



# Geophysical fluid dynamics



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## Simulations of the 2D Navier-Stokes equation

*This study focuses on the development of an efficient two-dimensional Navier-Stokes solver, including differential rotation. Due to the thin and predominantly stably stratified atmosphere, atmospheric motions are essentially horizontal. Consequently, a first approximation is to study two-dimensional turbulent flows affected by differential rotation. Turbulent motions occurring at planetary scales are referred to as geostrophic turbulence, a subject which was theoretically investigated by Charney (1971). This study will be extended into a pseudo-three-dimensional code aimed to numerically investigate Charney's theory.*

### The model and method

The code is based on the two-dimensional Navier-Stokes equation, with the addition of a dynamically important  $\beta$ -plane approximation, which accounts for the effect of differential rotation due to Earth's sphericity, and the continuity equation. In spectral space, these can be formulated as

$$\frac{\partial \hat{\omega}(\vec{k})}{\partial t} = -ik_x u \hat{\omega} - ik_y v \hat{\omega} - \nu k^2 \hat{\omega}(\vec{k}) - i\beta k_x \hat{v}, \quad (1)$$

$$i\vec{k} \cdot \hat{\vec{U}}(\vec{k}) = 0, \quad (2)$$

The numerical method is a pseudo-spectral approach with periodic boundary conditions on a square grid. Hence, the time-stepping is performed in spectral space, whereas the nonlinear term is calculated in real space. To enable this procedure, a Fast Fourier Transform (FFT) is utilized with the use of a freely available package; FFTW. The Runge-Kutta 4<sup>th</sup>-order scheme with low-level storage has proven to be efficient and accurate, and is therefore adopted in the code. Viscosity is accounted for by exact integration via an integrating factor. A cost-effective 9/8-dealiasing by padding has been introduced, rather than the full dealiasing of 3/2. Results show that quantities such as enstrophy and energy are accurately conserved in the inviscid case. Adaptive time-stepping is realized in the code from the CFL-condition.

### Parallelisation

The code has been developed in a single-processor environment, which makes extensive simulations unfeasible. However, a parallelised version of the code is under development allowing for larger problem sizes, since the calculations can be distributed to many processors. For this purpose, the Message-Passing Interface (MPI) will be a critical component of the code. MPI is a communication protocol used for parallel programs on a distributed memory system. Tests have been performed on the PDC Lenngren Intel Xeon system and an analytical performance model has shown that there is the potential for substantial speed-ups, especially in the computationally expensive FFTs, which demand  $N \log N$  operations.

### Preliminary results

A number of simulations have verified the conservation of the inviscid invariants. Figure 1 shows a snapshot of a freely evolving normalized random vorticity field without forcing. However, viscosity and a beta effect, affecting vortices of sufficiently large sizes, are present. Therefore, the filament-like structures are seen to be zonally elongated. Studies of the transfer of enstrophy indicates a transfer from low wave numbers toward high wave numbers (smaller scales). In two-dimensional turbulence, there is a tendency for energy to move to larger scales while enstrophy moves to smaller scales due to wave triad interactions (e.g., Pedlosky, 1987). This is in contrast to Richardson's view of the three-dimensional energy cascade, where the energy would move from large scales toward small scales due to vortex stretching (Pope, 2000). This behavior can be theoretically understood by reflecting the constraints of energy and enstrophy conservation (see e.g., Vallis, 2006).

