Feedback control & model reduction

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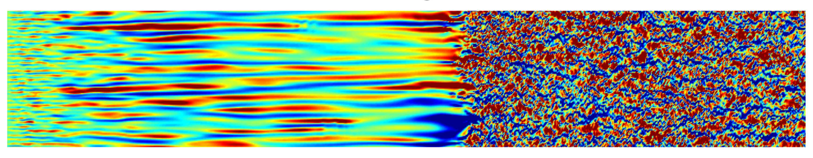
Introduction

The skin-friction drag associated with turbulent boundary layers constitutes a large fraction of the total drag on commercial vehicles such as aircraft and cargo ships. Therefore the delay of transition to turbulence implies a substantial reduction of operational and environmental costs. In flow control the union of modern control theory and fluid mechanics knowledge can provide a systematic way of achieving this objective. We consider feedback controllers in the Linear Quadratic Gaussian (LQG) framework. These controllers run online, measuring the state and feeding back control signals to the actuators. The optimal control feedback law is obtained as the solution of optimisation problems. Both tasks challenge the available computer hardware, since the numerical approximation of the flow equations yields a very high dimensional system. Hence there is need to construct reduced models that can accurately describe the connection between the sensors and actuators. A possible way to obtain a more computationally tractable problem is the classical assumption of horizontal spatial invariance of the boundary layer. This substantially reduces the complexity involved in solving the optimisation problems. However, the size of the online controller becomes as large as the original problem. We have applied this methodology to the so-called bypass transition scenario. Model reduction provides a way to reduce both the cost of solving the optimisation problems and the online actuation. In this setting, the full flow equations are projected onto a small subspace spanned by a set of suitable vectors. We present results using two different types of bases, namely the so called Balanced Truncation modes and the Global eigenmodes of the Navier–Stokes equations.

Optimal control of bypass transition

Bypass transition

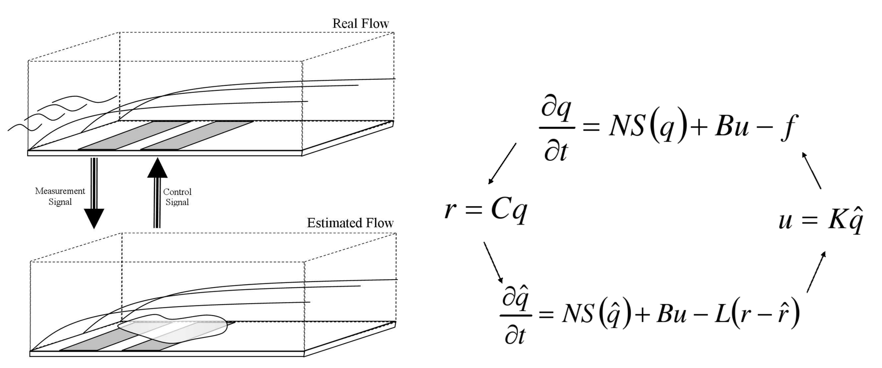
Laminar-turbulent transition in a boundary layer subject to high levels of free-stream turbulence is considered. Such a scenario is usually referred to as bypass transition since the transition occurs faster than the exponential growth of the Tollmien-Schlichting waves. In boundary layers strructures with the largest possible growth consist of streamwise counter-rotating vortices.



These vortices lift low-momentum fluid from the wall and push high-momentum fluid from the outer parts towards the plate, thus creating elongated regions of alternating accelerated and decelerated fluid, called streamwise streaks. After the primary energy growth, the flow is in a more complicated laminar state where strong nonlinear interactions can come into play, leading to transition to turbulence.

Optimal Control

A linear model-based feedback control approach is formulated where the Orr-Sommerfeld and Squire equations model the flow dynamics. An objective function that targets the perturbation kinetic energy is chosen. The requirement implicit in this formulation is the need of complete state information. However, the control problem can be combined with a state estimator to relax this requirement.

