Receptivity of 3D Boundary Layers

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Receptivity Study

Receptivity mechanisms in Falkner-Skan-Cooke boundary-layer flow are studied by numerical simulation.

Receptivity

Receptivity denotes the process by which external perturbations caused by e.g. surface roughness or free-stream turbulence are converted into boundary-layer disturbances. The initial disturbance amplitudes are established in the receptivity phase. The efficiency of the receptivity mechanism is measured in terms of a **receptivity coefficient** (=normalized receptivity amplitude).

Flow configuration

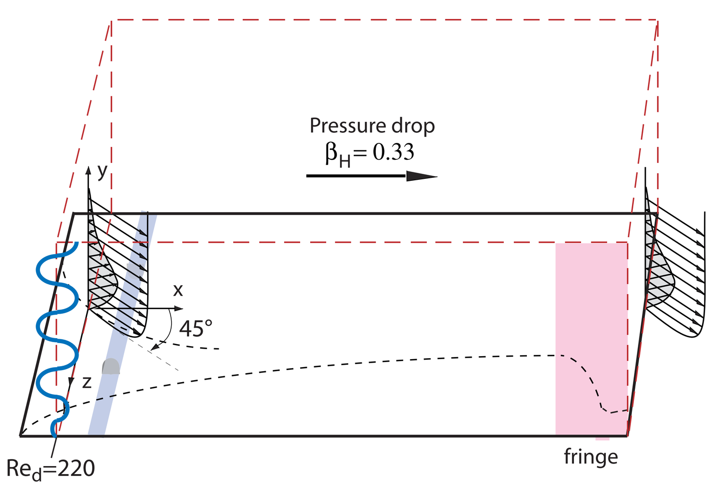


FIGURE 1 The 3D base flow is exposed to two types of perturbations: surface roughness and free-stream vorticity.

Receptivity to roughness

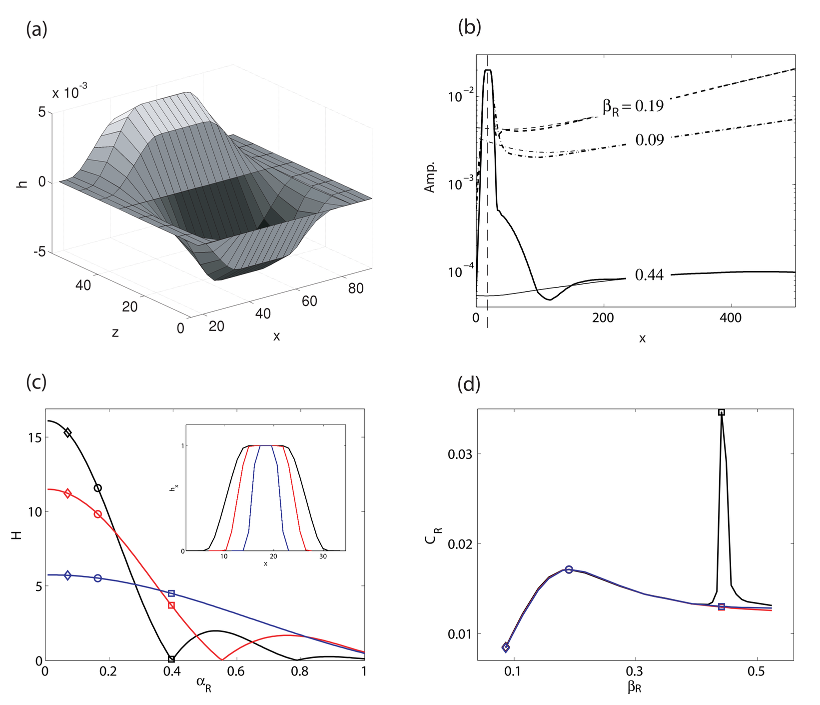


FIGURE 2 (a) Spanwise periodic localized roughness. (b) Boundary-layer response: Transient behavior around the roughness element and exponential growth of **steady cross-flow instability** further downstream. Strong dependence of the disturbance amplitude on the spanwise wavenumber β_R of the roughness. (c) Roughness contour in physical (insertion) and spectral space. (d) Receptivity coefficient is independent of the roughness shape for a large range of β_R. Maximum receptivity at $\beta_R = 0.19$

