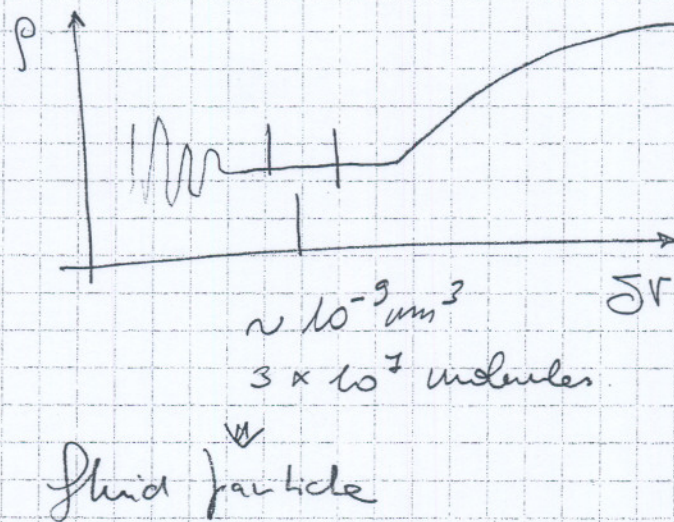


WIKI

- GALILEI PRINCIPLE (RECIPROCALITY)
- fluid can be considered a continuous medium
- any physical quantity $\rho, T, \nu \dots$ at P $p(P)$ is an average of a small volume whose center of gravity is P (δV)
- How big δV (?)
 - small enough to be considered infinitesimal with respect the variations
 - large enough to contain a sufficient number of molecules



- Fluid $\rightarrow$ substance that deforms itself under ~~shear stress~~ tangential stresses
 - liquid $\rightarrow$ eq. of state $p = \text{const}$
 - gases $\rightarrow$ eq. of state $p = \rho R T$ $R = 287 \frac{\text{m}^2}{\text{s}^2 \text{ } ^\circ\text{K}}$
- for gas in motion (compressible aerodynamics)
 - ρ varies when $v \sim a$ (speed of sound)

Kinematic properties

$\vec{V} \rightarrow$ mean of

$$\vec{a} = \frac{d\vec{V}}{dt} \quad (\text{material derivative})$$

$$\vec{\omega} = \frac{\text{curl}}{\text{rot}} \vec{V} \rightarrow 2\Omega$$

Transport properties

- streamlines
- pathlines
- streaklines

(2)

a = velocity of propagation of small disturbances.

$$a = \sqrt{\gamma R T}$$

$$\gamma = \frac{C_p}{C_v}$$

C_p = specific heat at constant pressure

$$M = \frac{V}{a}$$

$\rightarrow$ parameter which characterises compressibility

— density variations are of the order of $\frac{1}{2} M^2$

$$\Rightarrow M < 0.3 \quad \rho = \pm 5\%$$

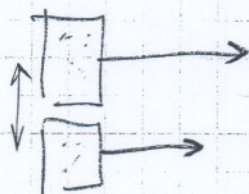
(1B)

When fluid is at rest particle is subjected to only normal stresses.

In motion there are also tangential stresses which describes the tendency to resist deformation.

This property is called viscosity!

$\rightarrow$ physical mechanism acting at molecular level.



— Viscosity stress can be normal $\rightarrow$ Stress Tensor

For Newtonian fluids
$$\tau_{ik} = \mu \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right)$$

(3)

- μ is function of Fluid
Temperature

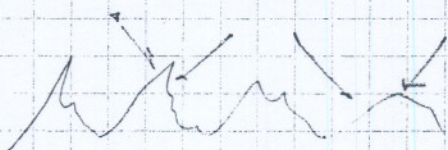
- luckily $\mu \approx \sqrt{T}$

if M is small variation of T *
then μ is constant

↓
Simplification

* When $\mu = 0 \rightarrow$ IDEAL FLUID $(\vec{V} - \vec{V}_p) \cdot \vec{n} = 0$
 $\mu \neq 0 \rightarrow$ REAL FLUID $(\vec{V} - \vec{V}_p) = 0$

Boundary conditions close walls are always like
 $(\vec{V} - \vec{V}_p) = 0$ R Fluids



Fundamental principles

Conservation of mass.

$$\frac{d}{dt} \int_V \rho dV = 0 \rightarrow \operatorname{div} \vec{v} = 0$$

Balance of momentum

$$\frac{d}{dt} \int_V \rho \vec{v} dV = 0 \rightarrow \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = - \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{v}$$

Conservation of energy.

$$f(T, \vec{v}, \phi) = 0$$

For incompressible flow we can decouple the system

→ Once is solved for $\phi, \vec{v}$ we can calculate all the forces on the car

ϕ , from $\vec{v} \rightarrow \vec{z}$ | pressure and friction coefficients c_p, c_f

Strong^{na} linearity → CANNOT BE SOLVED !!!

