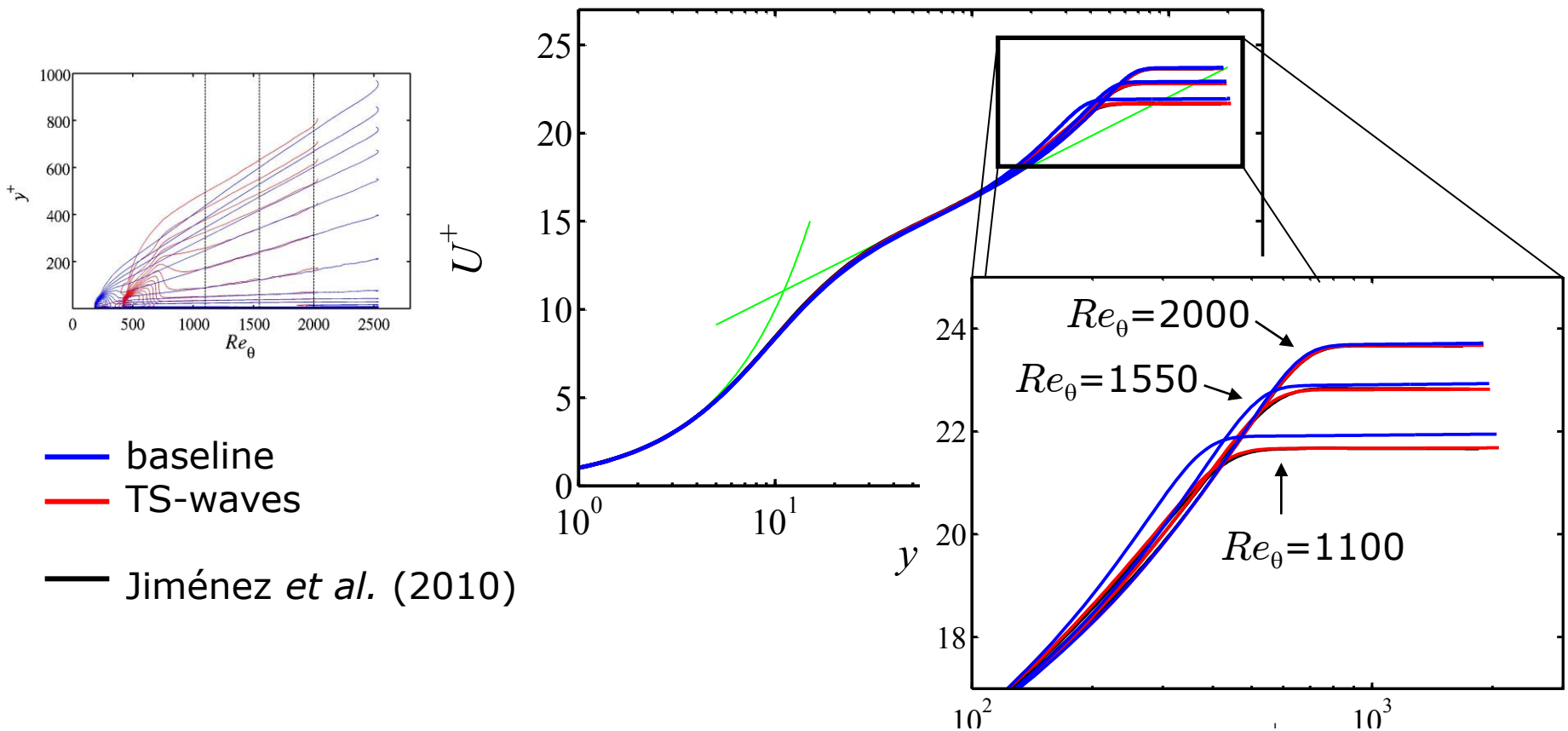


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# Inflow Length: Tripping

- Consider three  $Re_\theta = 1100, 1550, 2000$



Ref.: Örlü & Schlatter, ETC-13, 2011



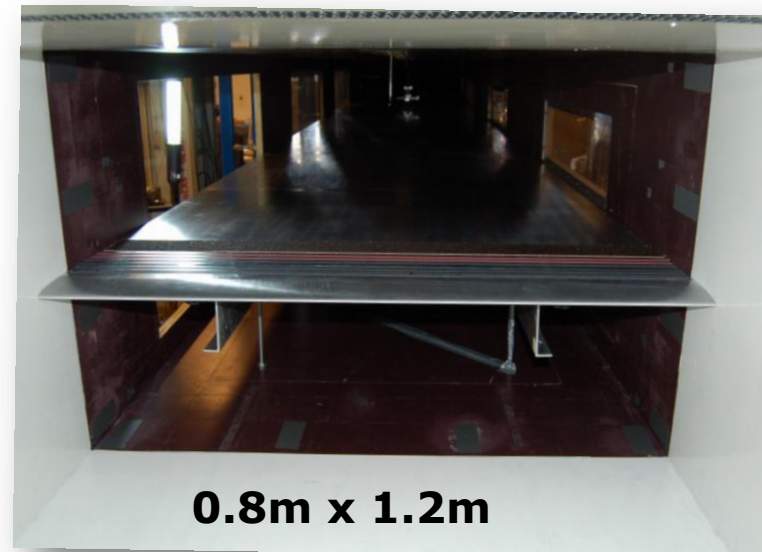
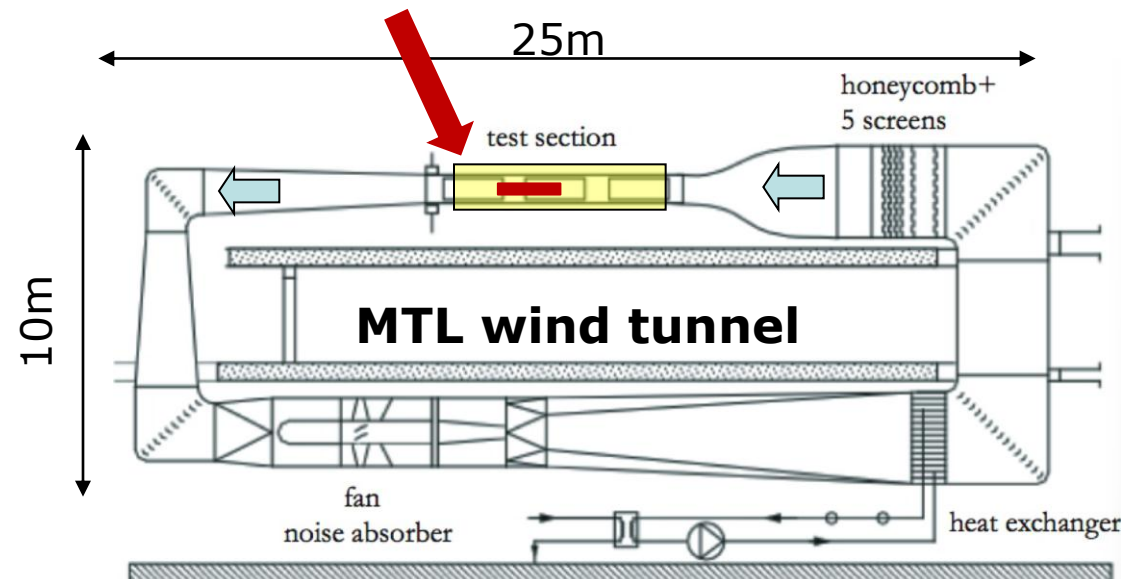


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# Let's compare DNS and experiments...

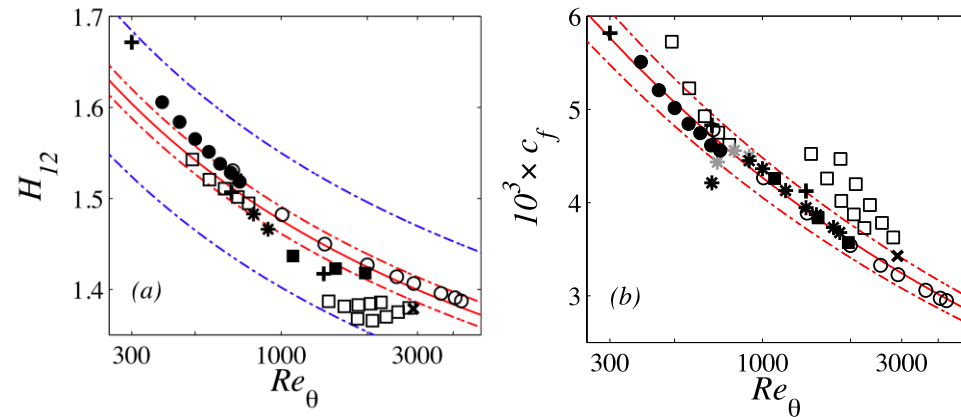
# New experiments at KTH

- ZPG TBL flow in the range  $2300 < Re_\theta < 7500$  (Örlü, 2009)
  - single hot-wire measurements at 1.65m from leading edge of a 7m long plate fulfilling “**equilibrium**” criteria (*à la Chauhan et al. 2009*)
  - independent skin friction measurements by means of **oil-film interferometry**
  - **DNS** corresponds to a 2m stretch... —

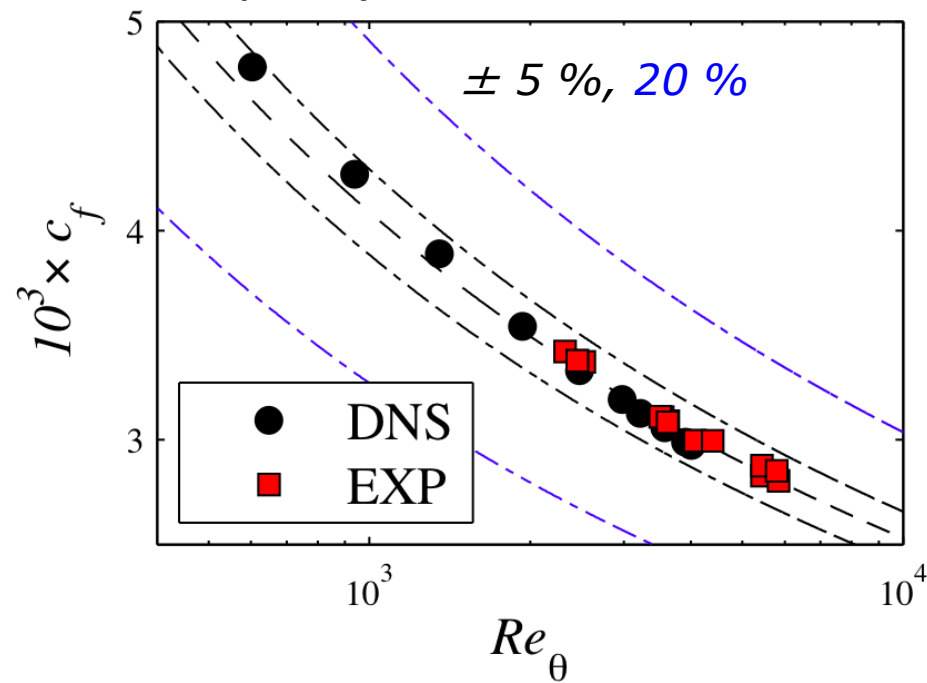
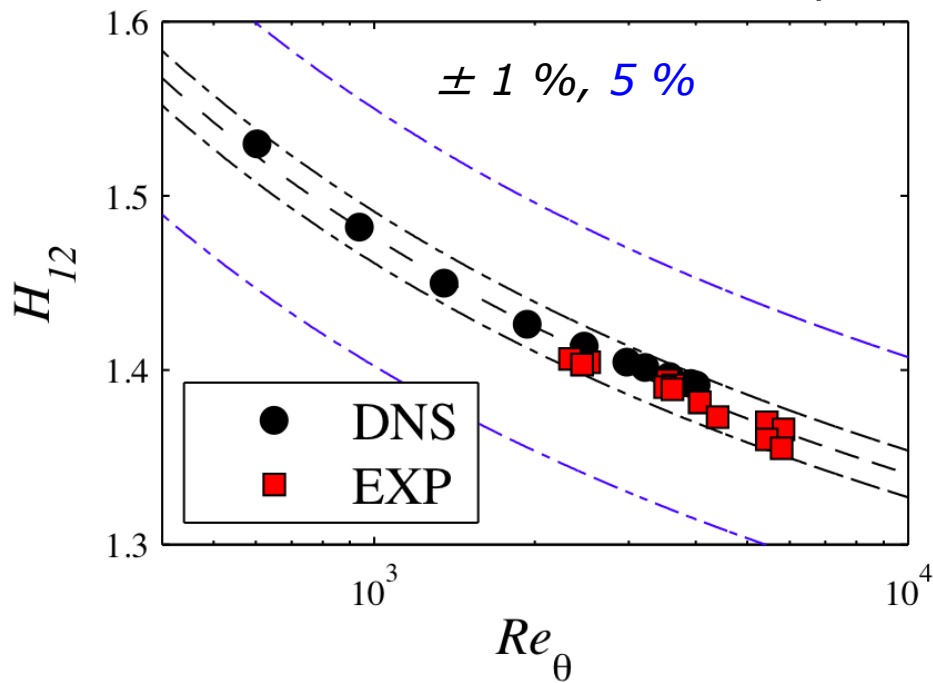


# Shape factor & skin friction coefficient

**Recall**



--- Correlation by Chauhan et al. (2009)