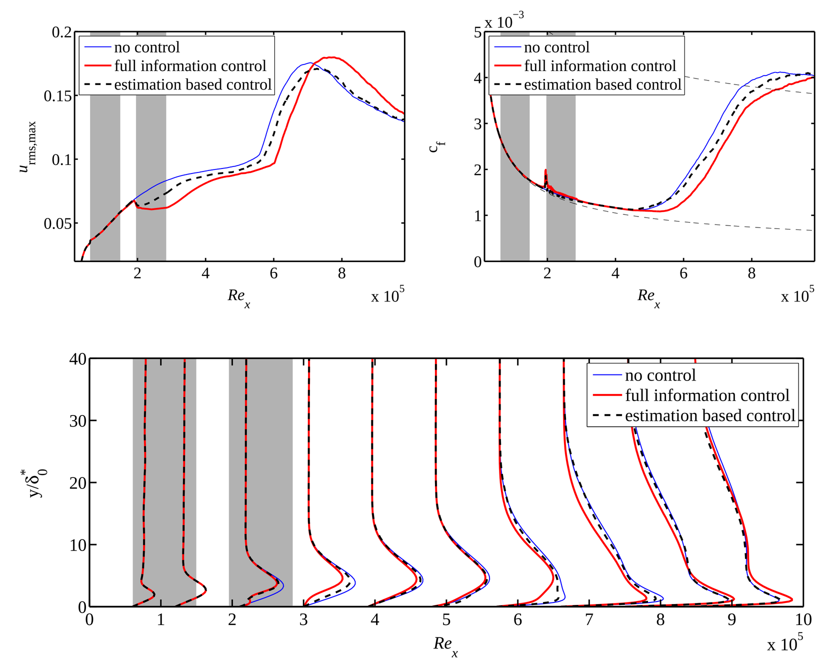


The feedback control loop sketched above is described here. Measurements taken from the real flow are sent to the estimator where they are used to compute the signal needed to reproduce the real flow. The actuation control signal is then computed from the estimated flow and is applied to both the estimated and the real flow.

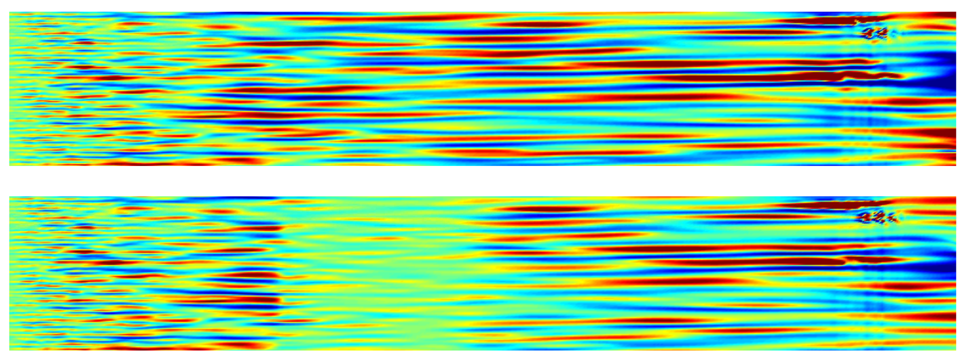
Optimal control of bypass transition

Results

Results on linear feedback control of a boundary layer subject to free-stream turbulence are reported next. In particular *rms* values of the perturbation velocity and the skin-friction coefficient are shown.



The controller is able to reduce the energy of the streaks, which through their secondary instabilities are responsible for the transition process. The delay achieved is of order of the streamwise extent of the control region. This relatively fast recovery of the streamwise streaks downstream of the control region was also observed in a recent experimental study.

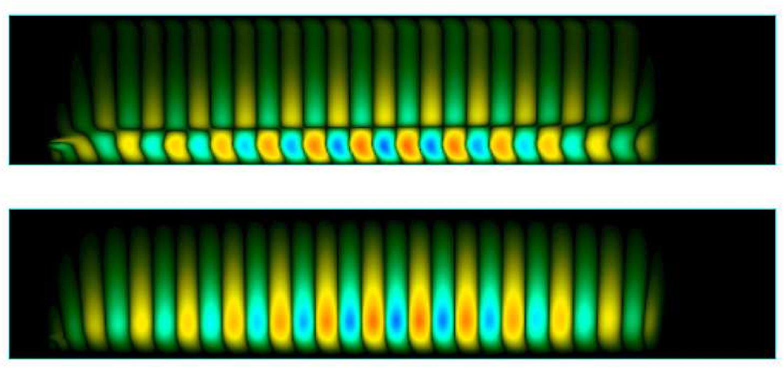


The above figures show a part of a plane parallel to the wall of the instantaneous streamwise velocity with & without control applied.

Balanced truncation

Balanced truncation

Balanced truncation is a systematic method of model reduction which preserves only the flow components that are influenced by the forcing and that can be observed by the sensors. The technique for approximating balanced truncation for very large systems using the method of snapshots is considered in order to obtain a low-dimensional model of the 2D flat-plate boundary layer. This projection basis (see the figure below) is computed from a composite snapshot set consisting of the impulse response of both the direct and adjoint linearised Blasius flow. The reduced model is then obtained by projecting the linearised Navier-Stokes on these modes.