It is necessary to look for simplification!

Let's introduce the vorticity vector

$$\vec{\omega} = \text{curl } \vec{V}$$

Maybe velocity field can be derived by vorticity.

Some algebra:

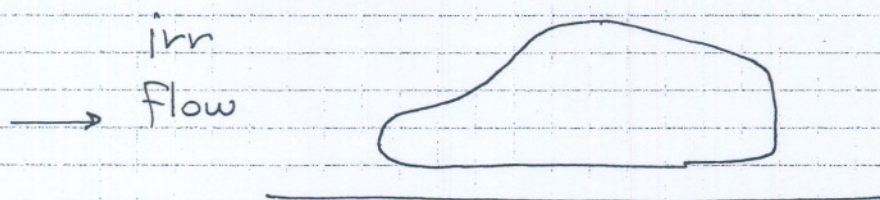
$$\nabla^2 \vec{V} = \nabla \text{grad}(\text{div } \vec{V}) - \text{curl } \vec{\omega}$$

$$\vec{V} \cdot \nabla \text{grad } \vec{V} = \vec{\omega} \wedge \vec{V} - \text{grad} \left(\frac{V^2}{2} \right)$$

$$\vec{f} = -\text{grad } \Omega$$

then

$$\left\{ \begin{array}{l} \text{div } \vec{V} = 0 \\ \frac{\partial \vec{V}}{\partial t} + \vec{\omega} \wedge \vec{V} = -\text{grad} \left(\Omega + \frac{p}{\rho} + \frac{V^2}{2} \right) - \nu \text{curl } \vec{\omega} \\ \text{energy eq.} \quad \text{c.c.} \quad \vec{V}|_{\text{surface}} = 0 \end{array} \right.$$



What happen if $\vec{\omega} = 0$ everywhere?

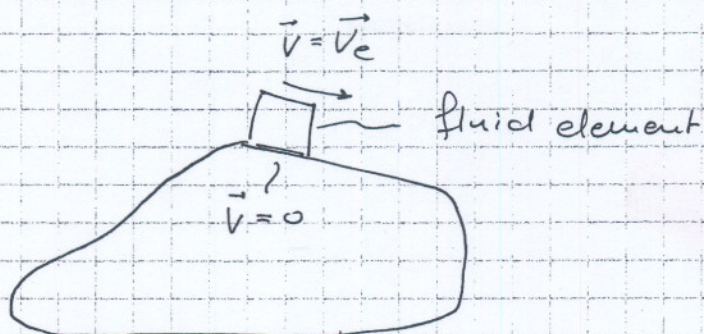
$$\text{if } \vec{\omega} = 0 \quad \text{curl } \vec{V} = 0 \Rightarrow \vec{V} = \text{grad } \varphi$$

$$\left\{ \begin{array}{l} \nabla^2 \varphi = 0 \\ \text{grad } \frac{\partial \varphi}{\partial t} = -\text{grad} \left(\Omega + \frac{p}{\rho} + \frac{V^2}{2} \right) \\ + \text{c.c.} \quad \text{grad } \varphi|_{\text{surface}} = 0 \end{array} \right.$$



No solutions !!!

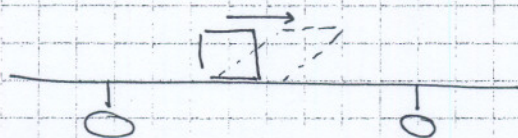
WHY? What happens to the surface



Fluid element rotates

Surface is a source of vorticity.

ORIGIN AND DYNAMICS OF VORTICITY



Flat car sudden move

If we integrate the vorticity as $y \rightarrow \gamma(x)$

$$\gamma(x) = \int_0^y -\frac{\partial u}{\partial y} dy = -U$$

In the element total vorticity is $\gamma(x)dx$

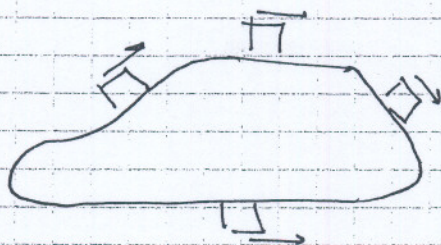
On the car $\Gamma = \int_0^L \gamma(x) dx = -UL$

The same results can be obtained using the Stokes - theorem.

$$\vec{\omega}_z(x,y) dx dy = \oint \vec{v} \cdot d\vec{e} = -U dx$$

Integrate On the car $\rightarrow -UL$

This can be extended to any type of body. 4



Globally Wu Theorem (1984)

Vorticity is constant everywhere $\int_V \vec{\omega} dV = \text{const}$

We must take into account the rotation of the bodies immersed in the fluid.