Örlü and Schlatter, iTi 2010

# Shape factor & skin friction coefficient

In the following slides data from DNS at

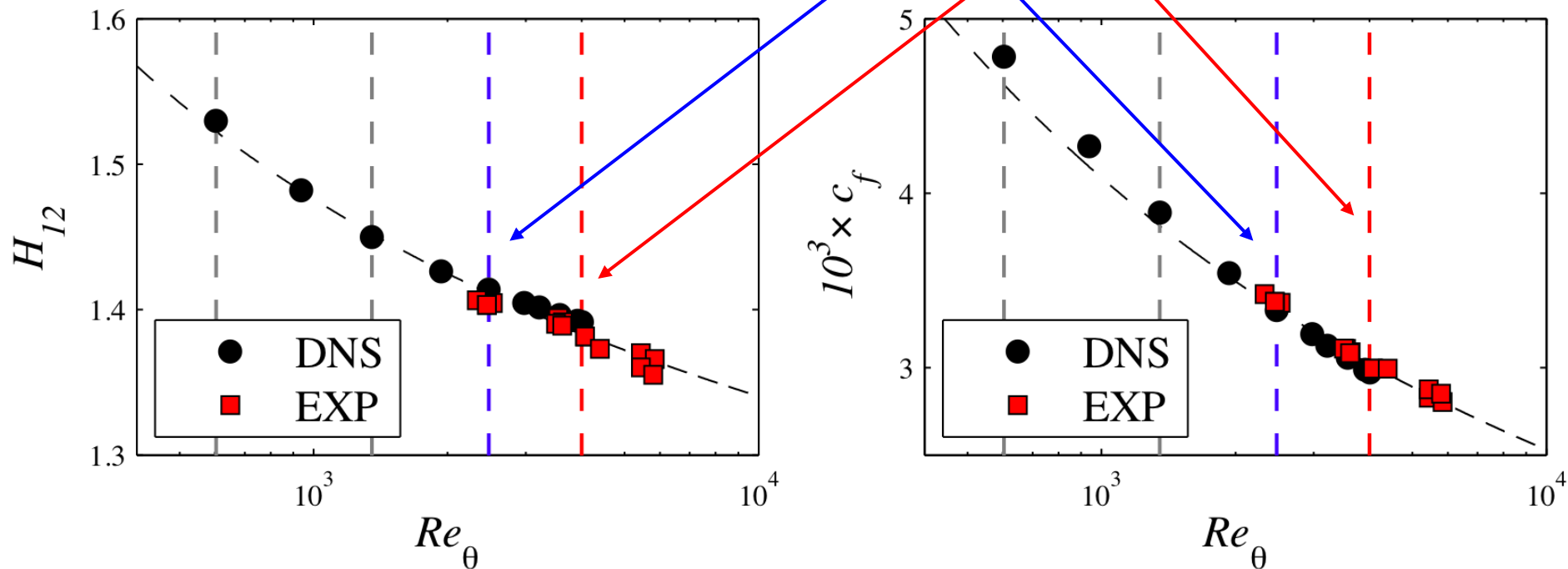
$$Re_\theta = 670, 1410, 2500, 4000$$

and experiments at

$$Re_\theta = 2500, 4000$$

will be shown and data at  
compared

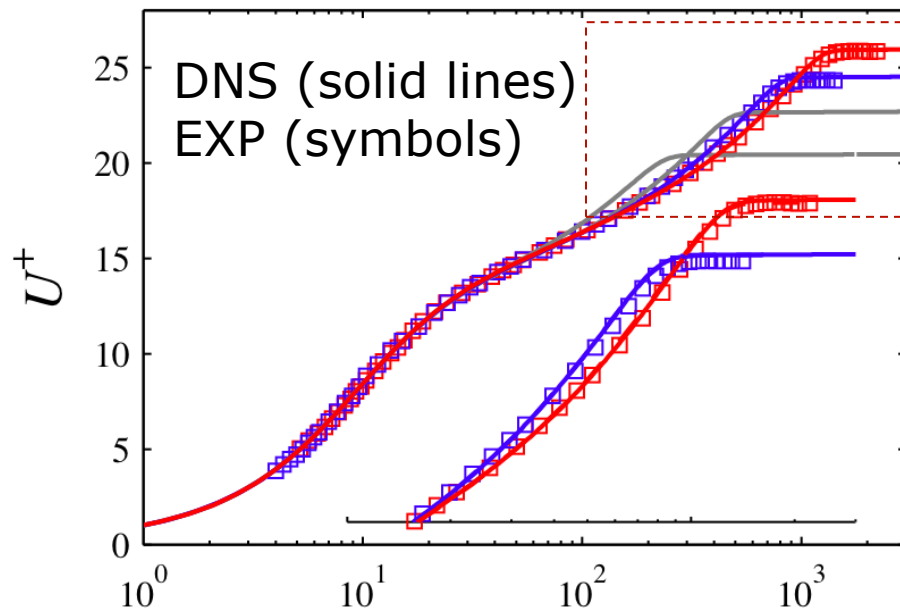
will be



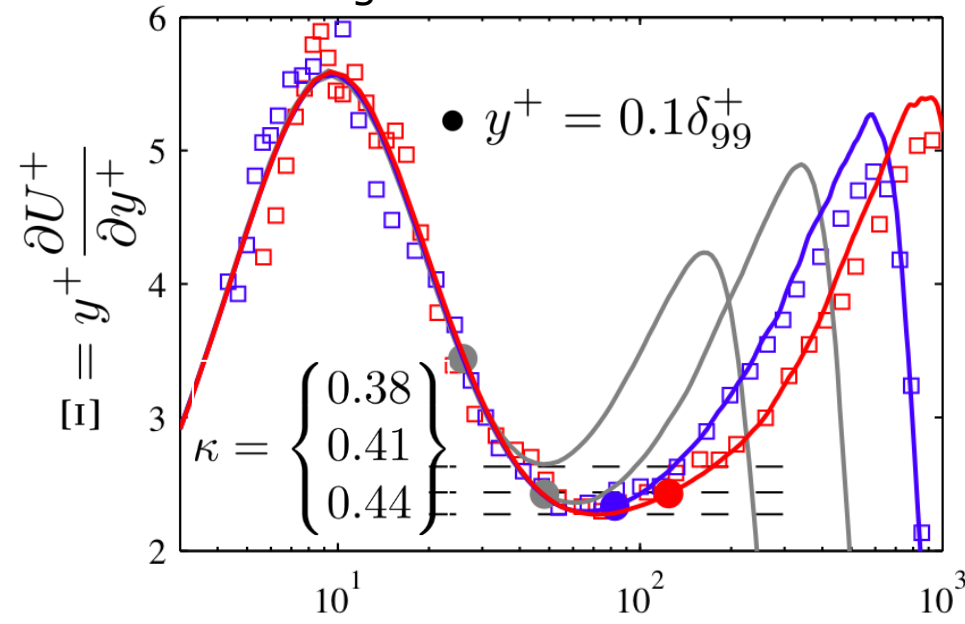
Örlü and Schlatter, iTi 2010

# Mean streamwise velocity profiles

Mean velocity profile



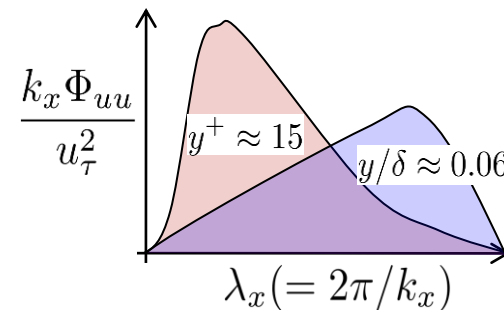
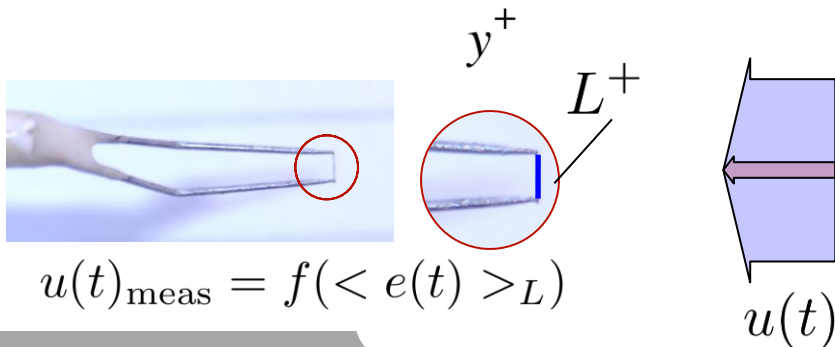
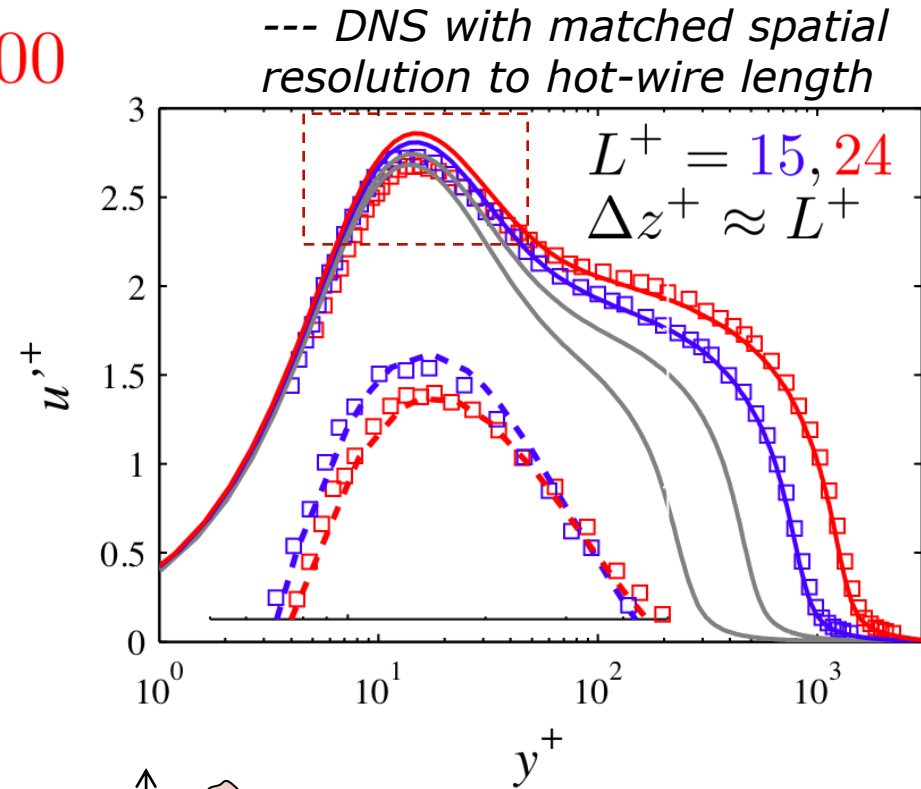
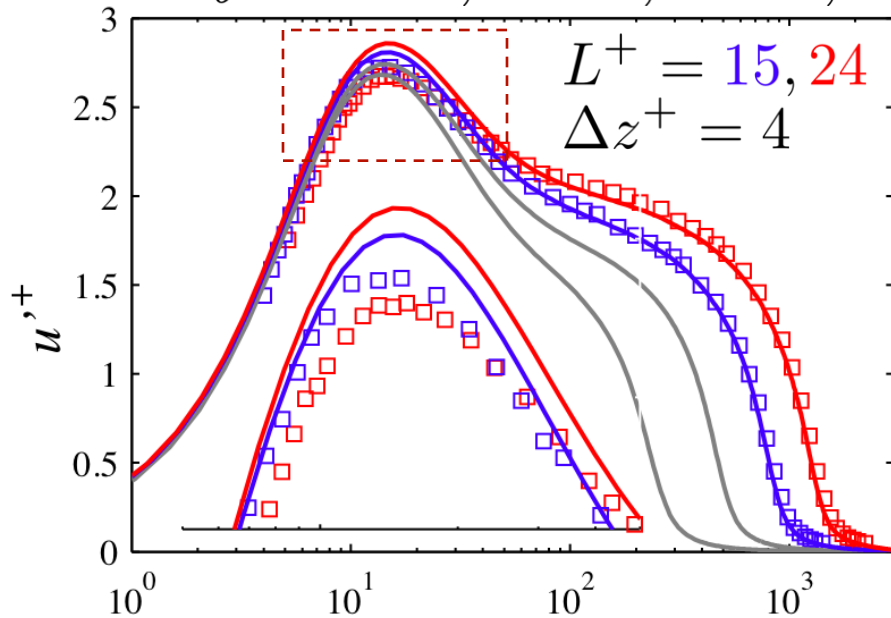
Log-law indicator function



$$y^+ \quad Re_\theta = 670, 1410, 2500, 4000 \quad y^+$$

# Turbulence intensity profiles

$Re_\theta = 670, 1410, 2500, 4000$



(e.g. Örlü & Alfredsson, EF, 2010)