Receptivity to free-stream vorticity

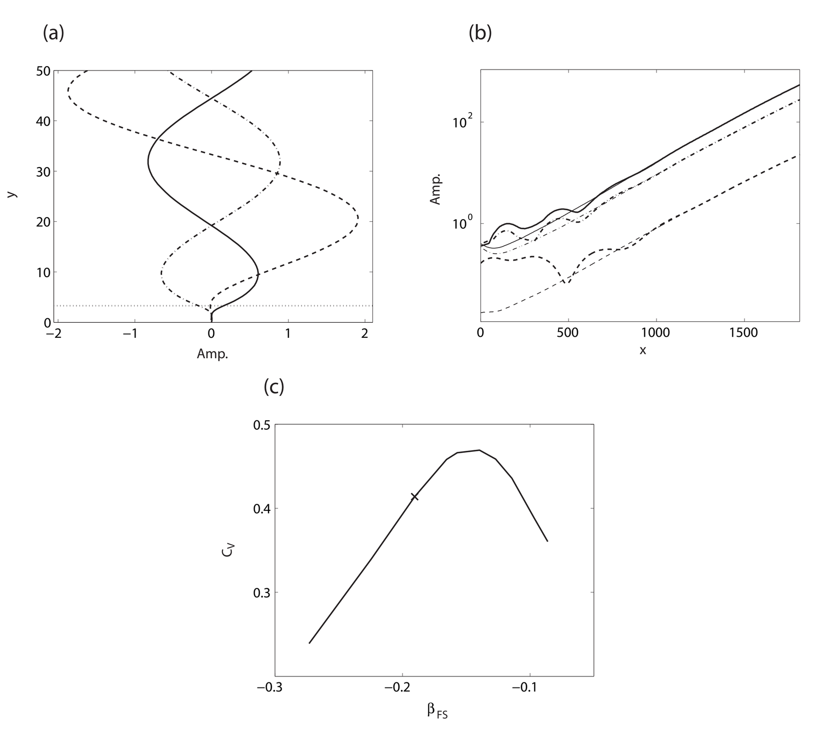
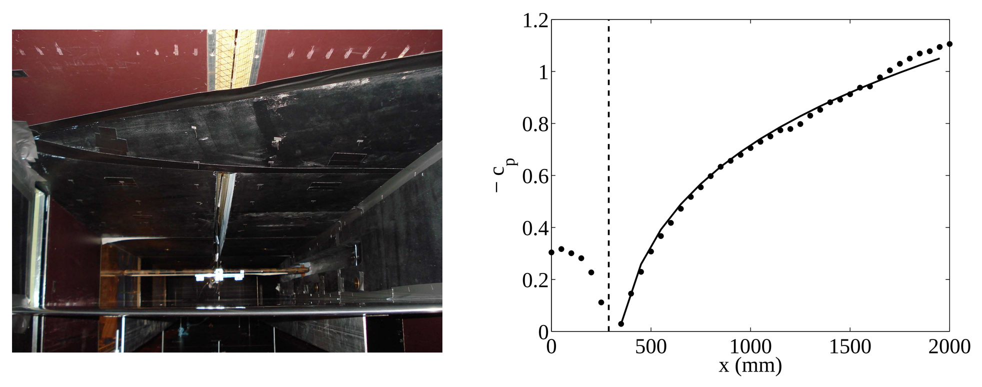


FIGURE 3 (a) Continuous-spectrum Orr-Sommerfeld mode as a model for vortical free-stream disturbances. (b) Response of the base flow: Transient behavior followed by exponential growth of **unsteady cross-flow instability**. (c) Receptivity coefficient versus spanwise wavenumber of the free-stream mode. Maximum receptivity at $\beta_{FS} = -0.14$

