Passive scalars in turbulent boundary layers

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Outline

1. Introduction
2. DNS on passive scalar transport
   1) Effects of the molecular Prandtl number
   2) Effects of the Reynolds number
   3) Inner and outer interactions
3. DNS on small-scale scalar mixing
   1) Effects of the Reynolds number
4. DNS databases in a channel flow with passive scalar transport

Introduction (1/8)

- Passive scalars

(e.g.) temperature, pollutant, chemical species

encountered in many engineering applications involving heat and mass transfer and atmospheric flows

For small scalar differences and the low concentration, the scalar field does not affect the flow dynamics.

- Role of turbulence

transporting and mixing the scalar effectively

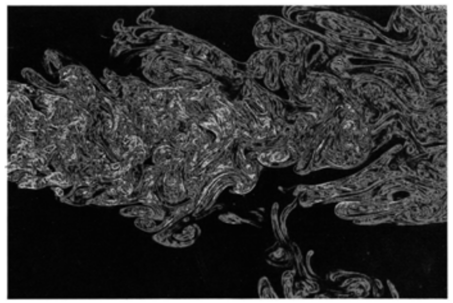
1) large scales
   playing a major part in transporting the scalar
2) small scales
   indispensable in implementing the mixing at the molecular level

Introduction (2/8)

- Visualizations

1. Experiment in a turbulent jet

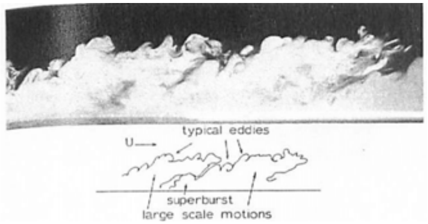
Sreenivasan (1991)



Introduction (3/8)

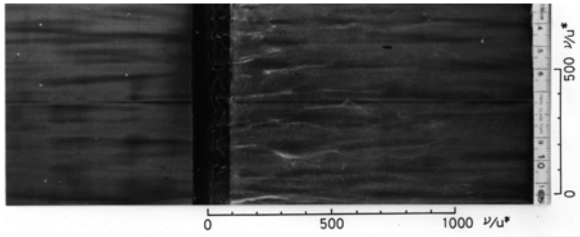
2. Experiment in a turbulent boundary layer

Falco (1977)



Iritani et al. (1992)

Thermal streaks   Momentum streaks



Introduction (4/8)

- Energy (conservation) equation

$$\frac{\partial \Theta}{\partial t} + U_j \frac{\partial \Theta}{\partial x_j} = \frac{1}{Pe} \frac{\partial^2 \Theta}{\partial x_j^2} \qquad (Pe = Re \cdot Pr \;\; or \;\; Pe = Re \cdot Sc)$$

U_j : Instantaneous velocity component
Θ : Instantaneous scalar
x_i : streamwise (x_1 or x) direction,
wall-normal (x_2 or y) direction,
spanwise (x_3 or z) direction
Pe : Peclet number
Re : Reynolds number
Pr (=ν/a) : Molecular Prandtl number
Sc (=Pr) : Schmidt number
ν : Kinematic viscosity
a : Thermal diffusivity

1) $Pr \ll 1$ liquid metals
2) $Pr \geq 1$ gas and liquids

(mercury: $Pr = 0.025$
air: $Pr = 0.71$
water: $Pr = 5 - 7$)

Introduction (5/8)

- Reynolds analogy
(viz. similarity between the momentum and scalar fields)
  1) $Pr \approx 1$: the analogy holds reasonably
  (e.g. spectral analogy by Antonia et al. 2009)
  2) $Pr \gg 1$ or $Pr \ll 1$: the analogy breaks down

- Reynolds decomposition

$$\underbrace{\Theta}_{Instantaneous\ scalar} = \underbrace{\overline{\Theta}}_{Mean\ scalar} + \underbrace{\theta}_{Scalar\ fluctuation}$$

[$\overline{}$: averaging in space and time]

- Averaged energy (conservation) equation

$$\frac{\partial \overline{\Theta}}{\partial t} + \overline{U}_j \frac{\partial \overline{\Theta}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{1}{Pe} \frac{\partial \overline{\Theta}}{\partial x_j} - \overline{u_j \theta} \right)$$

$-\overline{u_j \theta}$ (turbulent scalar flux) needs to be modeled.