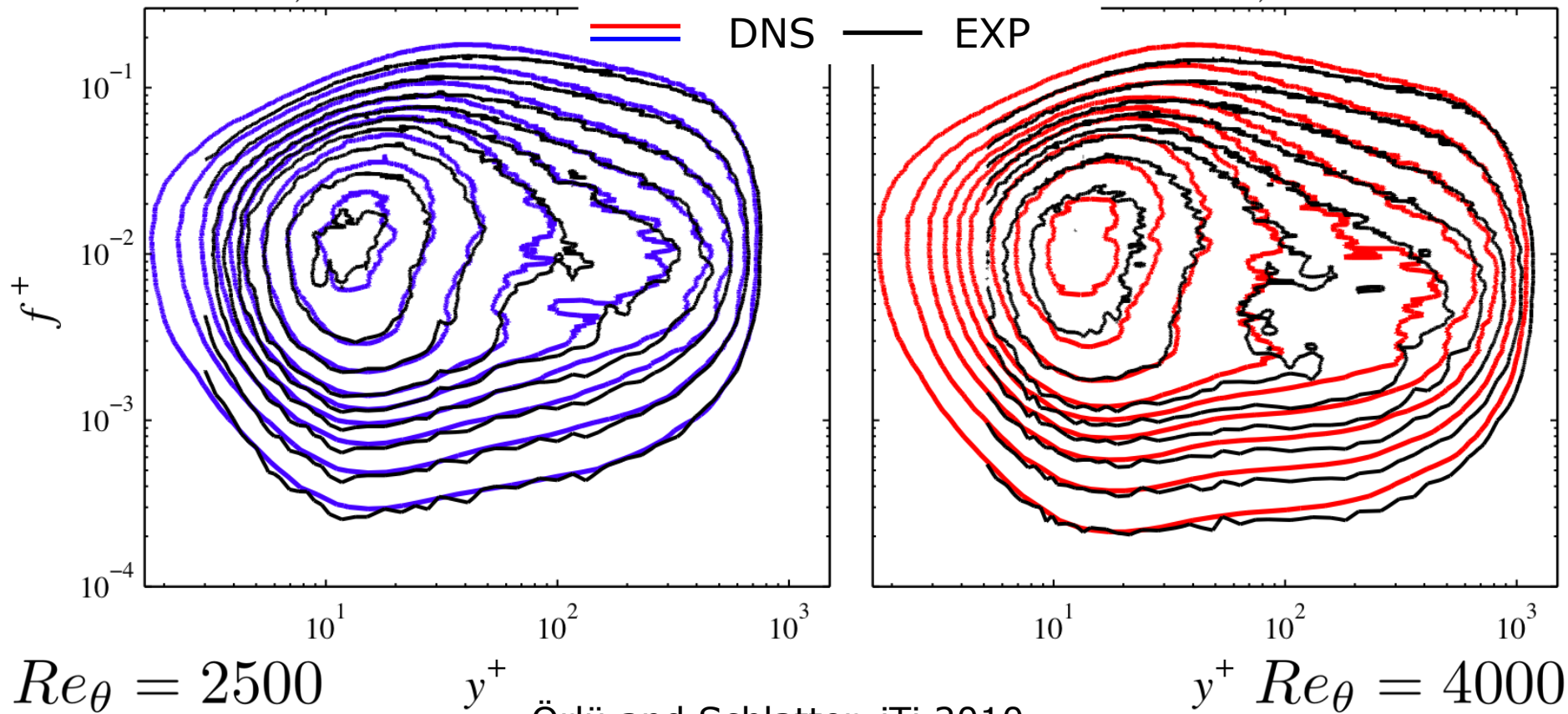
# Pre-multiplied spectral map

$$\Delta z^+ = 4, \Delta t^+ = 0.30$$

$$L^+ = 15, \Delta t^+ = 0.27$$

$$\Delta z^+ = 4, \Delta t^+ = 0.27$$

$$L^+ = 23, \Delta t^+ = 0.66$$



Örlü and Schlatter, iTi 2010

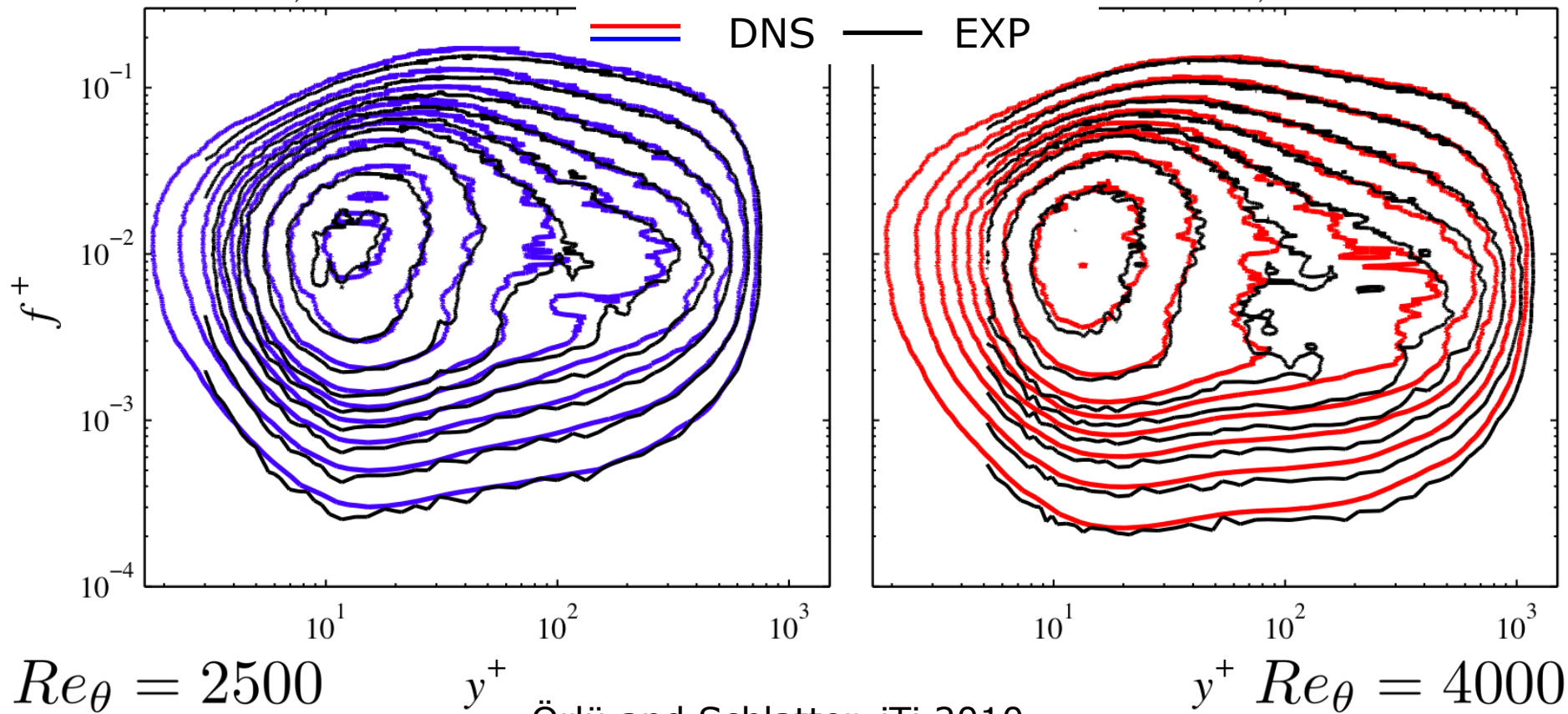
# Pre-multiplied spectral map

$$\Delta z^+ \approx L^+, \Delta t^+ = 0.30$$

$$L^+ = 15, \Delta t^+ = 0.27$$

$$\Delta z^+ \approx L^+, \Delta t^+ = 0.27$$

$$L^+ = 23, \Delta t^+ = 0.66$$



Örlü and Schlatter, iTi 2010





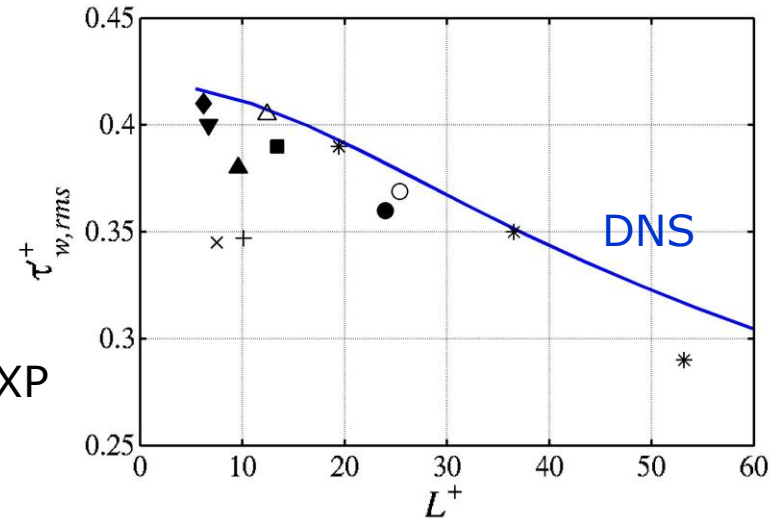
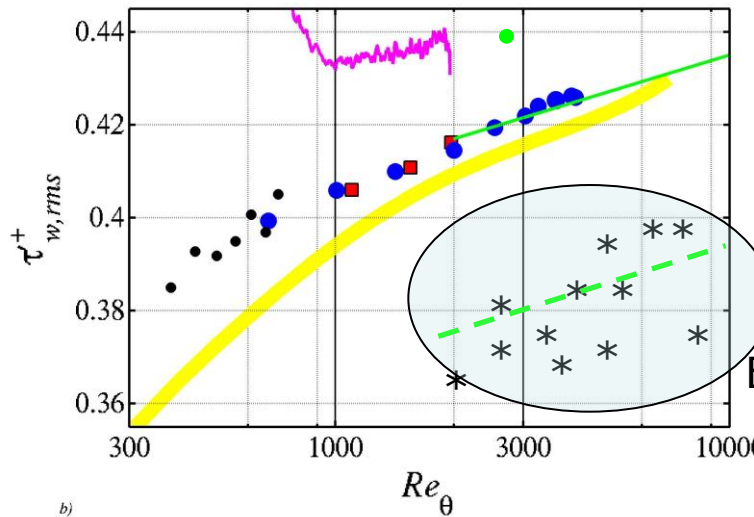
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Wall-shear stress  $\tau_w$

# Wall-shear stress fluctuation

Wall shear stress:  $\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_w$       Fluctuation:  $\tau_{w,rms} / \tau_w \equiv \tau_{w,rms}^+ = \left. \frac{u}{U} \right|_{y \rightarrow 0}$

Main reference: Alfredsson *et al.* (1988): Explanation with spatial resolution...



- Channel flows
- Skote (2001)
- Ferrante & Elghobashi (2005)
- Correlation Österlund (\*)
- Schlatter & Örlü (2010)
- Jiménez *et al.* (2010)
- Wu & Moin, *Phys. Fluids* 2010

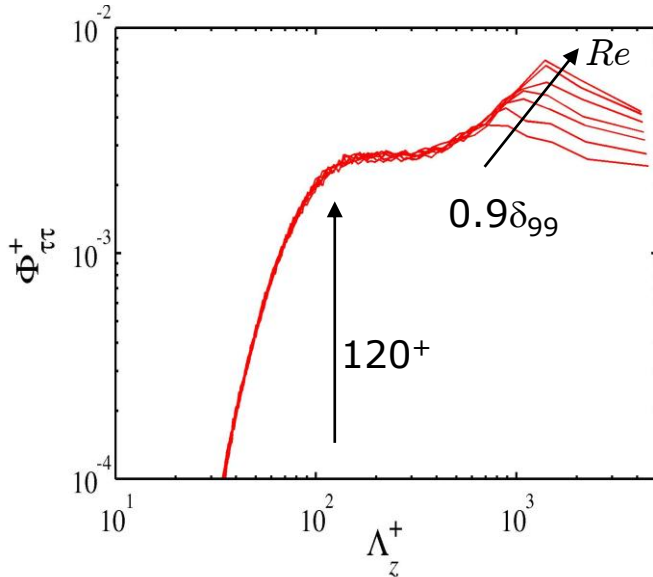
Ref.: Örlü & Schlatter, *Phys. Fluids*, 2011

# Wall-shear stress fluctuation

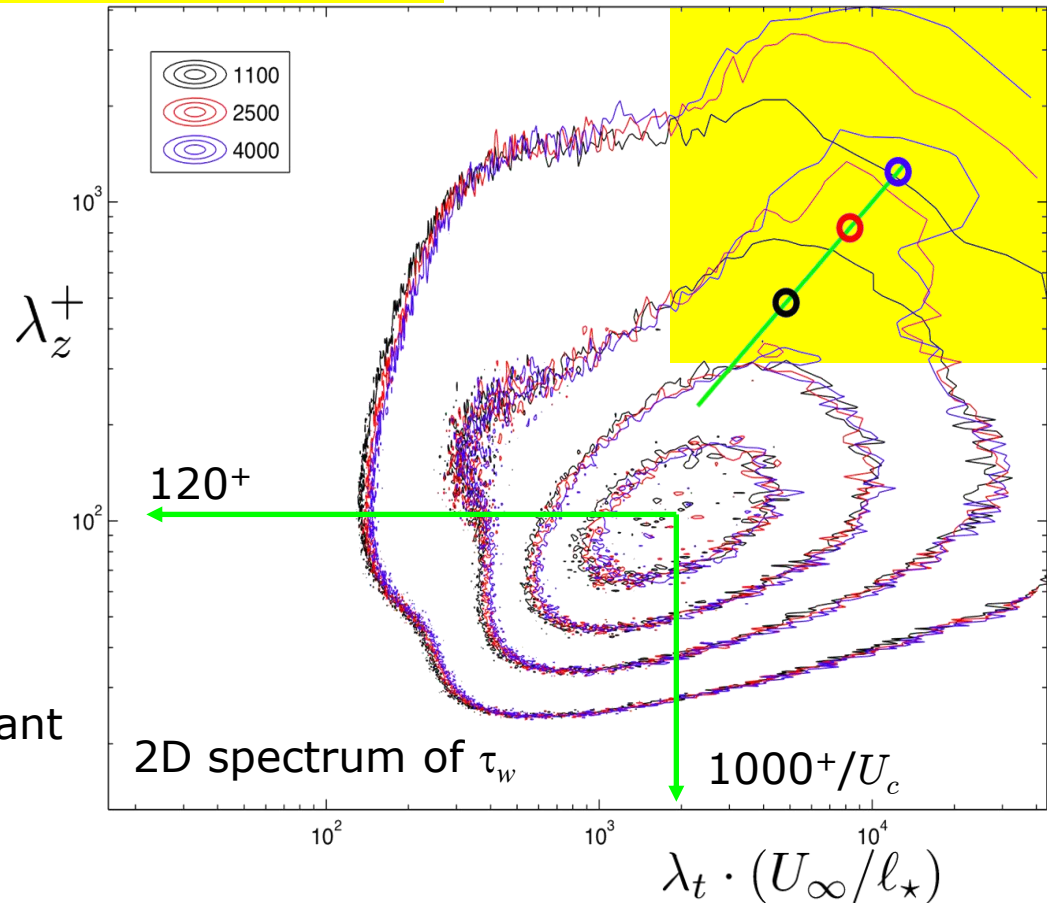
- Why increasing for all Re, even for low Re?