Where the vorticity goes?

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \text{grad} \vec{v} = -\frac{1}{\rho} \text{grad} p + \nu \nabla^2 \vec{v}$$

make the curl

$$\frac{\partial \vec{\omega}}{\partial t} + \vec{v} \cdot \text{grad} \vec{\omega} = \vec{\omega} \cdot \text{grad} \vec{v} + \nu \nabla^2 \vec{\omega}$$

being $\text{curl}(\vec{A} \wedge \vec{B}) = -\vec{A} \cdot \text{grad} \vec{B} + \vec{B} \cdot \text{grad} \vec{A}$
and $\text{curl} \text{grad} \vec{A} = 0$

$$\Rightarrow \frac{d\vec{\omega}}{dt} = \underbrace{\vec{\omega} \cdot \text{grad} \vec{v}}_{\text{Vortex stretching}} + \nu \nabla^2 \vec{\omega}$$

Vortex stretching

VORTEX STRETCHING

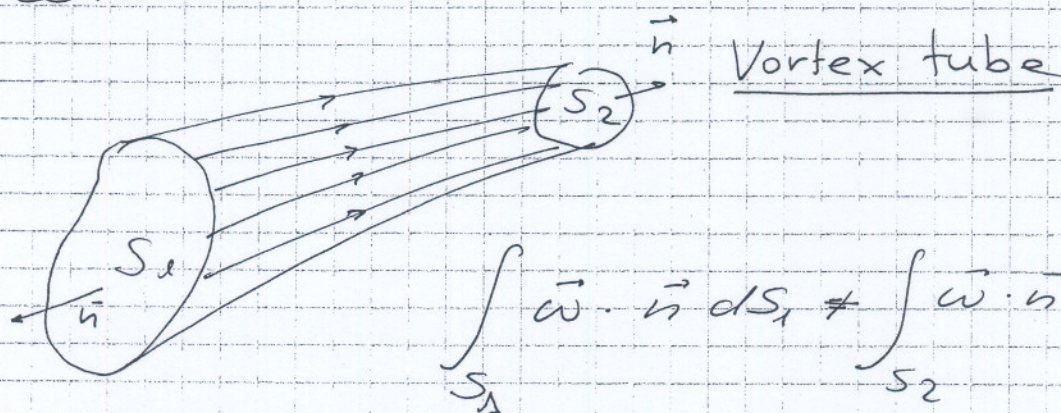
5

$$\operatorname{div} \operatorname{curl} \vec{A} = 0$$

$$\Rightarrow \operatorname{div} \vec{\omega} = 0 \quad \text{everywhere}$$

$$\int_V \operatorname{div} \vec{\omega} dV = \int_S \vec{\omega} \cdot \vec{n} dS = 0$$

Green theorem



$$\int_{S_1} \vec{\omega} \cdot \vec{n} dS_1 \neq \int_{S_2} \vec{\omega} \cdot \vec{n} dS_2$$

$$-\vec{\omega}_1 S_1 + \vec{\omega}_2 S_2 = 0$$

$$\vec{\omega}_1 S_1 = \vec{\omega}_2 S_2$$

but vorticity lines are material lines
(Helmoltz theorem)

Therefore if you stretch the tube
you decrease S_2 and you increase $\vec{\omega}_2$

$$\text{In 2D} \quad \vec{\omega} = (0, 0, \omega_z)$$

$$\vec{v} = (u, v, 0)$$

$$\boxed{\vec{\omega} \cdot \operatorname{grad} \vec{v} = 0}$$

In 2D

6

$$\frac{D\vec{\omega}_z}{Dt} = \nu \nabla^2 \vec{\omega}_z$$

DIFFUSION
EQUATION

So

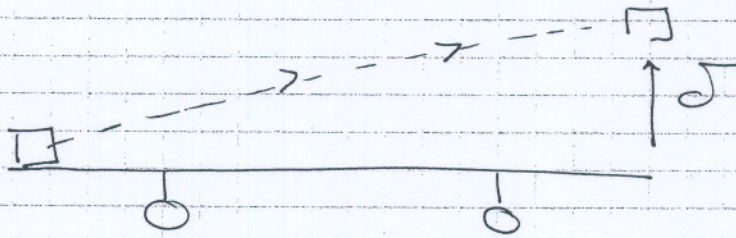
$$\frac{Dc}{Dt} = k_D \nabla^2 c$$

~~So~~ c instantly diffuses everywhere

but δ (where you have approx 90%)

$$\delta \approx (k_D t)^{1/2}$$

What about time?



$$t = \frac{L}{U} \rightarrow \delta \approx \left(\nu \frac{L}{U} \right)^{1/2}$$

$$\frac{\delta}{L} \approx \left(\frac{\nu}{LU} \right)^{1/2} = \frac{1}{\sqrt{Re_L}} \quad \text{VERY THIN !!!}$$

We can call BOUNDARY LAYER the region where we have 99% of vorticity.