Introduction (6/8)

- Gradient-diffusion model

$$-\overline{u_j \theta} = a_t \frac{\partial \overline{\Theta}}{\partial x_j}$$

1) zero-equation model

$$a_t = \nu_t / Pr_t \qquad \text{(e.g. } Pr_t \approx 0.9\text{)}$$

2) two-equation model

$$a_t = c_\lambda f_\lambda \frac{k^2}{\varepsilon} R^p \left(= c_\lambda f_\lambda k \tau_m\right) \qquad \left[R = (k_\theta / \varepsilon_\theta)/(k/\varepsilon) \right]$$

1) p=1/2 $\tau_m = \sqrt{(k/\varepsilon)(k_\theta/\varepsilon_\theta)}$ (Nagano & Kim 1988)
2) p=2 $\tau_m = k_\theta / \varepsilon_\theta$ (Yoshizawa 1988)
3) p=0 $\tau_m = k/\varepsilon$ (Pr_t=constant)

- Batchelor scale (η_B)

$$\eta_B = \eta Pr^{-1/2} \qquad \left[\eta \left(\equiv (\nu^3/\varepsilon)^{1/4} \right) : \text{Kolmogorov scale} \right]$$

η_B is smaller than η when $Pr > 1$.

Introduction (7/8)

- Experiments of wall turbulence

1) Heat transfer experiments in grid turbulence
   - Corrsin (1952)
   - Warhaft & Lumley (1978)
   - Tavoularis and Corrsin (1981)

2) Heat transfer experiments in a turbulent boundary layer
   - Perry & Hoffmann (1976)
   - Chen & Blackwelder (1978)
   - Subramaniann & Antonia (1981)
   - Antonia et al. (1988)
   - Iritani et al. (1992)

- Difficulties in experimental data acquisition
  1) near-wall region
  2) very high and low *Pr* fluids

DNS on passive scalar transport

Introduction (8/8)

- DNSs (Direct Numerical Simulations) of wall turbulence
  1) The pioneering DNSs
     Kim, Moin & Moser (1987) in a channel flow and Spalart (1988) in a turbulent boundary layer
  2) The first DNS with passive scalar transport
     Kim & Moin (1989) in a turbulent channel flow

     h^+ $(=u_\tau h/\nu)$ =180 and Pr=0.1, 0.71, 2 $\left(h^+ \equiv Re_\tau \right)$

     (The superscript + denotes the normalization by wall variables.)

- Advantage of DNS
  providing accurate information at any time and locations

- Disadvantage of DNS
  1) limitation of the Reynolds and Prandtl numbers
  2) difficulties in direct comparisons with experimental data

Effects of the molecular Prandtl number (1/12)

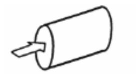
- DNSs of a turbulent channel flow

- wider range of *Pr*
- several thermal boundary conditions

1) Kim and Moin (1989)
   h^+ =180 and Pr=0.1, 0.71, 2 (constant source heating)
2) Kasagi et al. (1992); Kasagi and Ohtsubo (1993)
   h^+ =150 and Pr=0.025, 0.71 (constant heat flux)
3) Kawamura et al. (1998)
   h^+ =180 and Pr=0.025 – 5 (constant heat flux)
4) Na and Hanratty (2000)
   h^+ =150 and Pr=1 – 10 (constant temp. difference)
5) Schwertfirm and Manhart (2007)
   h^+ =180 and Pr=3 – 49 (constant temp. difference)

Effects of the molecular Prandtl number (2/12)