$$\tau_{w,rms} = 0.298 + 0.018 \ln Re_\tau$$

Power spectrum of  $\tau$



- Inner part essentially invariant (viscous scaling)
- Outer peak scales in outer units and is increasing



Ref.: Örlü & Schlatter, *Phys. Fluids*, 2011



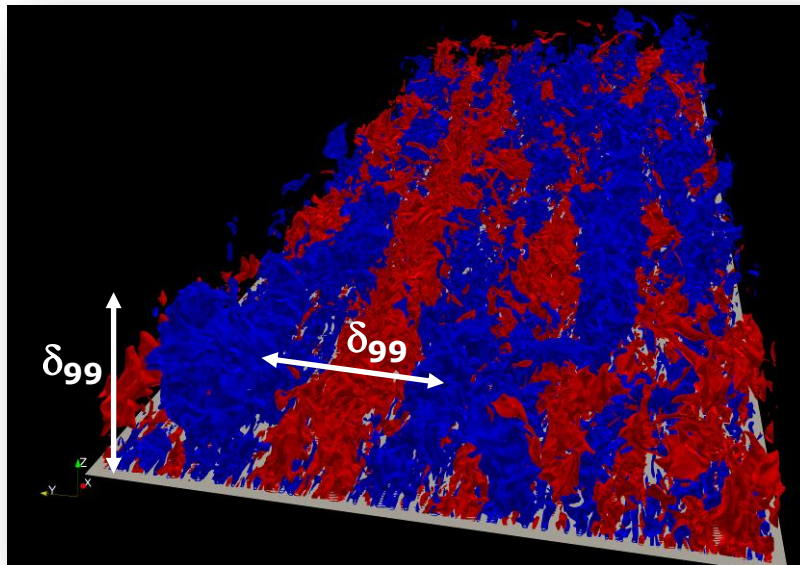
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# Spatial Structures...

# Disturbance Velocity

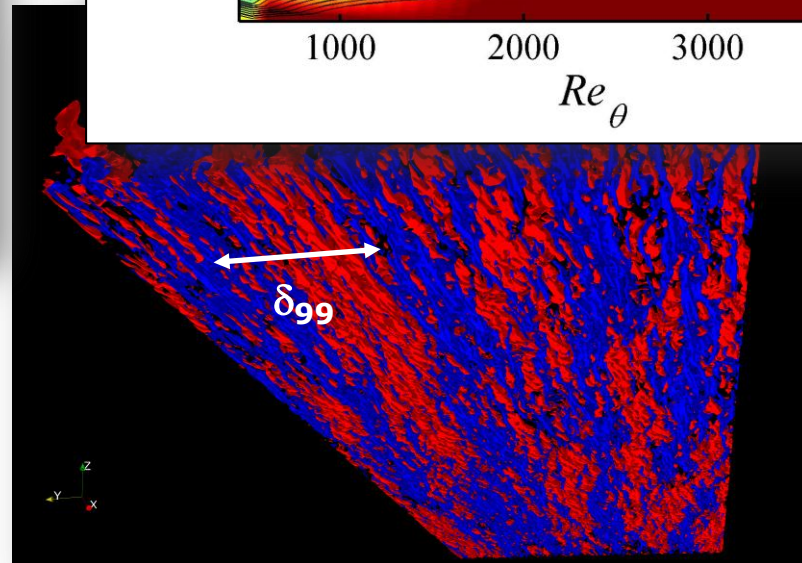
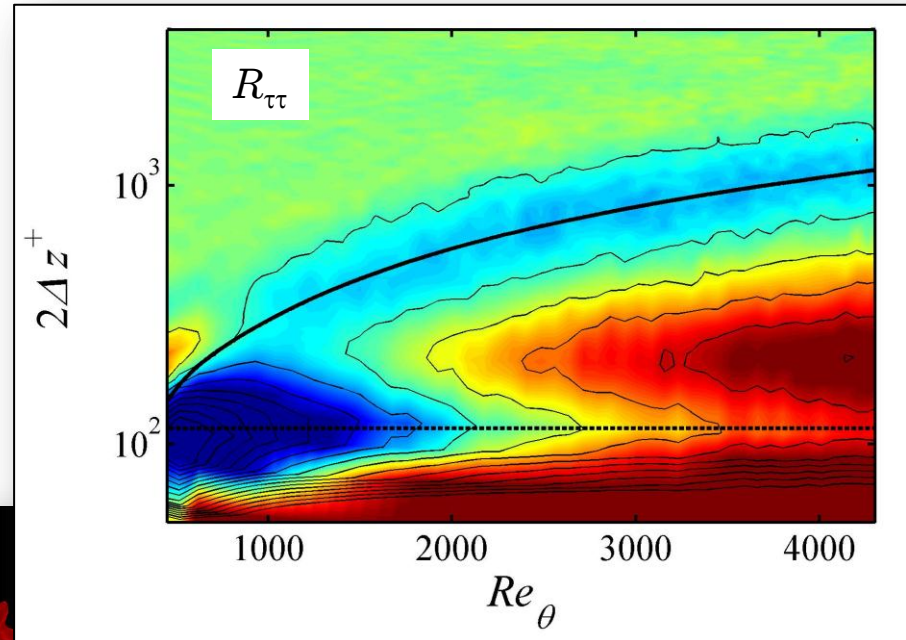
.....  $2\Delta z^+ = 115$   
 —  $2\Delta z/\delta_{99} = 0.85$

- **positive** and **negative** streamwise disturbance velocity ( $\pm 0.1U_\infty$ )



view from top

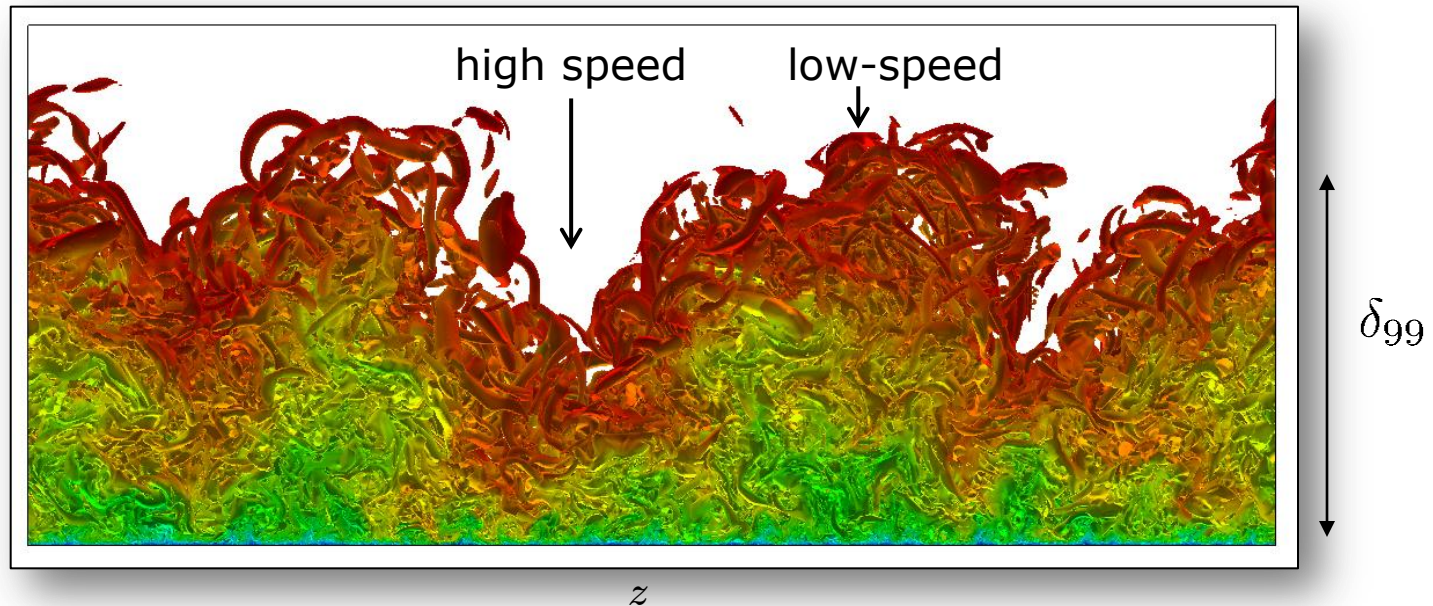
visualised domain:	at $Re_\theta = 4000$
length	$7000^+ \quad 5.0 \delta_{99}$
width	$4500^+ \quad 3.2 \delta_{99}$



view from bottom

# Amplitude Modulation

- Cross-stream cut through the boundary layer



vortical structures, coloured by streamwise velocity

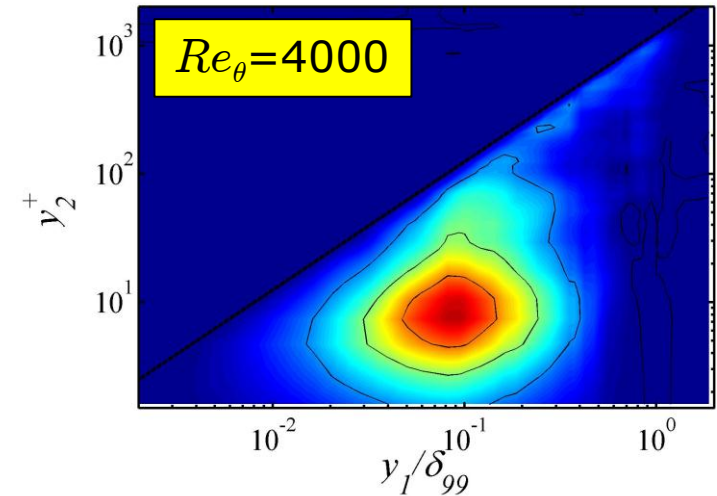
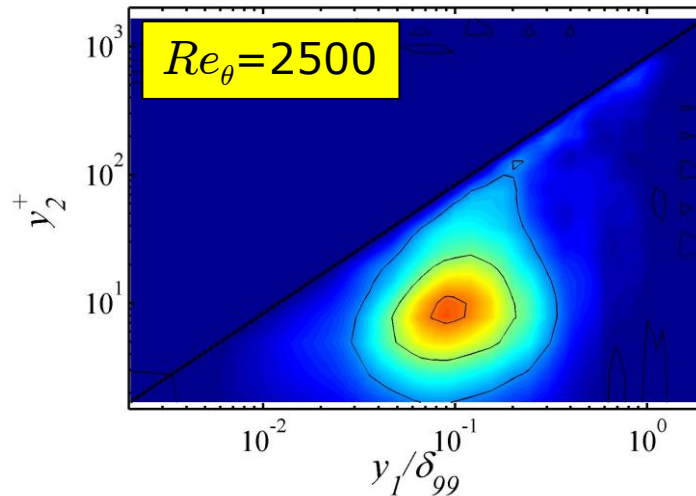
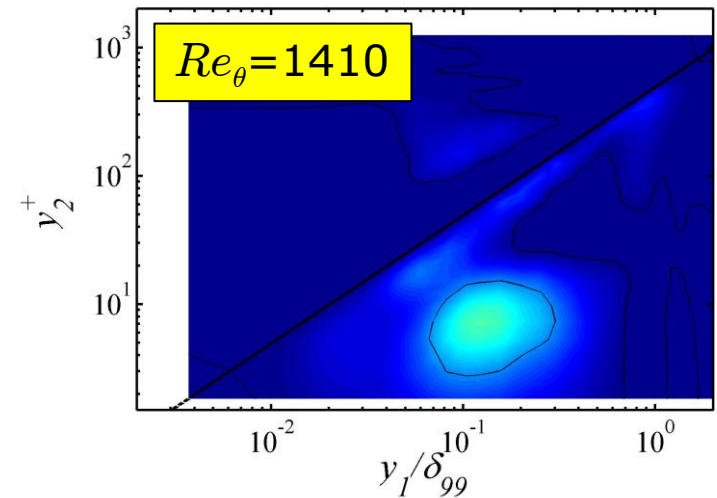
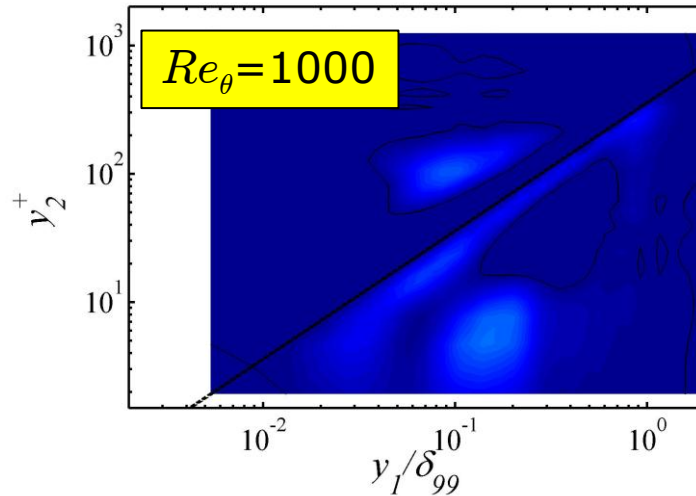
- corrugated edge of the boundary layer
- clear modulation of the whole boundary layer (including near-wall region)