And a new definition of Reynolds number

$$Re = \frac{\text{convective term}}{\text{diffusive term}} = \frac{\vec{v} \cdot \text{grad } \vec{\omega}}{\nu \nabla^2 \vec{\omega}}$$

PRANDTL THEORY

7

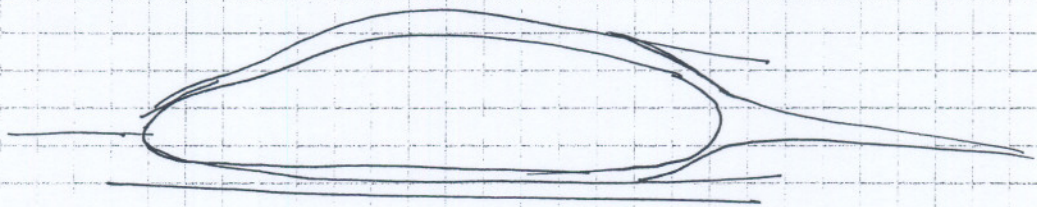
If this is the case.

boundary layer very thin! then

$$\left| \frac{\partial u}{\partial x} \right| \ll \left| \frac{\partial u}{\partial y} \right| ; \quad \left| \frac{\partial^2 u}{\partial x^2} \right| \ll \left| \frac{\partial^2 u}{\partial y^2} \right|$$

$$\begin{cases} \text{div } \vec{v} = 0 \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \end{cases}$$

$\left| \frac{\partial p}{\partial y} \approx 0 \right| \quad !!! \quad \Leftarrow \text{VERY IMPORTANT RESULT !!}$
eq. eq.



~~ITERATIVE PROCEDURE~~

1) $\delta = 0$

2)
$$\begin{cases} \nabla \phi = 0 \\ \frac{\partial \phi}{\partial t} + \frac{p}{\rho} + \frac{v^2}{2} = f(t) \end{cases}$$

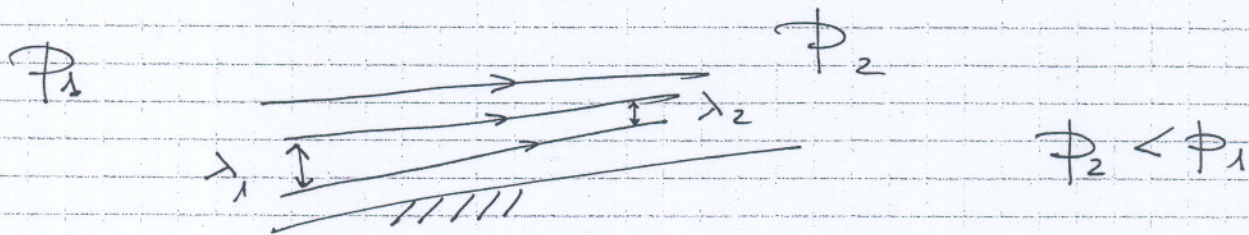
Very easy to perform!

but

do we know the boundary of the inviscid flow?

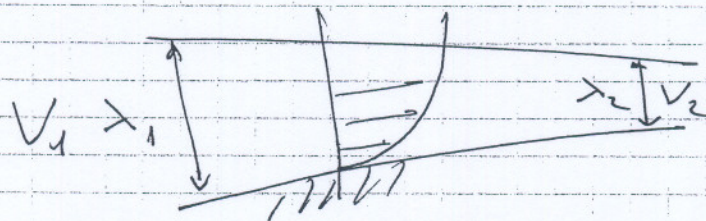
In a general body, i.e. a car, the b.l. remains thin?

But! Does it remains thin the region flat contains the vorticity?



$\bar{V}$ increases $\rightarrow \lambda$ decreases

see continuity equation



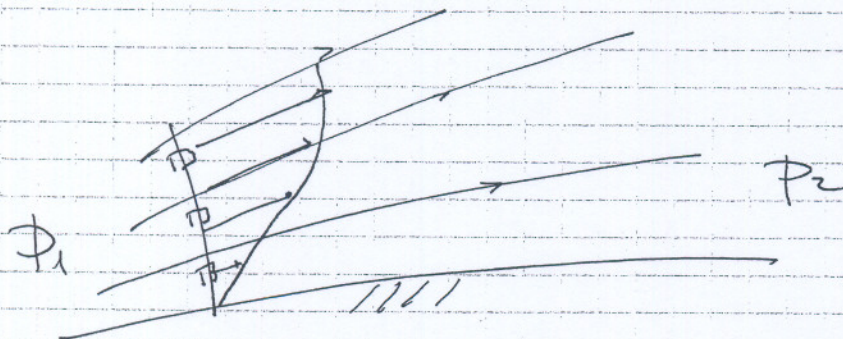
curvature of the velocity profile

FAVOURABLE PRESSURE GRADIENTS.