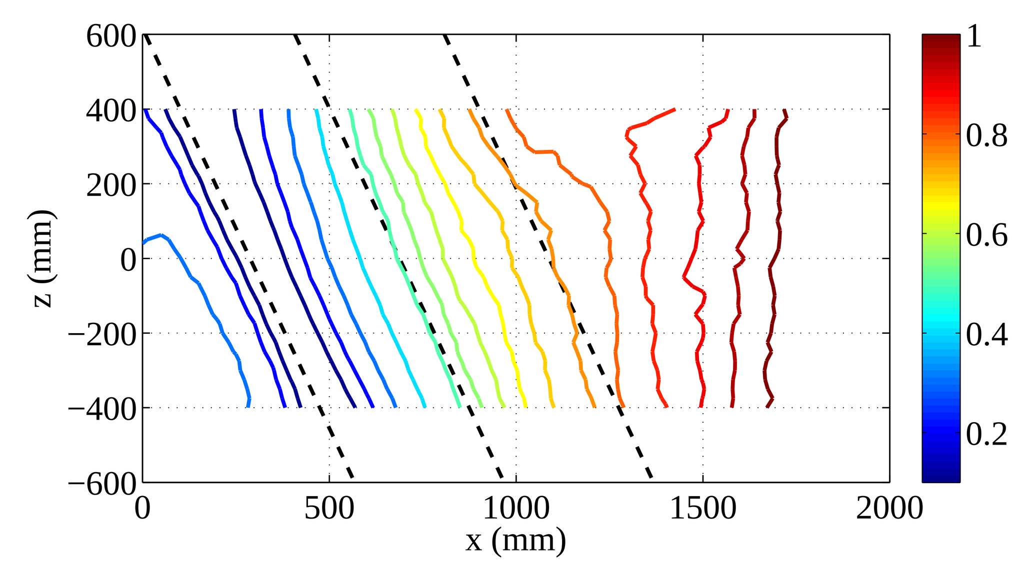
Experimental Study

Experiments will be carried out at the Minimum Turbulence Level (MTL) wind tunnel facility located at KTH Mechanics. Taking into account the expected growth of the crossflow instability, the size of the boundary layer and the overall blockage needed, the free stream velocity close to the leading edge was chosen to be 13.2 m/s at a sweep angle of 25°. Below to the left is a photo of the setup viewed from the front.



The black regions are made from styrofoam and are placed in the tunnel to provide a strong acceleration at the leading edge, which will excite the cross flow instability. The Hartree parameter, β_H, is related to the acceleration of the free stream velocity in a pressure gradient. The figure to the above right shows the resulting pressure distribution with the dashed line corresponding to the leading edge. In this case $\beta_H = 0.19$, but this value can be adjusted slightly by means of a flap located at the trailing edge of the flat plate, which regulates the leading edge stagnation line.

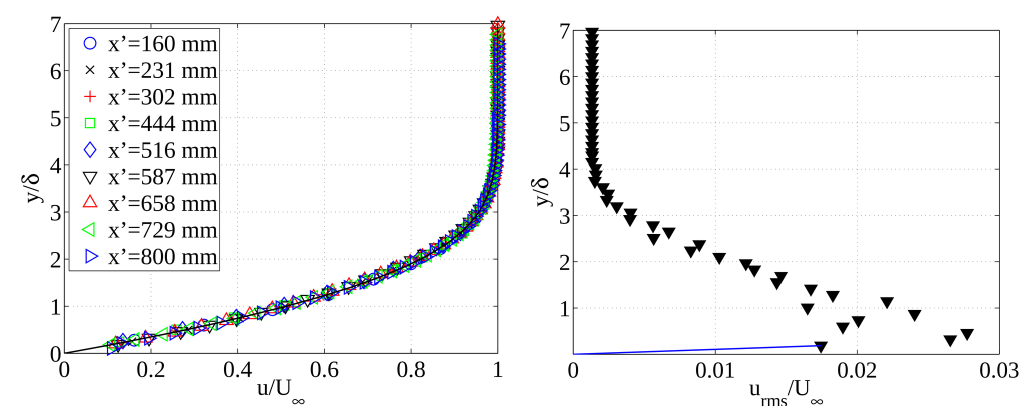
Fluent® calculations were done to design the side walls intended to redirect the flow and obtain a more uniform pressure distribution parallel to the leading edge. This was done by making the side walls the same shape as the inviscid streamlines of the flow at the center of the plate. The final measured base flow is shown below with the values corresponding to $-c_p$.



For the upcoming study several turbulence generating meshes were characterized in terms of turbulence decay and length scales.

Boundary Layer and Disturbance Profile

Several boundary layers were measured traversing chordwise over the flat plate. The profiles for several locations are shown below to the left. We see good agreement with the Falkner-Skan-Cooke boundary layer with $\beta_H = 0.19$ (solid line). A typical disturbance profile is shown below to the right, showing that we are able to resolve the peak in the u_{rms} level at a boundary layer thickness of 3 mm.



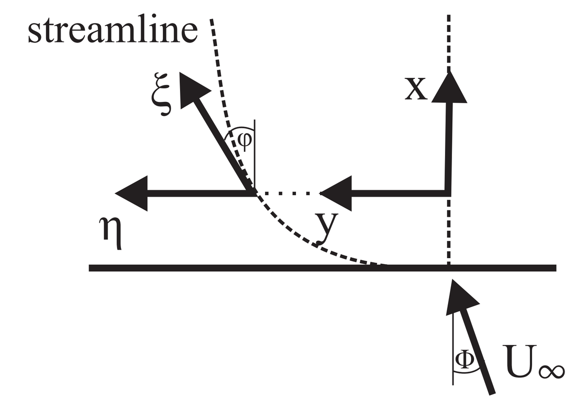
The next step will involve measuring the spanwise velocity component as well as the disturbance growth profile at different freestream turbulence and length scale conditions. Future work will also involve measuring the effect of surface roughness on the disturbance growth.