- DNSs of a turbulent pipe flow



• wider range of *Pr*
• one thermal boundary condition

1) Redjem-Saad, Ould-Rouiss & Lauriat (2007)
   R^+=180 and Pr=0.026 – 1.0 (constant heat flux)

- DNSs of a turbulent boundary layer

• several values of *Pr*
• two thermal boundary conditions

1) Bell & Ferziger (1993)
   $\delta_{99}^+ \approx 140 - 280$ and Pr=0.1, 0.71 , 2 (isothermal)
2) Kong, Choi & Lee (2000)
   $\delta_{99}^+ \approx 140$ and Pr=0.71 (isothermal and isoflux)
3) Tohdoh, Iwamoto & Kawamura (2007)
   $\delta_{99}^+ \approx 140$ and Pr=0.71 , 2 (isothermal)

Effects of the molecular Prandtl number (3/12)

- Here, we focus on two DNSs in a turbulent channel flow by Kim and Moin (1989) and Kawamura et al. (1998)
- Thermal boundary conditions for the latter two DNSs

$$\frac{\partial \Theta^+}{\partial t^\#} + U_j^+ \frac{\partial \Theta^+}{\partial x_j^\#} = \frac{1}{h^+ \cdot Pr} \frac{\partial^2 \Theta^+}{\partial x_j^{\#2}} + Q^+ \quad (\Theta = 0 \text{ at } y=0 \text{ and } 2h)$$

(1) Constant source heating (Kim and Moin 1989)
(Scalar is created internally and removed from both walls)

$$Q^+=2$$

(2) Constant heat flux (Kawamura et al. 1998)
(Both walls are heated with constant heat flux, where the bulk mean temperature increases linearly with x)

$$Q^+ = U_1^+ \frac{2}{\int_0^2 \bar{U}_1 dy^\#}$$

[The superscript # denotes the normalization by outer variable.]

Effects of the molecular Prandtl number (4/12)

– Turbulent channel flow –

Mean temperature

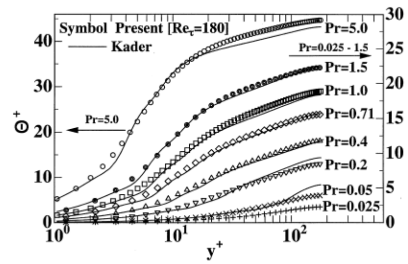


Fig. 2. Mean temperature profile with an emphasis on the logarithmic region.

Kawamura et al. (1998)

Rms of the temperature fluctuation

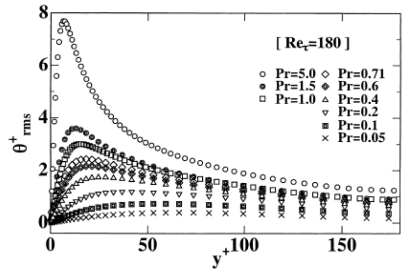


Fig. 15. Temperature variance for various Prandtl numbers.

Kawamura et al. (1998)

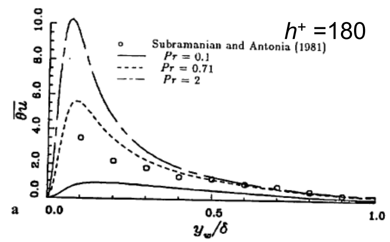
Kader denotes the empirical formula of Kader (1981).

Effects of the molecular Prandtl number (5/12)

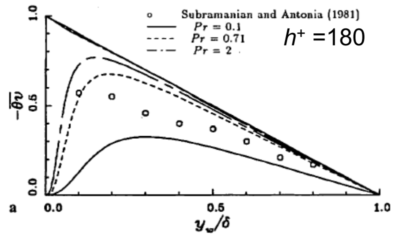
– Turbulent channel flow –

Turbulent heat flux

Kim & Moin (1989)