$$V_1 \lambda_1 = V_2 \lambda_2$$

So b.l. remains thin

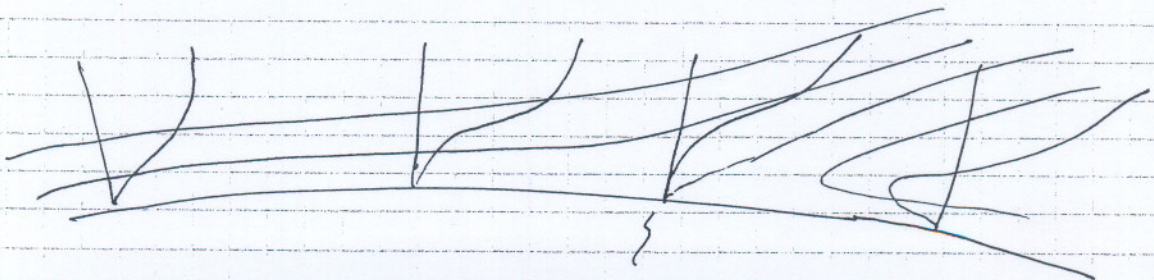
$$\left. \frac{\partial p}{\partial x} \right|_w = \nu \left. \frac{\partial^2 u}{\partial y^2} \right|_w$$



ADVERSE PRESSURE GRADIENTS

$$\left. \frac{\partial^2 u}{\partial y^2} \right|_w < 0$$

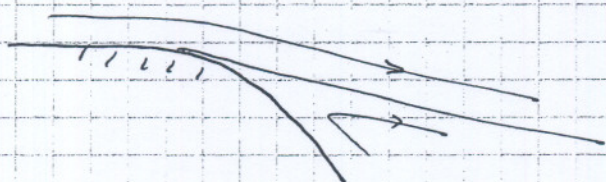
Thickness increases and



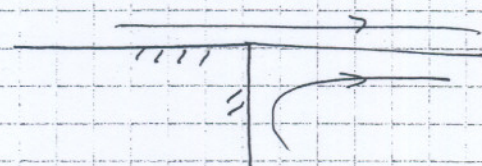
SEPARATION.

Separation = $f\left(\frac{\partial p}{\partial x}, \text{geometry}\right)$

geometry



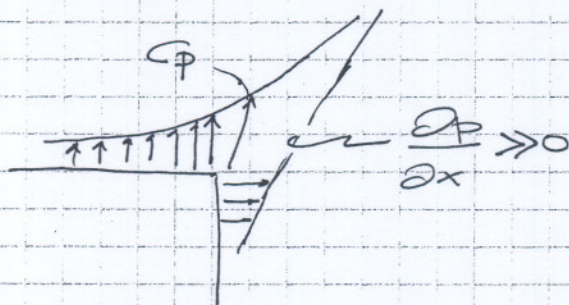
free separation



fixed separation

Actually this geometry dependence can be viewed as a dependence on $\frac{\partial p}{\partial x}$