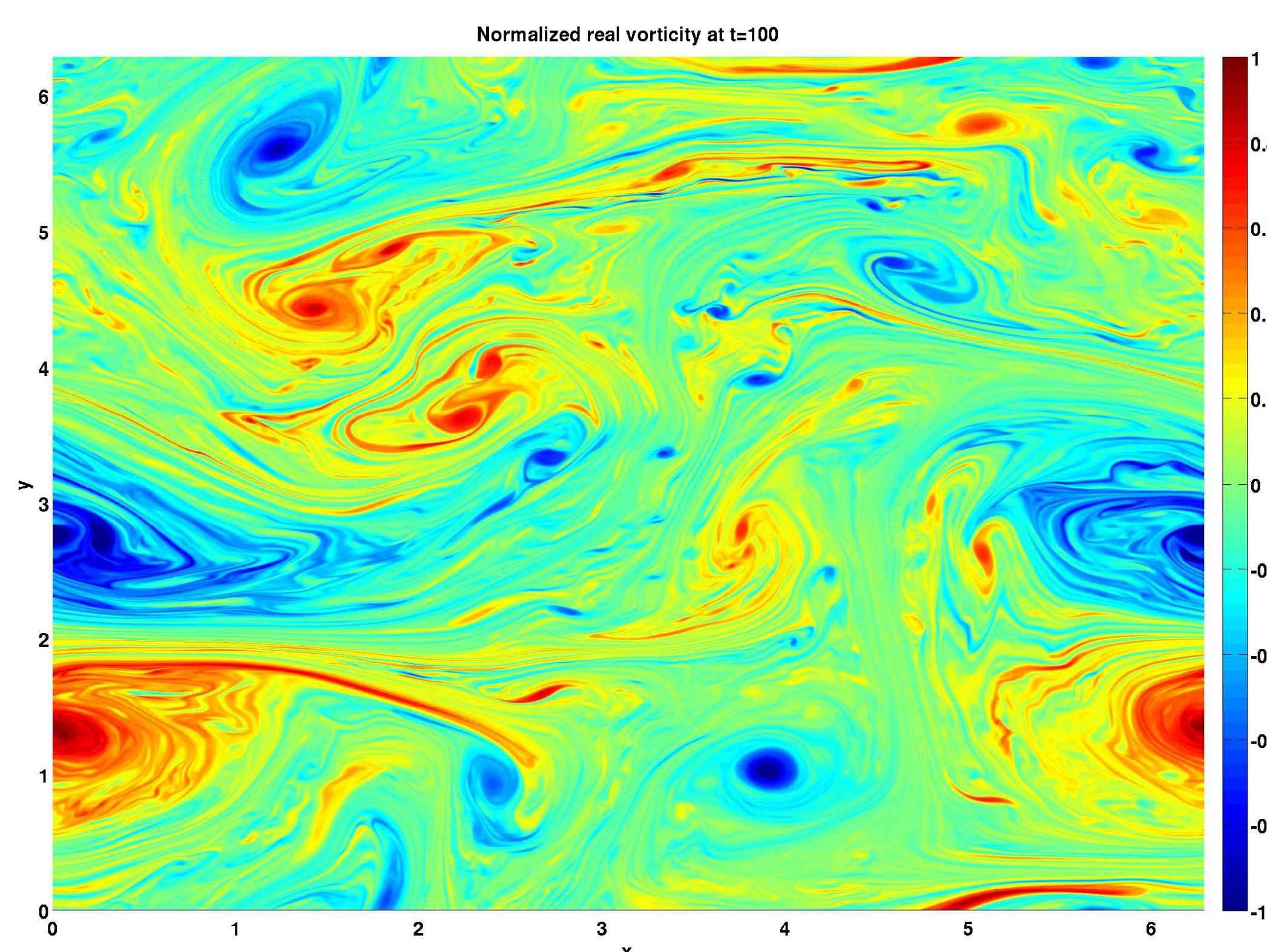


Figure 1: A snapshot of a freely evolving real vorticity field in presence of a beta effect.

### To be continued

The model will be further improved by adding a vertical dimension in order to investigate the vertical modes and to compare with Charney's expectations. Furthermore, the code will be optimized to suit the technical infrastructure of the upcoming climate super computer.

## Climate in a 100,000 year time perspective

*Planning for long-term storage of radioactive waste products from Swedish nuclear power plants requires careful consideration of many aspects including variations in the climate. In a 100,000-year time frame, climate conditions in Sweden may change between cold conditions with or without an ice cap and warm conditions like today or possibly even warmer in a future climate influenced by an amplified greenhouse effect.*

*This project intends to identify the climatic extremes within which climate may vary over a 100,000 year time span. Based on forcing conditions which have yielded extreme conditions during the last glacial-interglacial cycle climate models will be applied to reproduce climate variables for these climatic extremes.*

### Global and regional climate modelling

The climate modelling involves a global model (CCSM3) for producing boundary conditions that will be used by a regional climate model (RCA3). The regional model in turn will produce detailed information on climate variables like the near-surface air temperature and precipitation with an emphasis on Scandinavia. The regional climate model data will be compared to existing paleo-data for Northern Europe.

### Three cases - forcing and boundary data

The specific time periods involved are: a Stage 3 stadial representing a cold period with a relatively small ice sheet over Scandinavia (Permafrost) and the Last Glacial Maximum with an extensive ice sheet over northern Europe (LGM). As an example of a warm climate a future time period with a strong forcing from additional greenhouse gases in the atmosphere and with a complete loss of the Greenland ice sheet will be investigated (Warm). Forcing conditions are taken either from proxy data or model simulations. In the LGM case we have restricted ourselves to follow the set up of the global model inter-comparison project PMIP2. Forcing conditions in the GCM and in the RCM will be taken so that they are as similar/consistent as possible. The extent and thickness of ice sheets is taken from simulations with the ICE5G model (LGM) and from earlier SKB project combined with a simulation with CLIMBER-2 and ICE5G (Permafrost). The major differences in forcing and boundary conditions as compared to present day conditions are shown in the table below.

	Permafrost	LGM	Warm	Present day
Orbital year	44 kyr BP	21 kyr BP	1990	1990
CO <sub>2</sub> (ppm(v))	200	185	750	380
CH <sub>4</sub> (ppb(v))	420	350	1714	1714
N <sub>2</sub> O (ppb(v))	225	200	311	311
Aerosols	PI<PD	PI<PD	PI<PD	PD
Sea level (m)	-70	-120	+7	0

PI=pre-industrial, PD=present day, kyr BP= 1000 years before present.