Optimal Disturbances

The algebraic growth of spanwise periodic, stationary disturbances in a Falkner-Skan-Cooke boundary layer is studied and the optimal disturbances associated with the maximum energy growth are calculated. To study the receptivity mechanisms these optimal disturbances have to be related to free stream turbulence.

Governing equations

Starting from the linearized, incompressible disturbance equations, the aim is to derive a set of parabolic equations. Therefore a scaling is introduced where the disturbances are assumed to be periodic in spanwise direction and weakly varying, nonoscillatory along the streamlines. Because these are curved a nonorthogonal coordinate system (ξ, η, ζ) according to the figure below has to be introduced to apply the appropriate scaling.



Using Jacobian transformation the disturbance equations are expressed in the nonorthogonal curvilinear coordinate system. The disturbances are assumed to be of the form

$$\mathbf{q}'(\xi,\eta,\zeta) = \hat{\mathbf{q}}(\xi,\zeta)\exp(i\beta\eta) \quad \text{where} \quad \mathbf{q} = (u,v,w,p). \qquad (1)$$

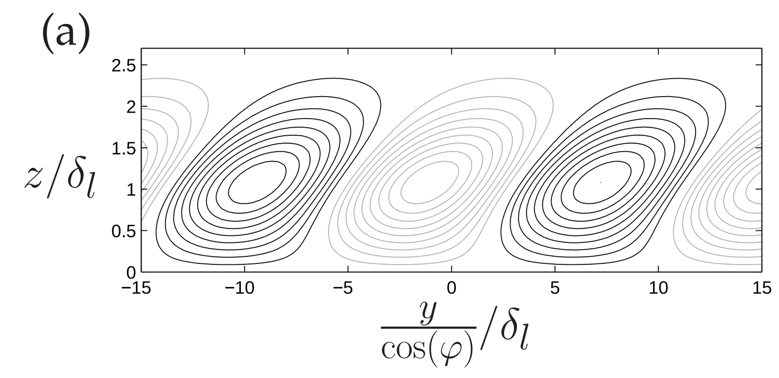
Introducing (1) into the transformed disturbance equations, applying the above described scaling and neglecting terms of order higher than $\mathcal{O}(Re^{-1})$ leads to a set of parabolic equations which allows for the study of algebraic growth. Now that all assumptions and scalings were applied the equations can be transformed back to cartesian coordinates which gives

$$\mathbf{A}\hat{\mathbf{q}} + \mathbf{B}\frac{\partial \hat{\mathbf{q}}}{\partial z} + \mathbf{C}\frac{\partial^2 \hat{\mathbf{q}}}{\partial z^2} + \mathbf{D}\frac{\partial \hat{\mathbf{q}}}{\partial x} = 0. \qquad (2)$$

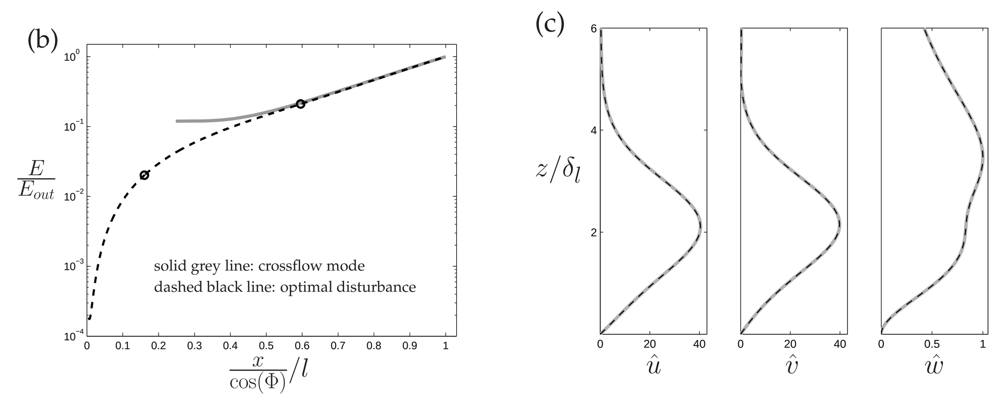
where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are linear operators. Adjoint based optimisation is carried out to compute the optimal disturbance for a given base flow and a specific spanwise wavenumber.

Results

Computations carried out by Martin Byström show that the optimal disturbance takes the form of tilted vortices in the cross-flow plane. Figure (a) shows contours of streamwise velocity for a favorable pressure gradient.



In figures (b) and (c) the energy growth and amplitude functions of a cross-flow mode are compared to the energy growth and the downstream response of the optimal disturbance. This reveals that the optimal disturbance evolves into a crossflow mode.



Acknowledgments

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