In fact flow at the corner accelerates, then pressure decreases

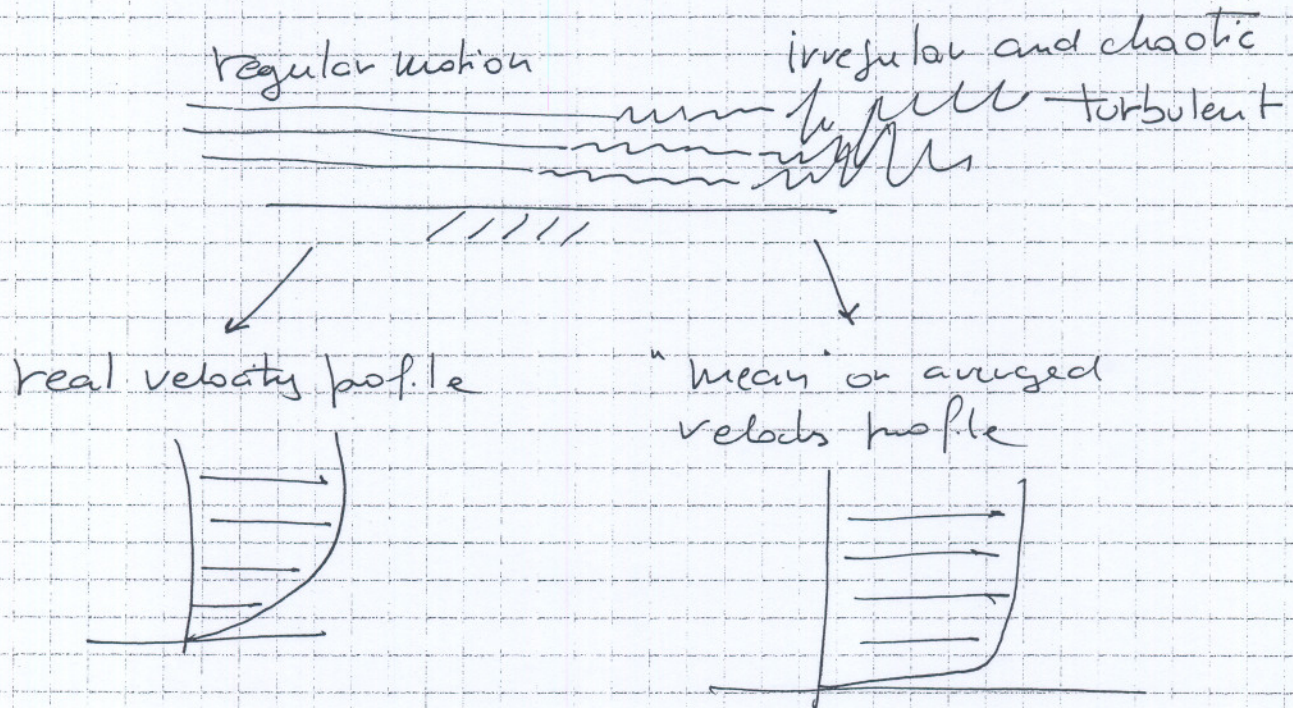


Separation also strongly depends on the Reynolds number -

Why?

LAMINAR SEPARATION TRANSITION

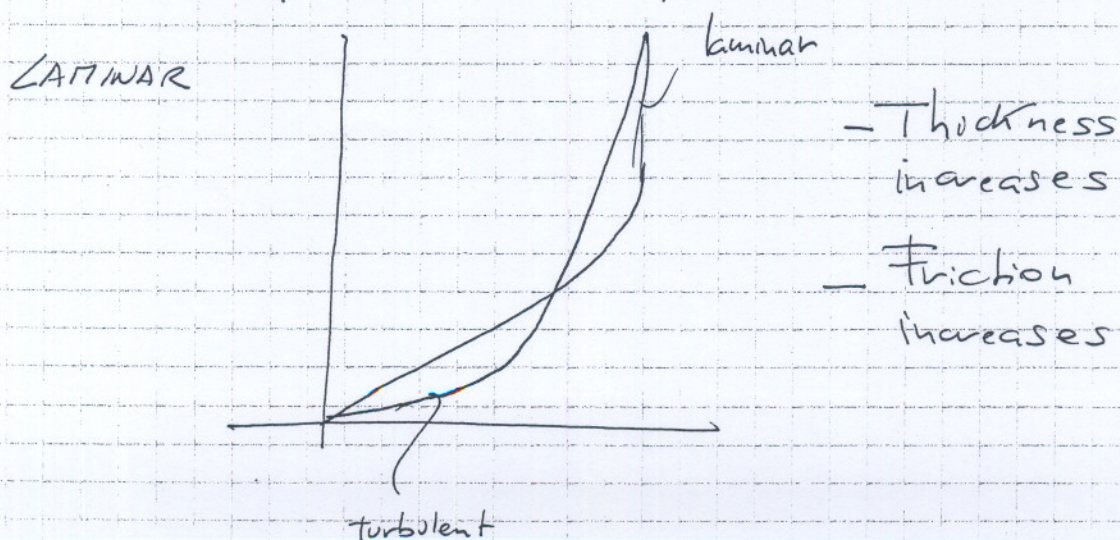
But there is another phenomenon



$$\text{trans.} = f \left(\text{disturbances, } Re, \frac{\partial P}{\partial x} \right) \quad \text{free stream turbulence}$$

roughness speed and distance shape

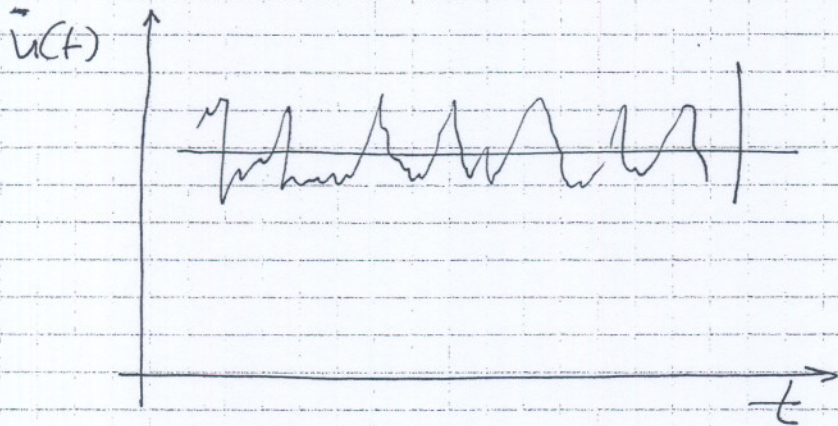
Comparison between profiles



Turbulence

The knowledge of the characteristics of the meaning turbulence is more interesting for people working in INDUSTRIAL AERODYNAMICS