Streamwise



Wall-normal

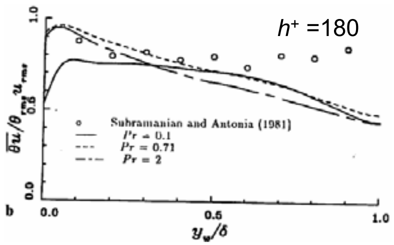
Unlike $\overline{v\theta}$, no limitation for the magnitude of $\overline{u\theta}$

Effects of the molecular Prandtl number (6/12)

ー Turbulent channel flow ー

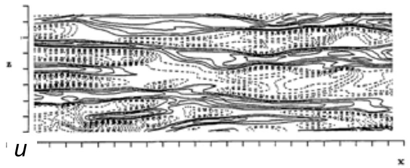
Correlation coefficients for

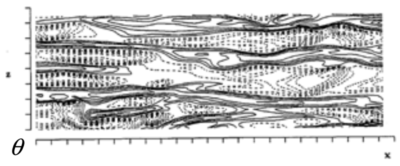
Kim & Moin (1989)



Excellent similarity between u and θ near the wall when $Pr \approx 1$

Instantaneous fields (h^+=180 and Pr=0.71 at y^+=5)





Effects of the molecular Prandtl number (7/12)

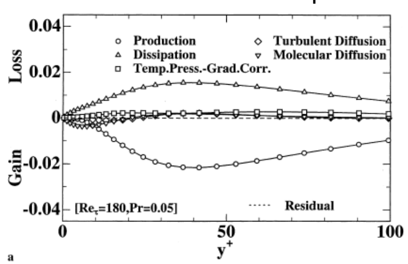
ー Turbulent channel flow ー

Budget of $\overline{v^+\theta^+}$

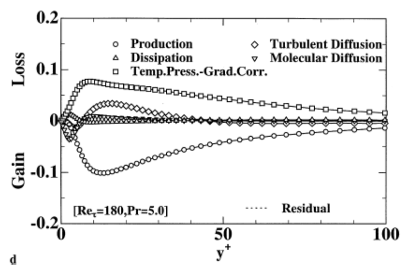
$$0 = \underbrace{-\overline{\theta^+ u_j^+}\frac{\partial \bar{U}^+}{\partial x_j^+} - \overline{u_2^+ u_j^+}\frac{\partial \bar{\Theta}^+}{\partial x_j^+} + \overline{u_1^+ u_2^+}\frac{\partial \langle \bar{T}_m \rangle^+}{\partial x_1^+}}_{Production} \underbrace{- \frac{\partial}{\partial x_j^+}\left(\frac{1}{2}\overline{\theta^+ u_2^+ u_j^+}\right)}_{Turbulent\ Diff.} \underbrace{- \overline{\theta^+ \frac{\partial p^+}{\partial x_2^+}}}_{TPG}$$

$$+ \underbrace{\frac{\partial}{\partial x_j^+}\left(\overline{\theta^+ \frac{\partial u_2^+}{\partial x_j^+}} + \frac{1}{Pr}\overline{u_2^+ \frac{\partial \theta^+}{\partial x_j^+}}\right)}_{Molecular\ Diff.} \underbrace{- \left(1 + \frac{1}{Pr}\right)\overline{\left(\frac{\partial u_2^+}{\partial x_j^+}\right)\left(\frac{\partial \theta^+}{\partial x_j^+}\right)}}_{Dissipation}$$

Dominant sink term: Dissipation



Dominant sink term: TPG



Kawamura et al. (1998)

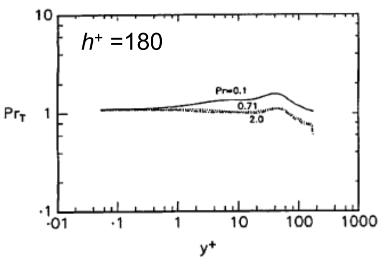
Effects of the molecular Prandtl number (8/12)

ー Turbulent channel flow ー

Turbulent Prandtl number (Pr_t)

$$Pr_t = \frac{\nu_t}{a_t} = \frac{\overline{uv}}{\overline{v\theta}}\frac{d\bar{\Theta}/dy}{d\bar{U}/dy}$$