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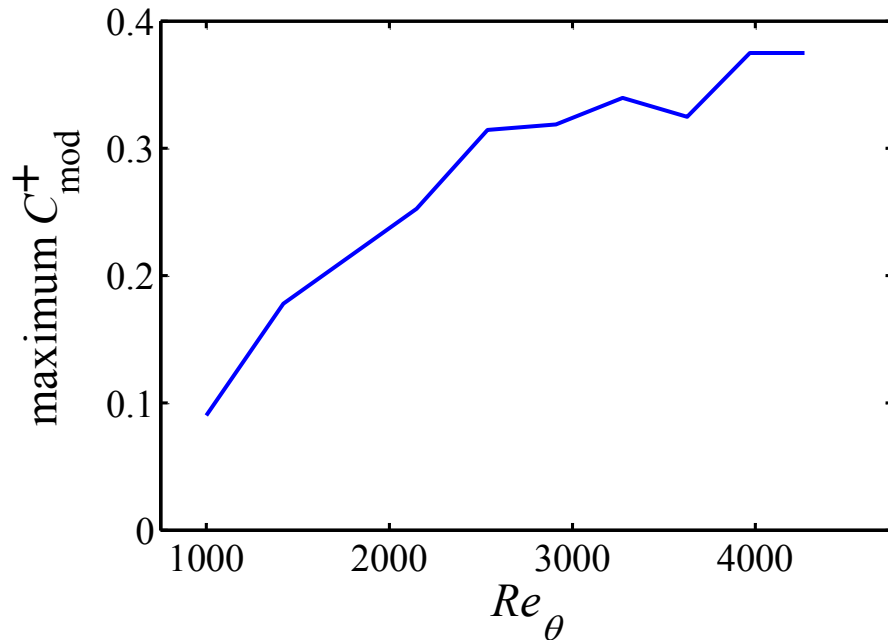
# Amplitude Modulation

Remove symmetric part:  $C_{mod}(y_1, y_2) = C_{AM}^{2p}(y_1, y_2) - C_{AM}^{2p}(y_2, y_1)$



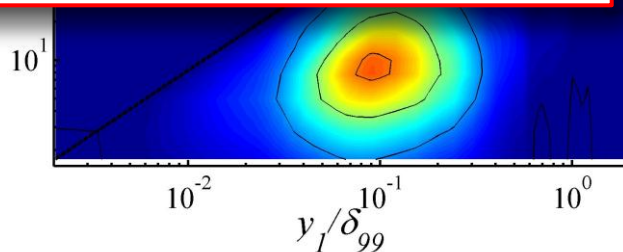
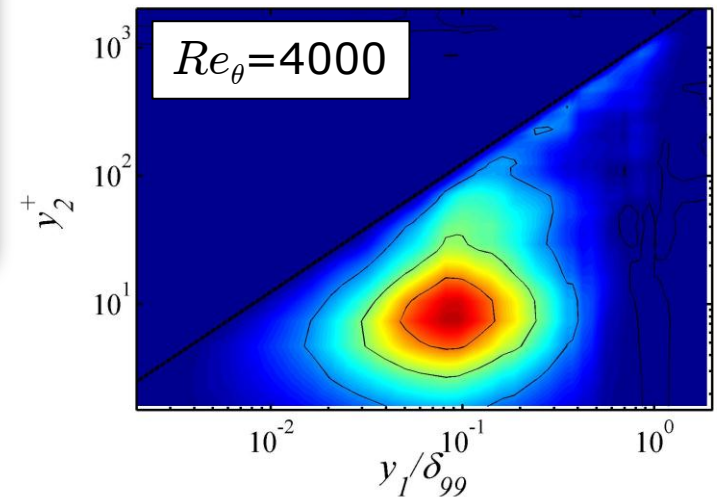
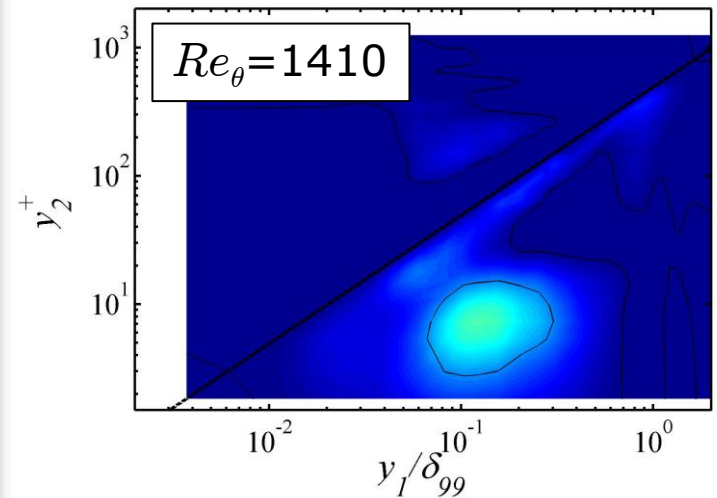
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# Amplitude Modulation



Increase of modulation coefficient in agreement with the findings by Bernardini & Pirozzoli 2011.

$$C_{AM}^{2p}(y_1, y_2) = C_{AM}^{2p}(y_1, y_2) - C_{AM}^{2p}(y_2, y_1)$$







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# Structures – Visualisation

# "Focus on Fluids" (May 2009)...

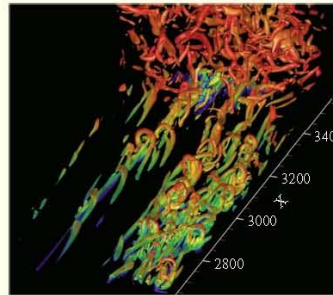
*Journal of Fluid Mechanics*

Focus  
on  
Fluids

## Unravelling turbulence near walls

IVAN MARUSIC<sup>1</sup>

<sup>1</sup>Department of Mechanical Engineering,  
University of Melbourne, Victoria, 3010, Australia

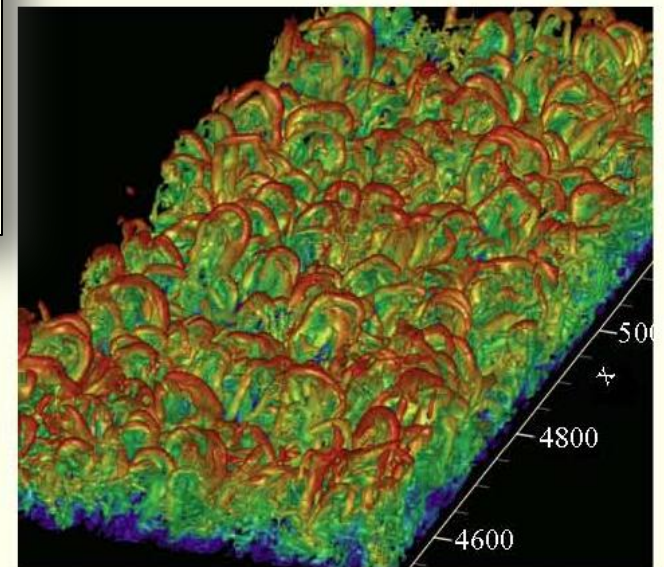


Turbulent flows near walls have been the focus of intense study since their first description by Ludwig Prandtl over 100 years ago. They are critical in determining the drag and lift of an aircraft wing for example. Key challenges are to understand the physical mechanisms causing the transition from smooth, laminar flow to turbulent flow and how the turbulence is then maintained. Recent direct numerical simulations have contributed significantly towards this understanding.

**Keywords.** Turbulent boundary layers, Transition

Figures:  
Wu & Moin (JFM 2009)

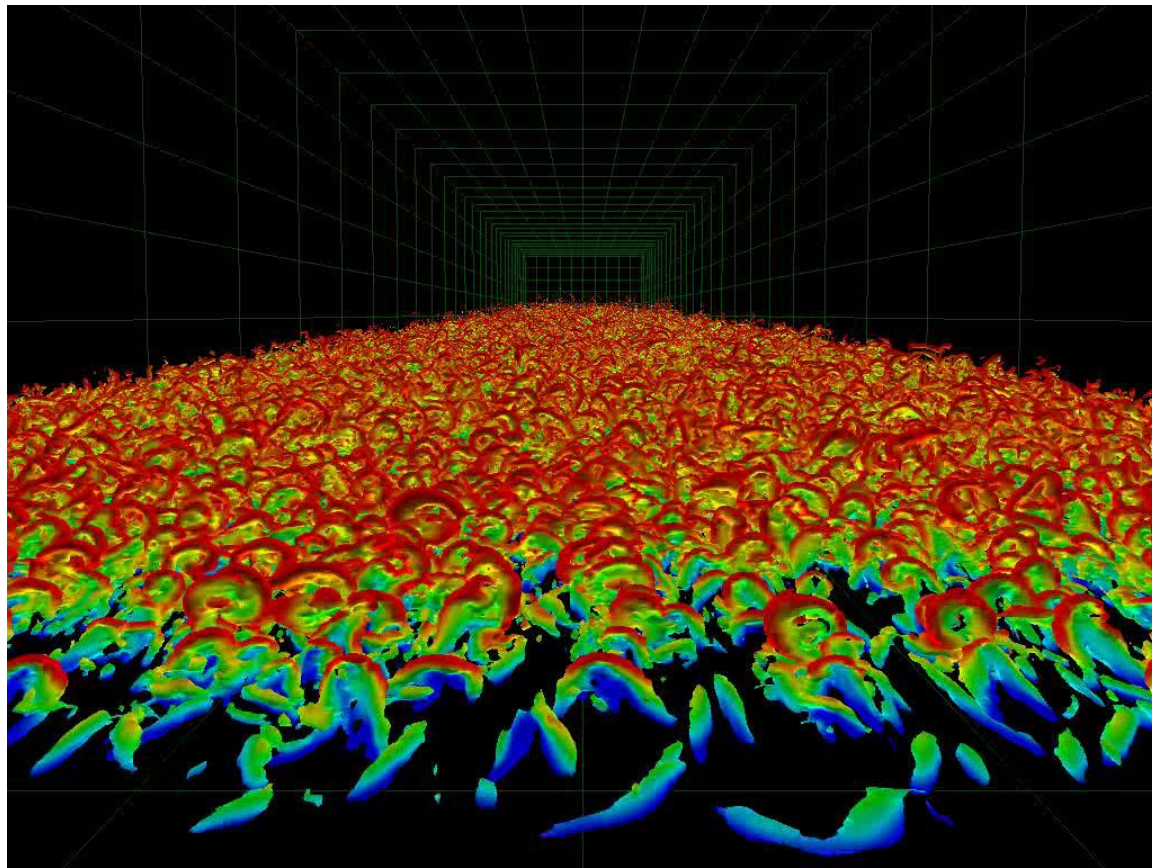
FIGURE 1. Instantaneous view of the coherent structures observed in the simulation of Wu & Moin in the fully turbulent region. The vivid appearance of hairpin-shaped structures is noted.



# Boundary Layer Visualisation



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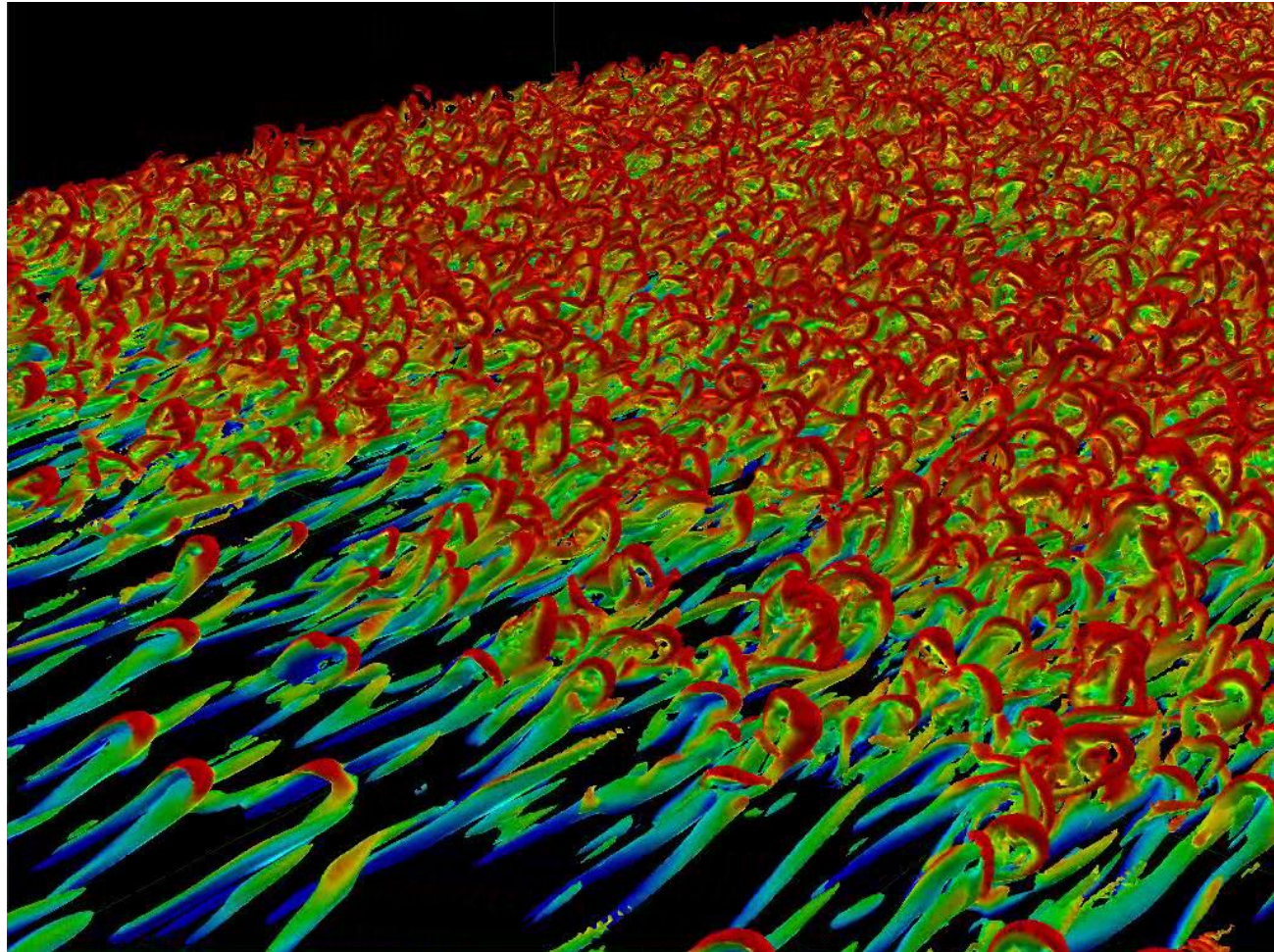