### Last glacial maximum climate variability

Although the initial conditions of the LGM global simulation were taken from a 400 year model simulation there is still a drift in the climate system towards colder temperatures during the next 350 years. This drift is associated with a drift in the sea ice extent primarily in the North Atlantic. The Northern Hemisphere climate undergoes rapid (10-20 years) variations during the new quasi-equilibrium which are also associated with variations in sea ice. The simulation will be continued for another 100-200 years to investigate the stability of the new quasi-equilibrium.

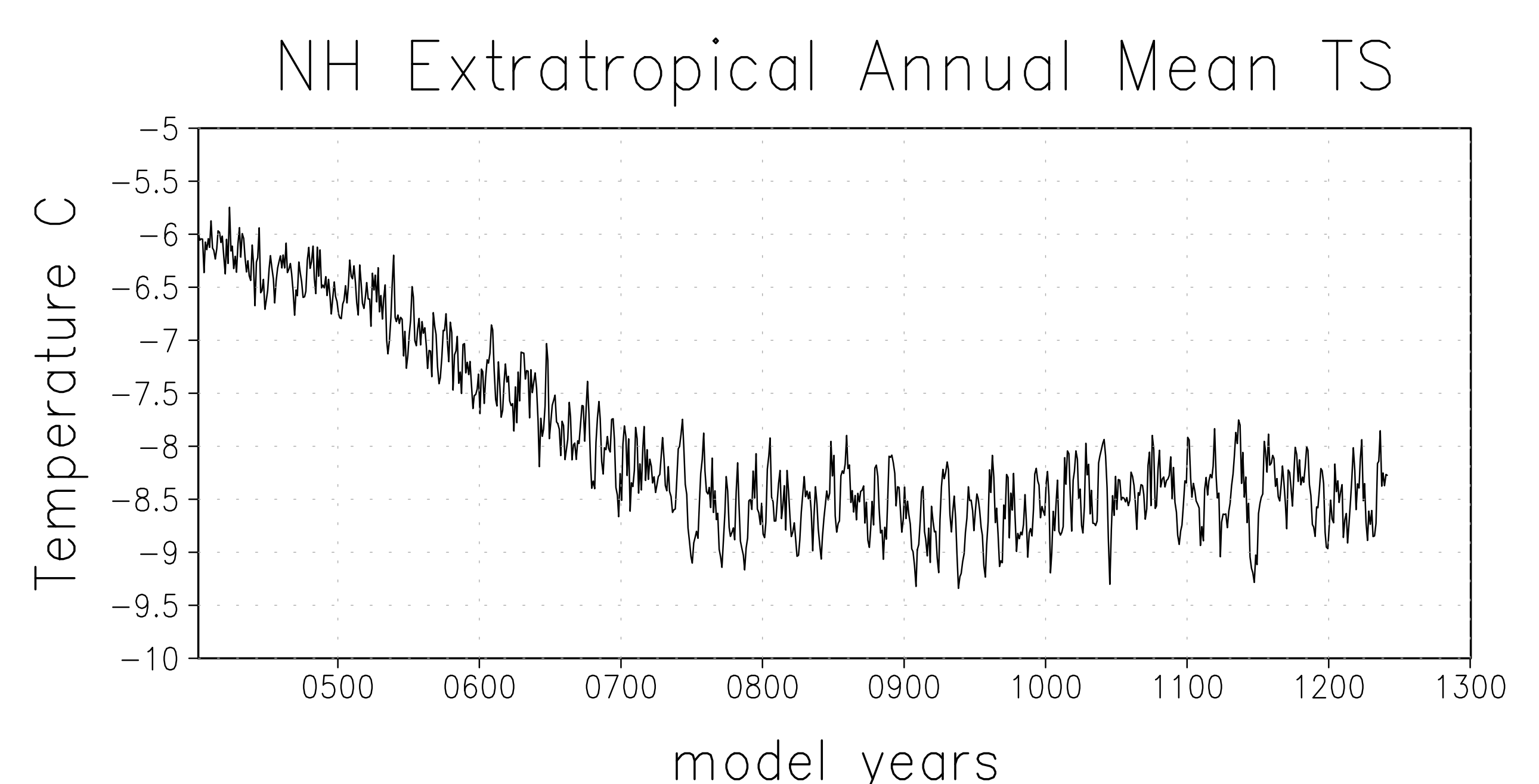


Figure 1: Northern Hemisphere Extratropical (30° – 90°) surface temperature (°C) in the Last Glacial Maximum simulation.

### To be continued

This is an ongoing study. The global simulations will be analysed and compared to other simulations and paleo-data during 2008.