Let's take a velocity signal $u(t) = \bar{U} + u'(t)$



We take a large T

$$\bar{U} = \frac{1}{T} \int_0^T u(t) dt$$

$$\overline{u'^2} = \sigma^2 = \frac{1}{T} \int_0^T (u')^2 dt \quad \text{variance}$$

while σ is called rms (root mean squared)

finally turbulence intensity

$$I_{\%} = \frac{\sqrt{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}}{\sqrt{\bar{U}^2 + \bar{V}^2 + \bar{W}^2}} \cdot 100 \quad \leftarrow \text{Mean velocity}$$

For airplanes $I_{\%}$ is normally very small.

For car sometimes cannot be neglected

It's easier to build a tunnel for automotive application.

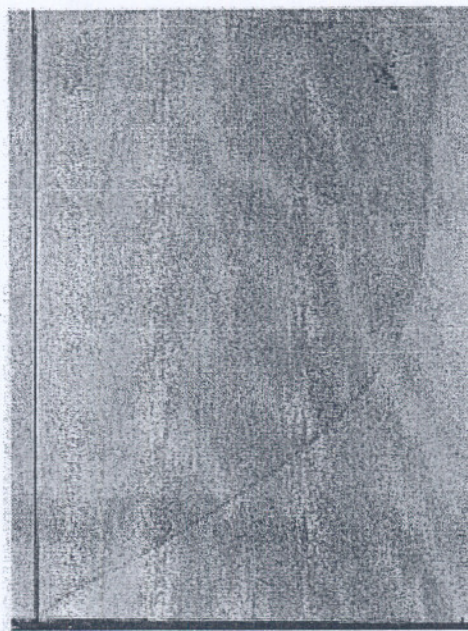


Fig. 1 – Boundary layer profile over a flat plate (from Van Dyke 1982).

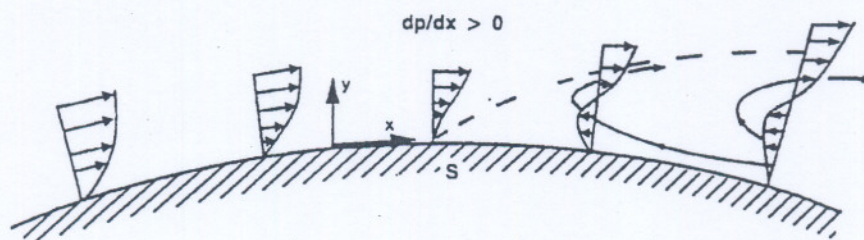


Fig. 2 – Boundary layer separation

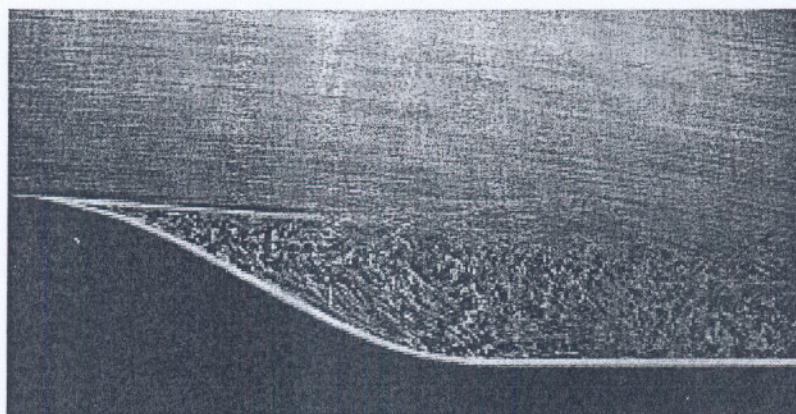


Fig. 3 – Example of flow separation from a curved wall (from Van Dyke 1982).



Fig. 4 – Example of turbulent boundary layer visualization (from Van Dyke 1982).