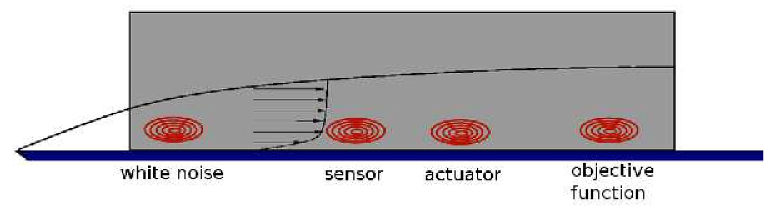


Feedback control of Blasius

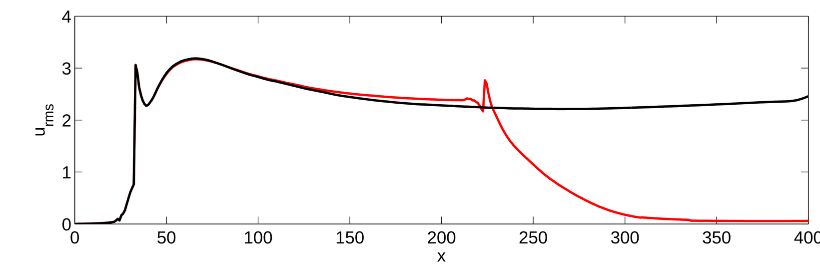
Once a reduced-order model is devised using balanced truncation we can design an LQG controller for this reduced model and couple it to the flow system. Consider the convectively unstable Blasius flow that is driven by white noise upstream of branch I. From the noisy measurements between branch I and II an estimated state is obtained by an optimal Kalman filter. Based on this estimate, the control signal is introduced downstream of the sensor.

Feedback control of blasius

A schematic representation of the Blasius boundary layer flow is shown below. The locations and shapes of the inputs and outputs are illustrated by the included contours.



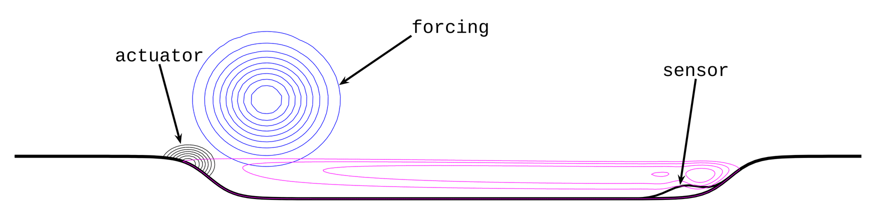
Below the rms values of the streamwise velocity component without control (black) and with control (red) as a function of the streamwise coordinate are depicted.



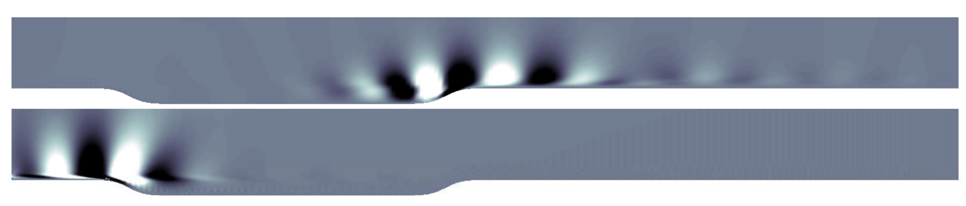
The rms value of the uncontrolled state grows exponentially as it enters the unstable domain at branch I; this growth prevails until the fringe region of the computation domain. The rms-value of the controlled state, however, decays downstream of the actuation.

Cavity control

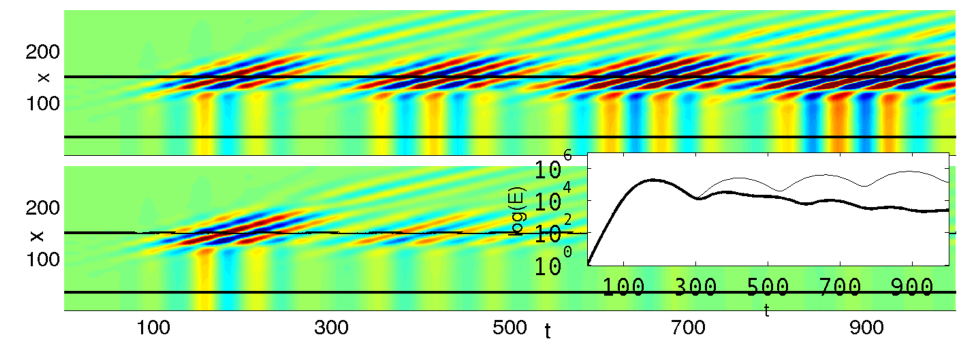
Consider a shallow cavity mounted on a flat plate with flow coming from left to right. The presence of the cavity creates a shear layer and a separation bubble as seen from the steady state streamlines. At the upstream lip of the cavity a volume forcing actuator is mounted and at the downstream cavity lip a sensor is placed.



A reduced order model for control is obtained by projecting the linearised Navier–Stokes equations on a few of the least stable left and right global eigenfunctions of the system.



A spatio-temporal diagram of the pressure field reveals that the uncontrolled flow is globally unstable. An initial wavepacket is propagating across the shear layer creating a global pressure pulse that through receptivity at the upstream cavity edge regenerates the disturbances. When the control is applied one still observes the vertical rays of the global pressure changes but the wavepacket regeneration is reduced, leading to a decrease in the levels of fluctuations at each cycle, i.e. flow stabilisation.



Conclusions

Optimal feedback control in the LQG framework has been applied to different flow cases. The control of bypass transition explores the limit of linear optimal control applied to a flow with strong non-linear interactions. A low dimensional model constructed from Balanced truncation modes is indeed capable of capturing the input output behaviour of the full flow equations. For the cavity flow, where self-sustained oscillations occur, the global eigenmodes of the flow system provide an attractive basis for model reduction.