(ν_t : turbulent eddy viscosity
a_t : turbulent eddy diffusivity)



Antonia & Kim (1991)

Pr_t = 1.1 (wall value) when Pr>0.1
(Kim & Moin 1989; Antonia & Kim 1991)

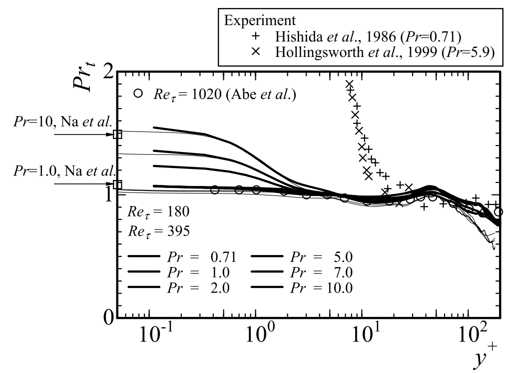
The similar trend was reported in DNSs of a pipe and TBL.

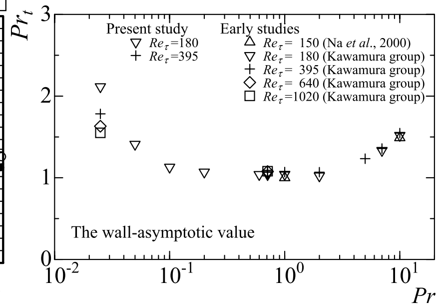
(Reynolds (1975)
For Pr >> 1 $\quad Pr_t \rightarrow$ finite constant
For Pr << 1 $\quad Pr_t \propto Pr^{-1}$ or $Pr_t \propto Pr^{-n}$ $(n<1)$)

Effects of the molecular Prandtl number (9/12)

ー Turbulent channel flow ー

Turbulent Prandtl number (Pr_t)





$0.2 < Pr < 2.0 \qquad Pr_t = 1.1$ (constant)

Kozuka, Seki & Kawamura (2009)

Effects of the molecular Prandtl number (10/12)

－ Turbulent channel flow －

Time scale ratio

$$R=\frac{\tau_\theta}{\tau_u}=\frac{k_\theta\overline{\varepsilon}}{\overline{\varepsilon_\theta}k}$$

At the wall, *R=Pr*

$$\begin{pmatrix} k_\theta=\frac{1}{2}\overline{b_\theta^2}y^2+\cdots \\ \overline{\varepsilon_\theta}=\kappa\overline{b_\theta^2}+\cdots \\ k=\frac{1}{2}\left(\overline{b_1^2}+\overline{b_3^2}\right)y^2+\cdots \\ \overline{\varepsilon}=\nu\left(\overline{b_1^2}+\overline{b_3^2}\right)+\cdots \end{pmatrix}$$

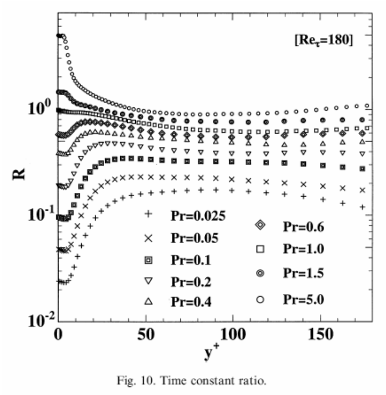


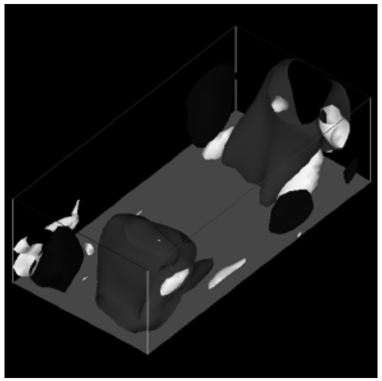
Fig. 10. Time constant ratio.