based on LES data

# Boundary Layer Visualisation



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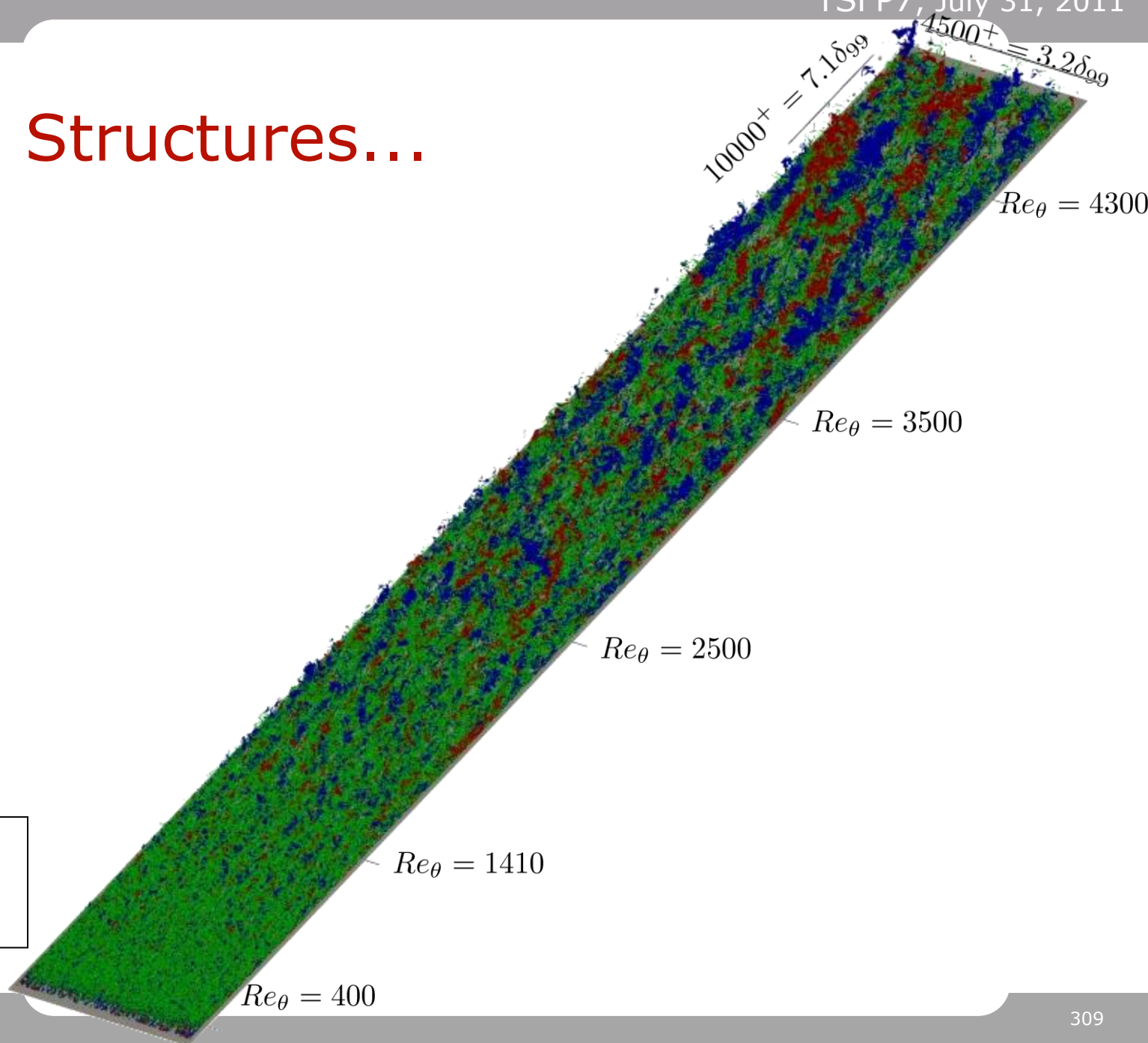


based on DNS data

# Structures...



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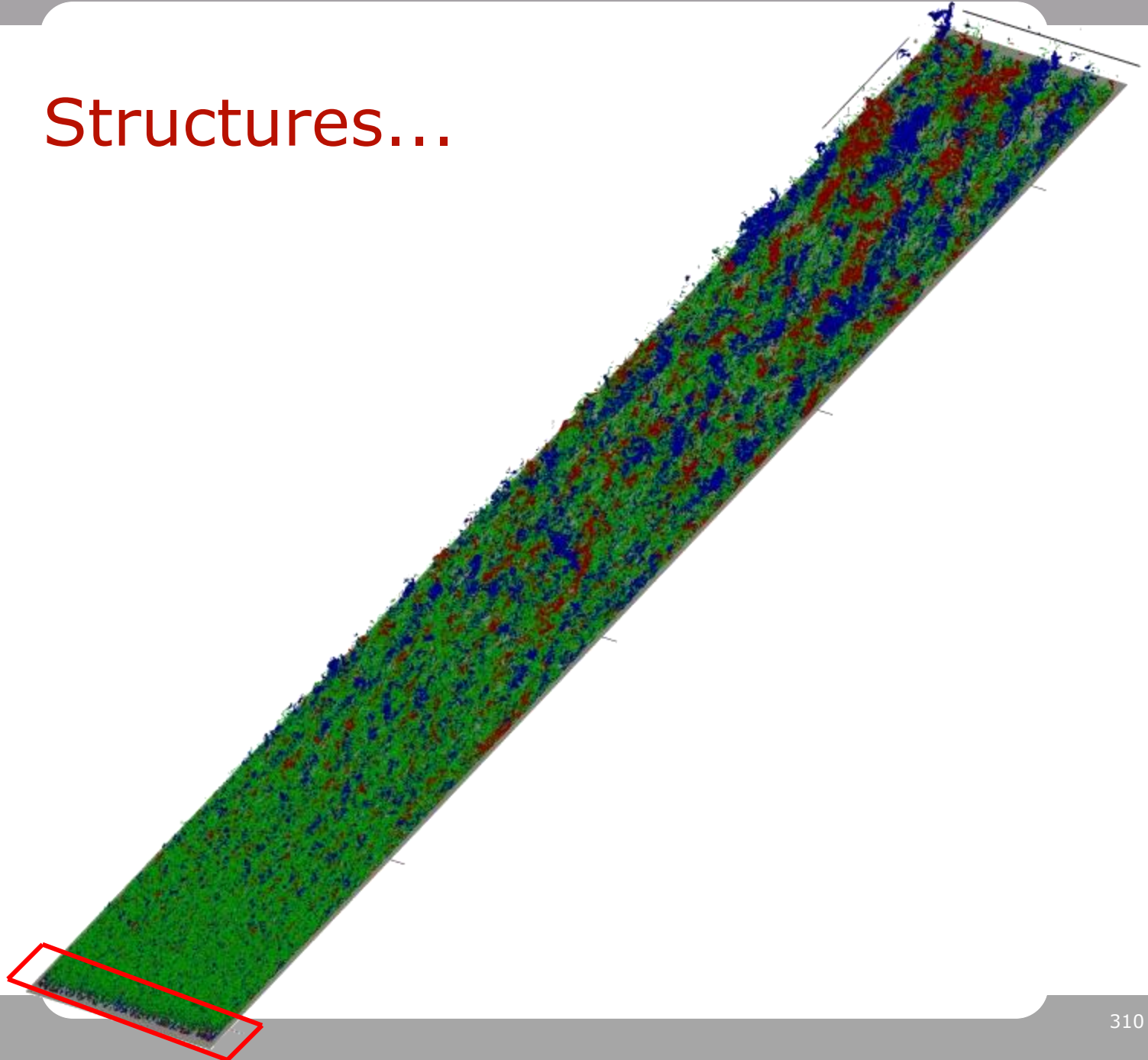


Isocontours of  
 negative  $\lambda_2$  and  
 positive / negative  
 disturbance velocity

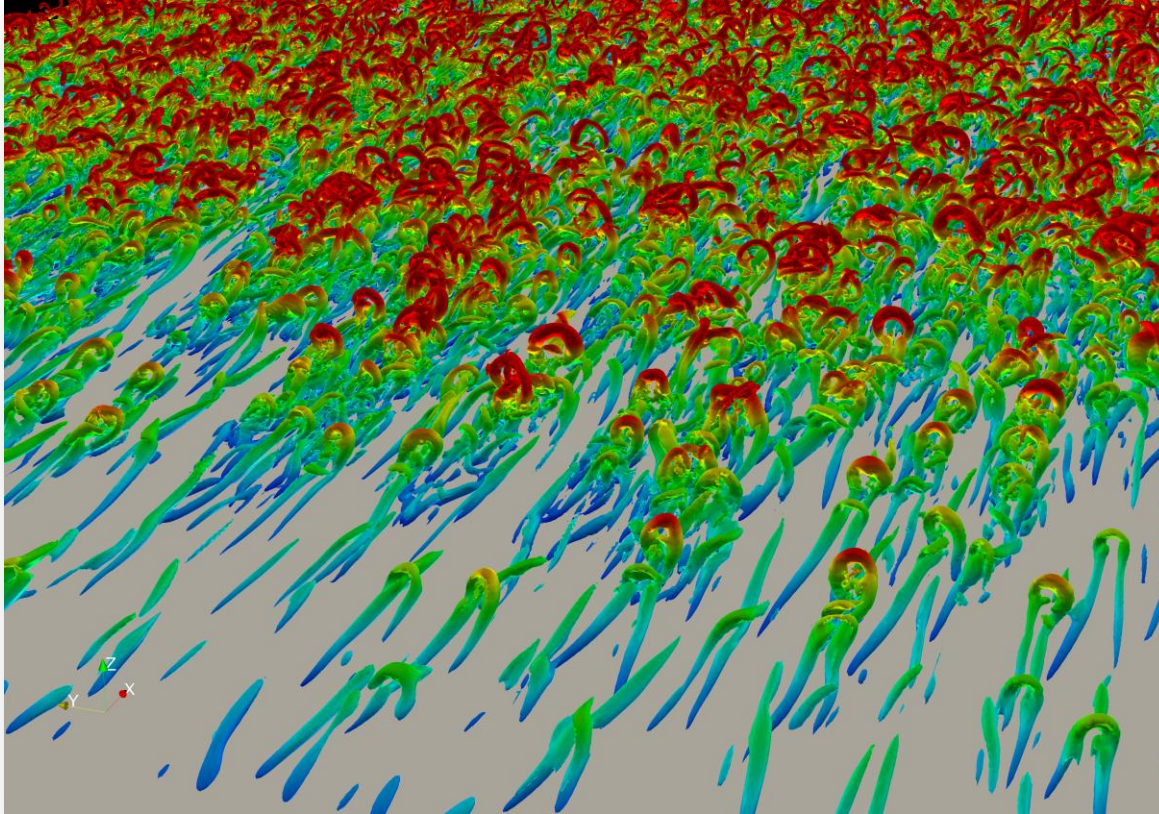
# Structures...



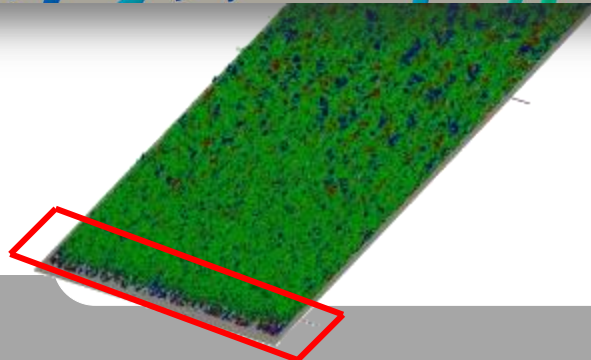
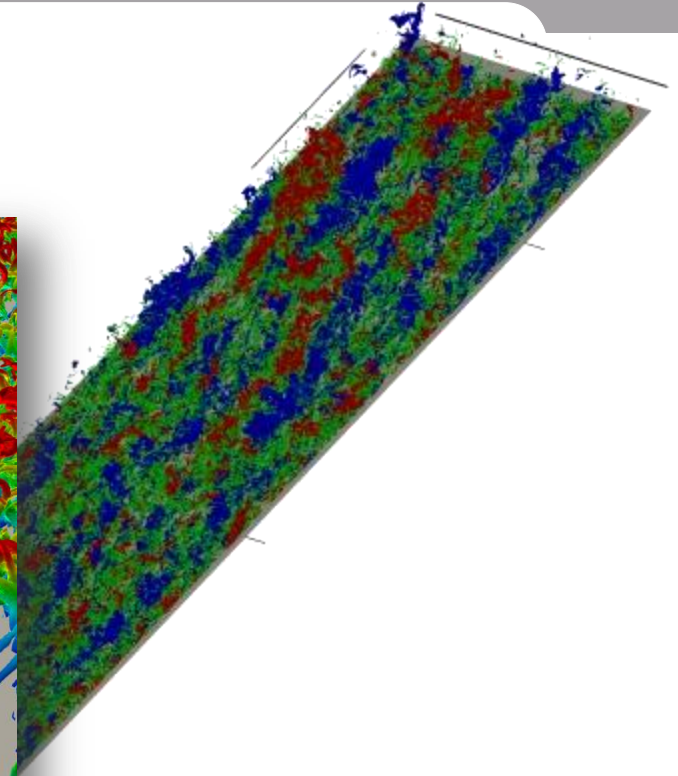
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# Structures...