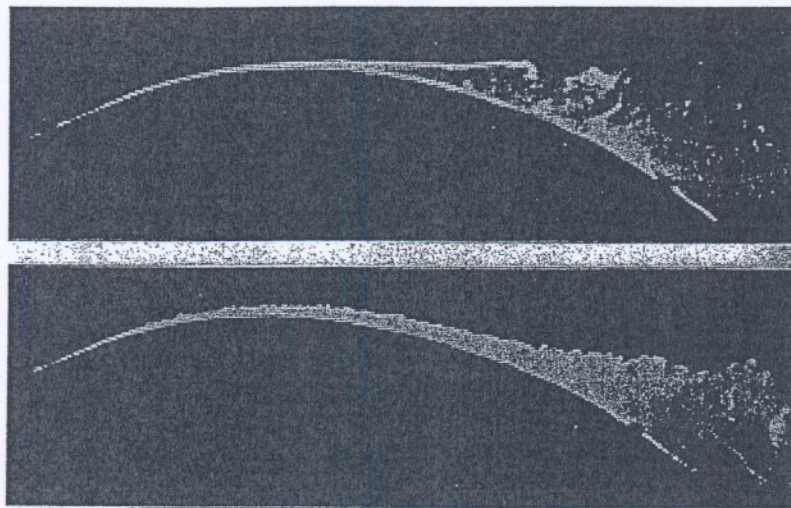


Fig. 5 – Example of laminar (upper photo) and turbulent (lower photo) separation over a curved surface (from Van Dyke 1982).

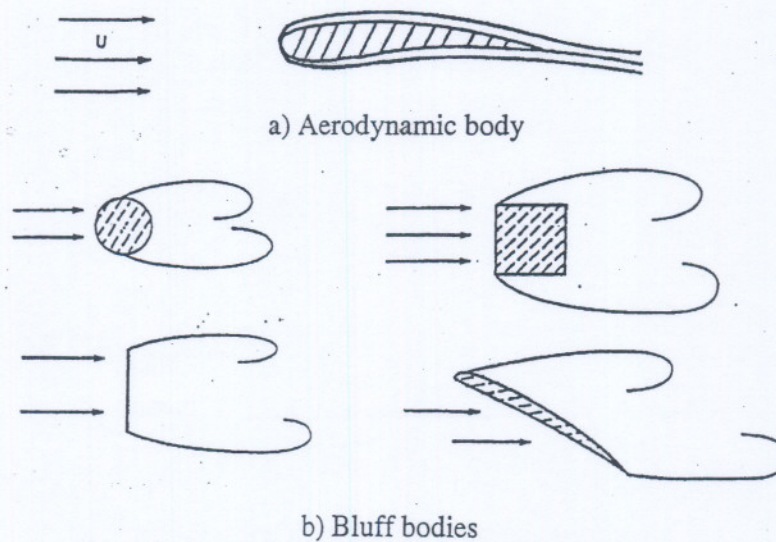
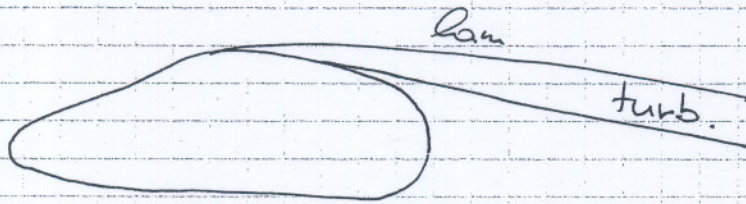


Fig. 6 – Examples of aerodynamic and bluff bodies

If profile is turbulent

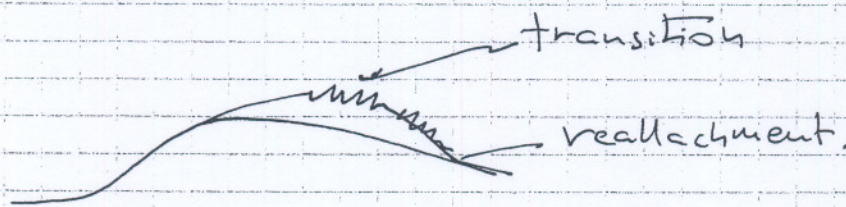
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→ SEPARATION IS DELAYED



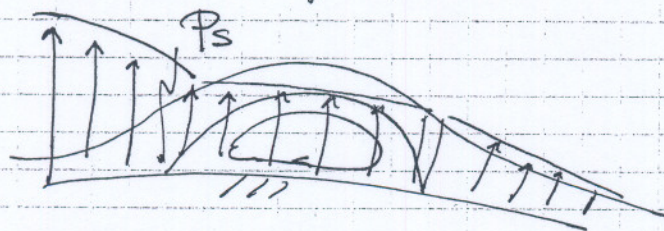
more energy
in the boundary
layer.

We can have REATTACHMENT



forming a separation bubble.

As a very first approximation it can be stated that the pressure in the separated region is constant and is the same as the one at the separation point !!



Now we are able to define what is aerodynamic and what is bluff.

AERODYNAMIC BODY →

boundary layer attached everywhere except a very small region at the end of the body.

BLUFF BODY

→ presence of ~~very~~ large separated regions.