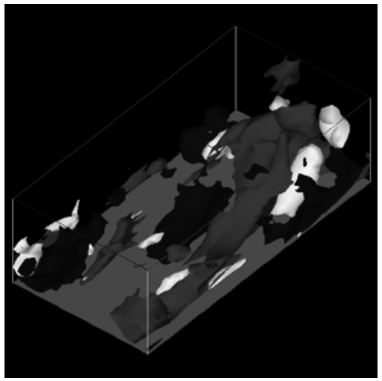
Kawamura et al. (1998)

Effects of the molecular Prandtl number (11/12)

－ Turbulent channel flow －

High and low temperature streaks





Pr = 0.025 $\quad$ Re$_\tau$ =180 $\quad$ *Pr* = 0.71

(red: $T'^+ > 0.23$, blue: $T'^+ < -0.23$, $p'^+ > -3.0$)

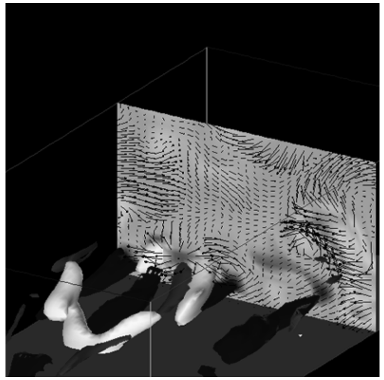
(red: $T'^+ > 3.0$, blue: $T'^+ < -3.0$, $p'^+ > -3.0$)

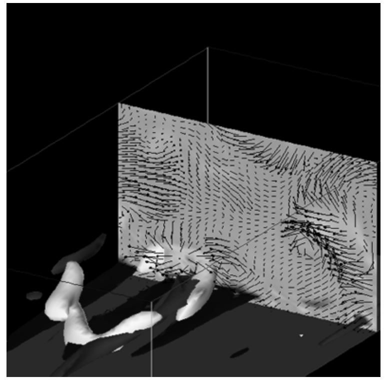
Kawamura et al. (1999)

Effects of the molecular Prandtl number (12/12)

－ Turbulent channel flow －

High and low temperature streaks





Pr = 1.0 $\quad$ Re$_\tau$ =180 $\quad$ *Pr* = 5.0

(red: $T'^+ > 4.0$, blue: $T'^+ < -4.0$, $p'^+ > -3.0$)

(red: $T'^+ > 10.0$, blue: $T'^+ < -10.0$, $p'^+ > -3.0$)

Kawamura et al. (1998)

Effects of the Reynolds number (1/11)

- DNSs of a turbulent channel flow
  1) Wikström & Johansson (1998)
     h^+ =265 and *Pr*=0.71
  2) Kawamura et al. (1999)
     h^+ =180, 395 and *Pr*=0.025, 0.2, 0.71
  3) Abe, Kawamura & Matsuo (2004)
     h^+ =180, 395, 640, 1020 and *Pr*=0.025, 0.71
  4) Kozuka, Seki & Kawamura (2009)
     h^+ =180, 395 and *Pr*=0.71 – 10
- DNSs of a turbulent boundary layer
  1) Hattori, Houra & Nagano (2007)
     $\delta_{99}^+ \approx 380$ and *Pr*=0.71
  2) Li et al. (2009)
     δ_{99}^+ =70 – 315 and *Pr*=0.1, 0.71 , 2