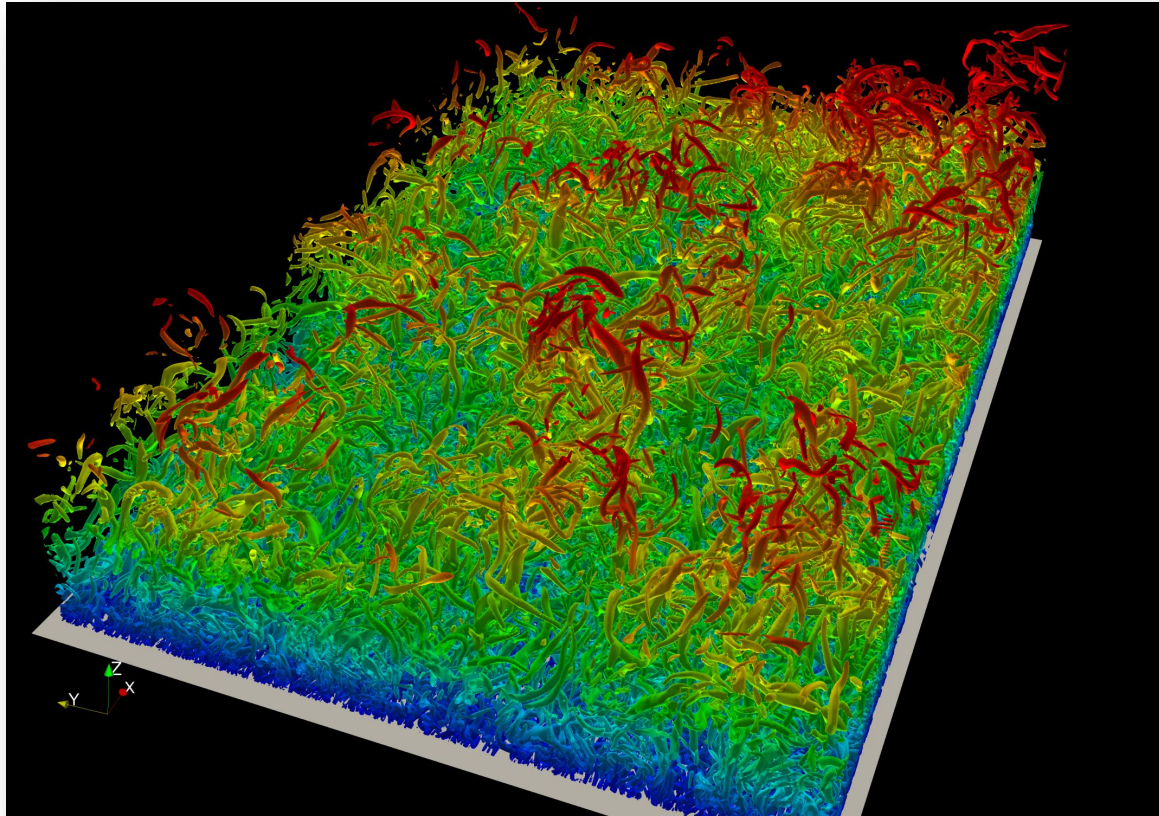


$Re_\theta = 300$

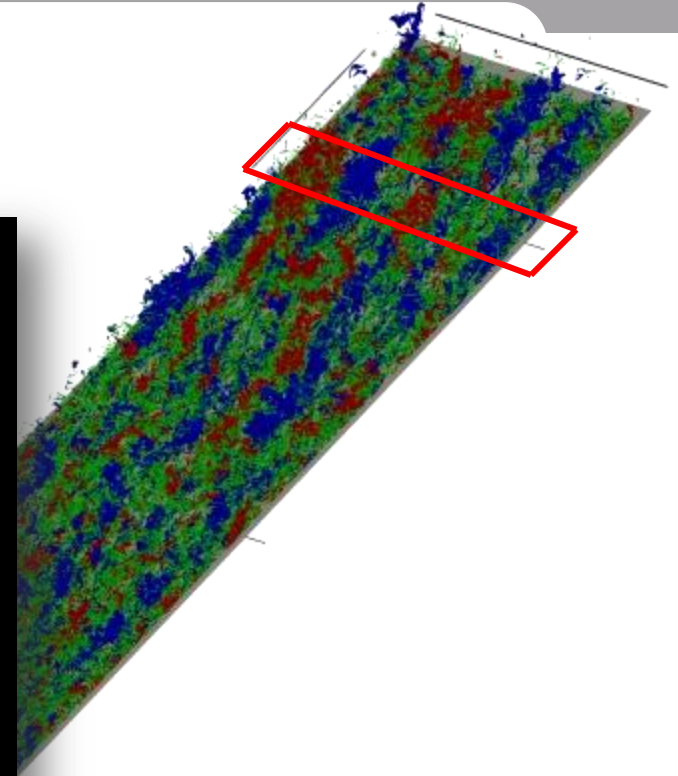


Isocontours of  $\lambda_2$ ,  
colour code  $\sim$  wall distance

# Structures...



$$Re_\theta = 4300$$



Isocontours of  $\lambda_2$ ,  
colour code  $\sim$  wall distance



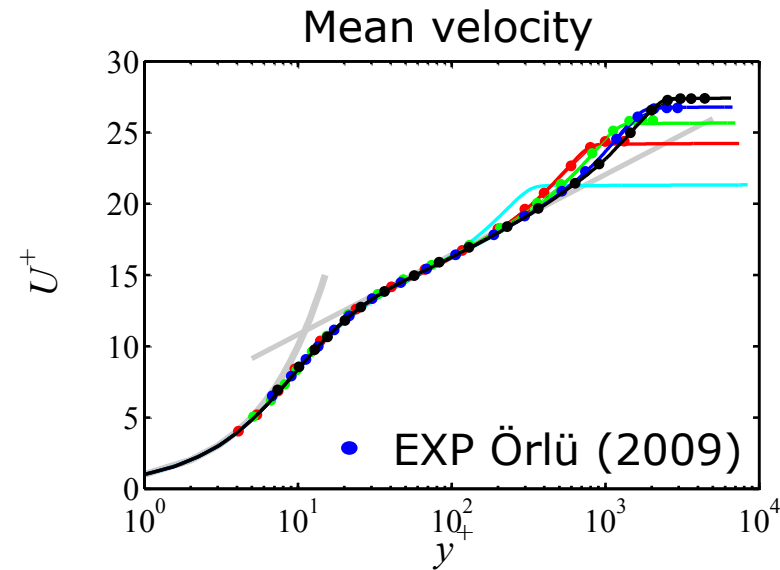
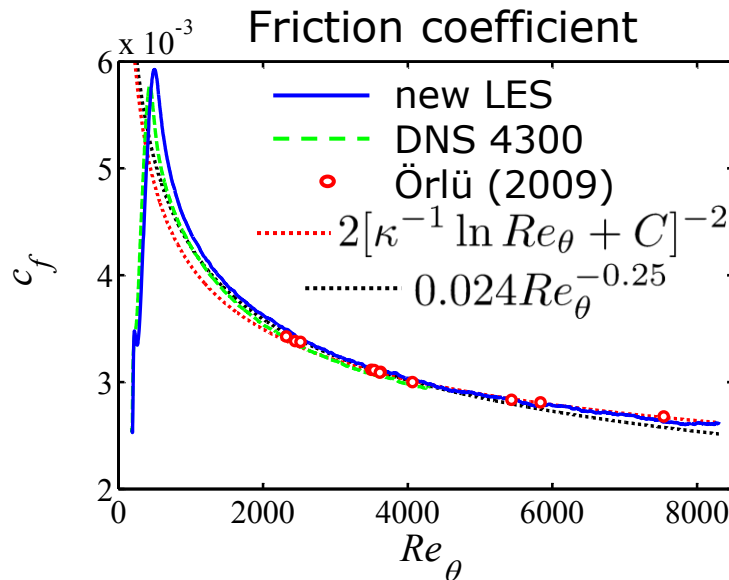


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Preliminary Results:  
Ongoing simulation  
up to  $Re_{\theta}=8300$

# LES up to $Re_\theta=8300$

- **Ongoing** LES (using ADM-RT)



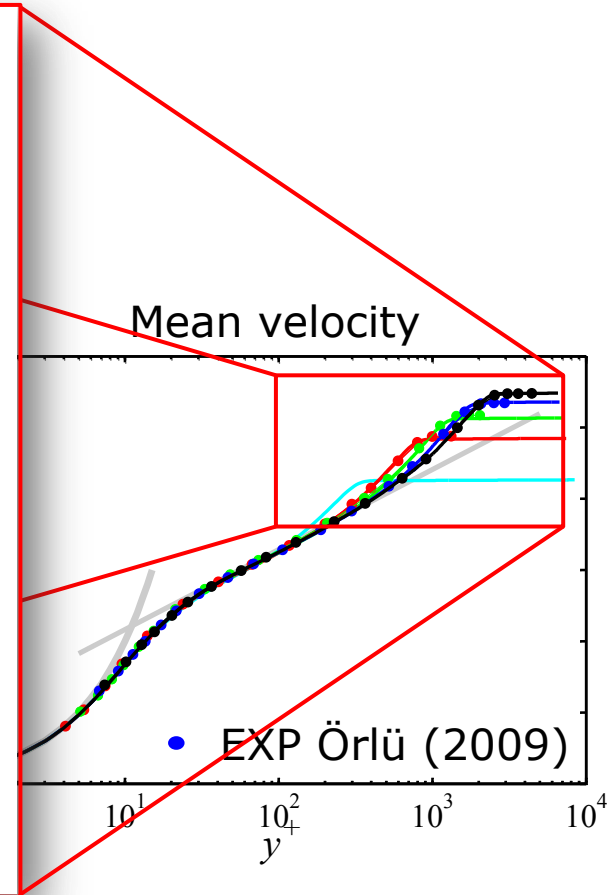
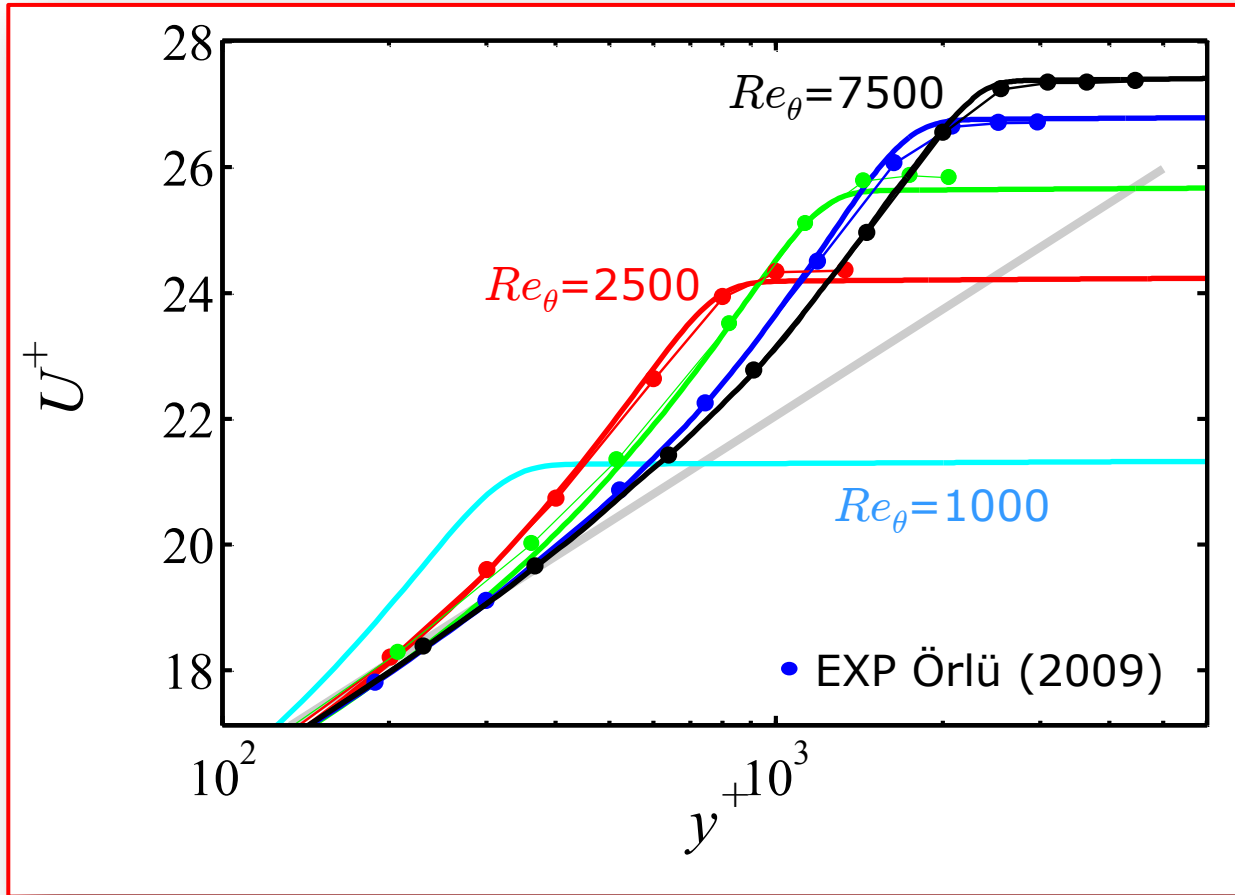
Domain: 13500 x 400 x 540 $\delta_0^*$