Effects of the Reynolds number (2/11)
－ Turbulent channel flow －

Mean velocity　　　　Mean temperature

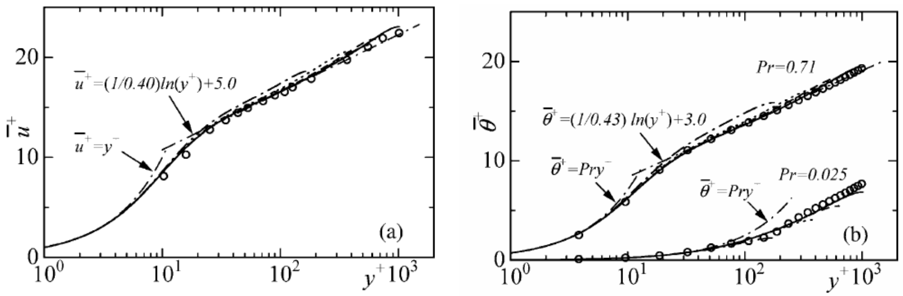
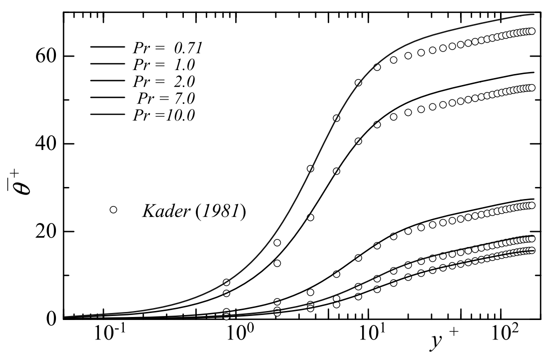


Fig. 1. Mean velocity and temperature distributions: —, $Re_\tau = 1020$; - - -, $Re_\tau = 640$; ···, $Re_\tau = 395$; –·–, $Re_\tau = 180$. (a) $\bar{u}^+$: –··–, Moser et al. (1999) for $Re_\tau = 590$; ○, Wei and Willmarth (1989) for $Re_\tau \approx 1017$; (b) $\bar{\theta}^+$: ○, Kader (1981) for $Re_\tau = 1020$.

Abe et al. (Int. J. Heat and Fluid Flow, 2004)

Effects of the Reynolds number (3/11)
－ Turbulent channel flow －

Mean temperature （ h^+=395 ）



The predictions of Kader (1981) seem to be poor for higher Pr fluids.

Kozuka, Seki & Kawamura (2009)

Effects of the Reynolds number (4/11)
－ Turbulent channel flow －

Rms of streamwise velocity and temperature fluctuations

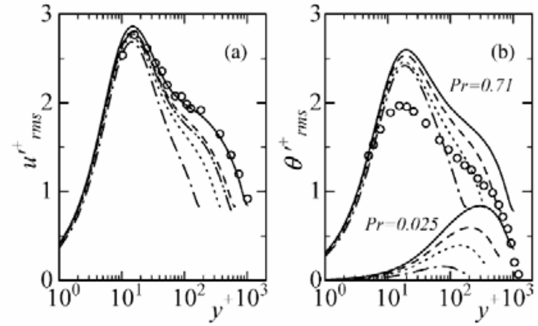


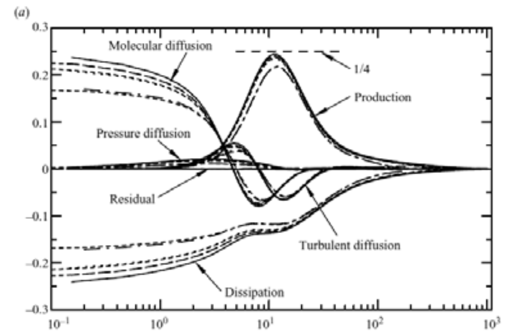
Fig. 2. Rms of streamwise velocity and temperature fluctuations: —, $Re_\tau = 1020$; - - -, $Re_\tau = 640$; ···, $Re_\tau = 395$; –·– $Re_\tau = 180$. (a) u'^+_{rms}: –··–, Moser et al. (1999) for $Re_\tau = 590$; ○, Wei and Willmarth (1989) for $Re_\tau \approx 1017$; (b) θ'^+_{rms}: ○, Subramanian and Antonia (1981) for $Re_\tau \approx 1055$.