$Re_\theta = 500-8300 ; Re_\tau = 2300$

Resolution: 9216 x 513 x 768  
(8.5 billion grid points)

$\Delta x^+ = 18, \Delta y^+ = 0.06-16, \Delta z^+ = 8$

—  $Re_\theta=1000$   
 —  $Re_\theta=2500$   
 —  $Re_\theta=4000$   
 —  $Re_\theta=5800$   
 —  $Re_\theta=7500$



Domain: 13500 x 400 x 540 $\delta_0^*$   
 **$Re_\theta = 500-8300 ; Re_\tau = 2300$**

Resolution: 9216 x 513 x 768  
 (8.5 billion grid points)  
 $\Delta x^+ = 18, \Delta y^+ = 0.06-16, \Delta z^+ = 8$

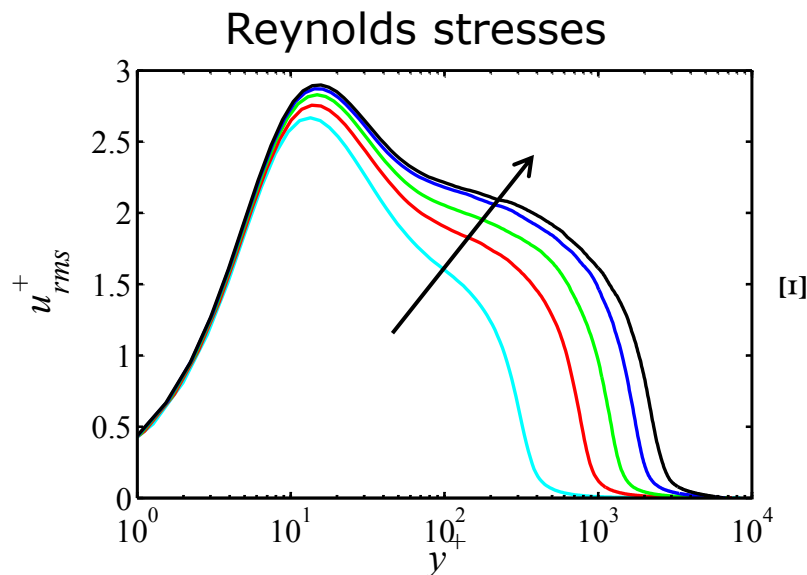
- $Re_\theta = 1000$
- $Re_\theta = 2500$
- $Re_\theta = 4000$
- $Re_\theta = 5800$
- $Re_\theta = 7500$

# LES up to $Re_\theta = 8300$

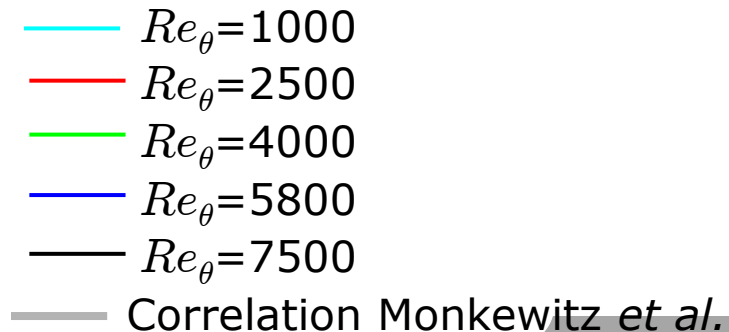
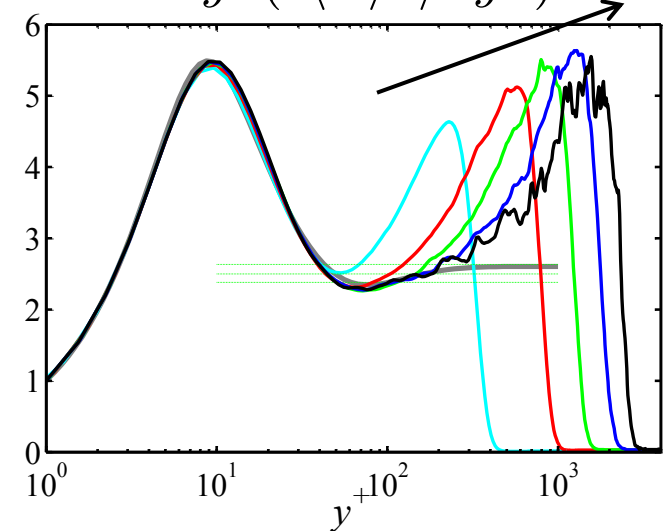
- **Ongoing** LES (using ADM-RT)



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Indicator function  
 $\Xi = y^+ (d\langle U \rangle^+ / dy^+)$



**$Re_\theta = 500-8300$  ;  $Re_\tau = 2300$**

- Good agreement at lower Reynolds number with other DNS/LES
- Good agreement with experiments at higher  $Re$
- Proper scaling behaviour at higher  $Re$

# Conclusions

DNS data for a **spatial turbulent zero pressure gradient turbulent boundary layer** from  $Re_\theta=180$  up to  $Re_\theta=4300$ , using  $\sim 7.5 \cdot 10^9$  grid points.

## NUMERICAL EXPERIMENT

1. **Statistics/budgets/spectra/PDF** etc. in excellent agreement with experiments
2. **Outer-layer convergence** and fully developed state for boundary layers. When do we have a "good" simulation?
3. Large-scale structures  $\mathcal{O}(\delta_{99})$  with **footprint/modulation** visible at the wall
4. Visualisation of **coherent structures** in high- $Re$  turbulent boundary layers: No clear hairpin vortices detected except for low- $Re$  (transition) region.

Data and visualisations at: [www.mech.kth.se/~pschlatt/DATA](http://www.mech.kth.se/~pschlatt/DATA)

Contact me: [pschlatt@mech.kth.se](mailto:pschlatt@mech.kth.se)

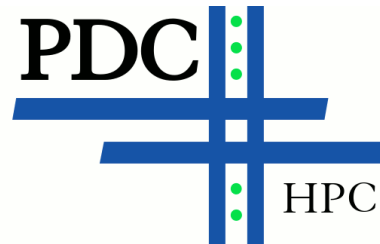


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# Acknowledgments:



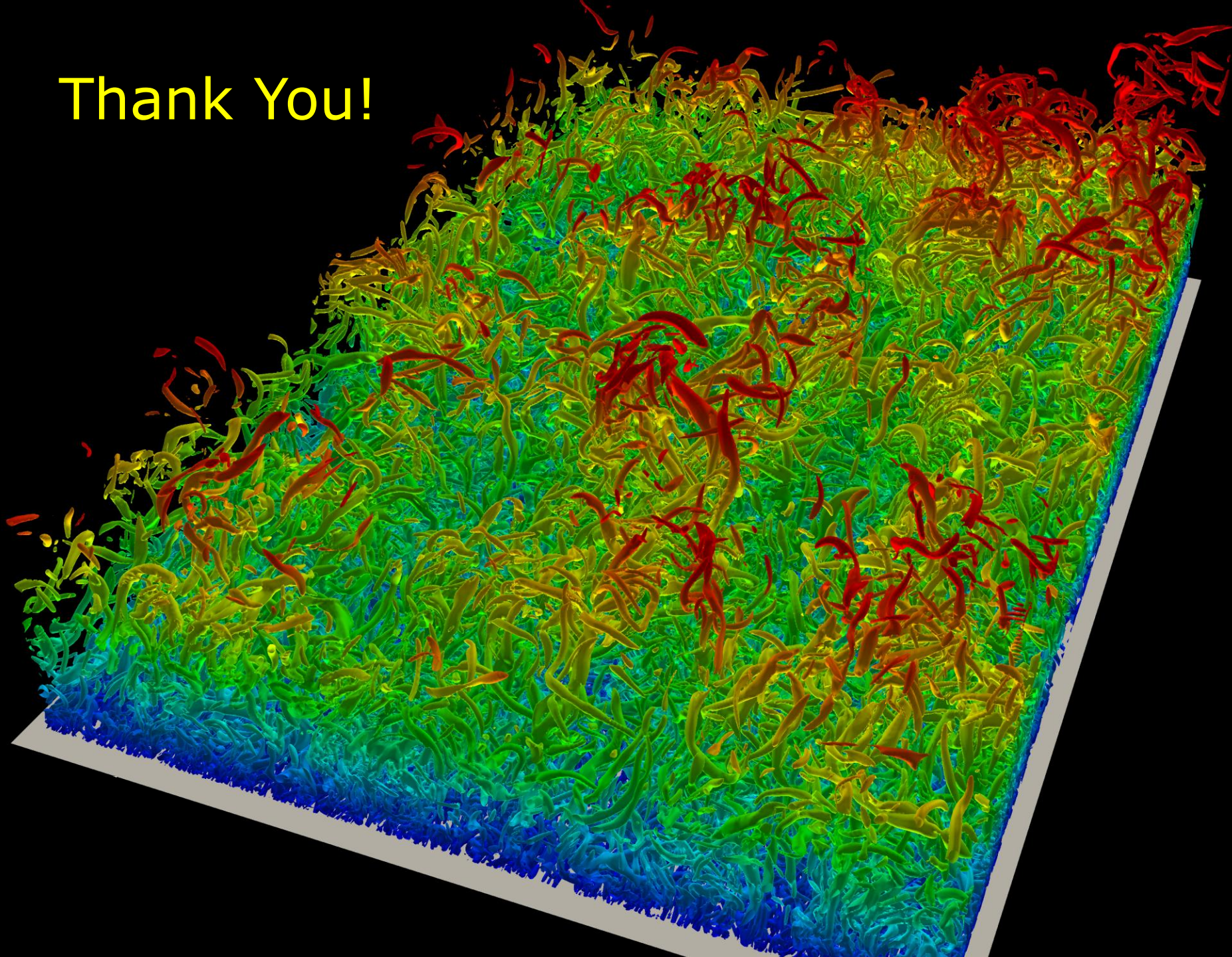
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*Knut och Alice  
Wallenbergs  
Stiftelse*



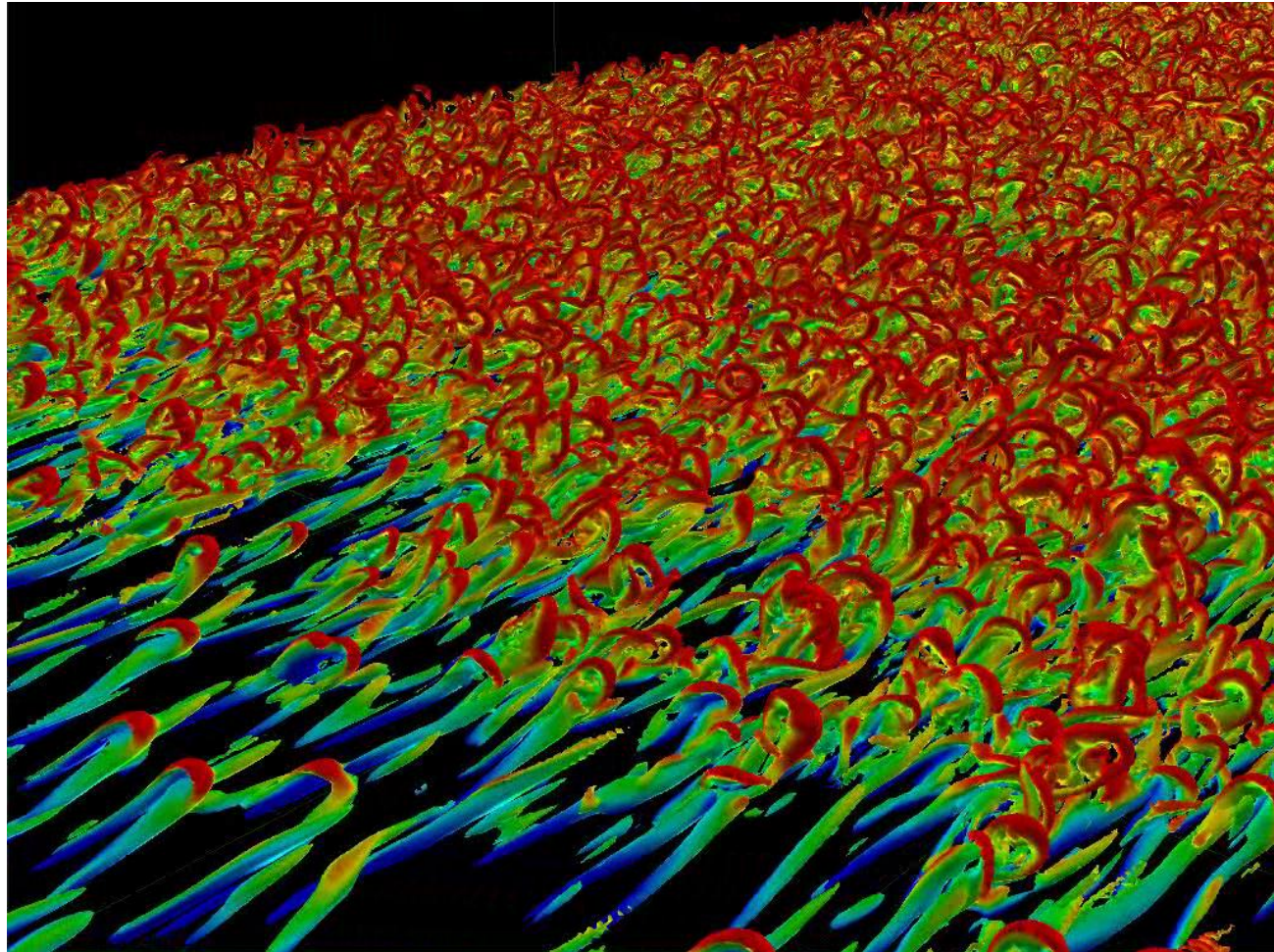
Thank You!



# Boundary Layer Visualisation



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based on DNS data