Abe et al. (Int. J. Heat and Fluid Flow, 2004)

Effects of the Reynolds number (5/11)
－ Turbulent channel flow －

Budget of $k^+\left(\equiv \overline{u_i^{+2}}/2\right)$

$$0 = \underbrace{-\overline{u_i^+ u_k^+}\frac{\partial \bar{U}_i^+}{\partial x_k^+}}_{Production} \underbrace{-\frac{\partial}{\partial x_k^+}\left(\frac{1}{2}\overline{u_i^{+2} u_k^+}\right)}_{Turbulent\ Diff.} \underbrace{-\frac{\partial}{\partial x_i^+}\left(\overline{u_i^+ p^+}\right)}_{Pressure\ Diff.} \underbrace{+\frac{\partial^2}{\partial x_k^{+2}}\left(\frac{1}{2}\overline{u_i^{+2}}\right)}_{Molecular\ Diff.} - \underbrace{\overline{\left(\frac{\partial u_i^+}{\partial x_k^+}\right)^2}}_{Dissipation}$$



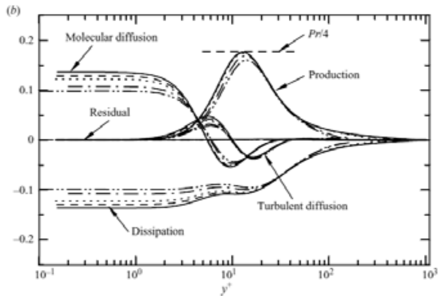
$P_{k,max}^+$=1/4 (theoretically)

Abe et al. (2009)

Effects of the Reynolds number (6/11)
─ Turbulent channel flow ─

Budget of $k_\theta^+\left(\equiv \overline{\theta^{+2}}/2\right)$

$$0 = \underbrace{-\overline{\theta^+ u_j^+}\frac{\partial\overline{\Theta}^+}{\partial x_j^+} + \overline{\theta^+ u_1^+}\frac{\partial\langle\overline{T}_m\rangle^+}{\partial x_1^+}}_{Production} \underbrace{-\frac{\partial}{\partial x_j^+}\left(\frac{1}{2}\overline{\theta^{+2}u_j^+}\right)}_{Turbulent\ Diff.} + \underbrace{\frac{1}{Pr}\frac{\partial^2}{\partial x_j^{+2}}\left(\frac{1}{2}\overline{\theta^{+2}}\right)}_{Molecular\ Diff.} \underbrace{-\frac{1}{Pr}\overline{\left(\frac{\partial\theta^+}{\partial x_j^+}\right)^2}}_{Dissipation}$$



$P_{k\theta,max}^+ = Pr/4$ (theoretically)

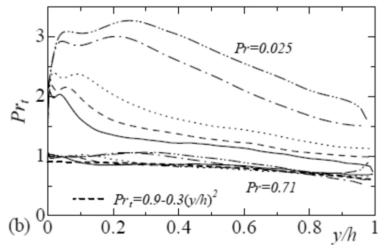
Abe et al. (2009)

Effects of the Reynolds number (7/11)
─ Turbulent channel flow ─

Turbulent Prandtl number (Pr_t)

$$Pr_t = \frac{\nu_t}{a_t} = \frac{\overline{uv}}{\overline{v\theta}}\frac{d\overline{\Theta}/dy}{d\overline{U}/dy}$$

ν_t : turbulent eddy viscosity
a_t : turbulent eddy diffusivity



Abe & Antonia (TSFP6, 2009)

1. *Pr*=0.71
   1) Pr_t = 1.1 (wall value)
   2) Pr_t = 0.9 ─ 1.1 (y^+=0 ─ 100)
   3) Pr_t = 0.9 - 0.3 $(y/h)^2$ $(y/h>0.2)$
      - i) analogous to the formula of Rotta (1964) in TBL
      - ii) applicable to water (*Pr*=5-7)
2. *Pr*=0.025
   1) Decrease with increasing *Re*
   2) Not scaled by either y^+ or y/h

Effects of the Reynolds number (8/11)
─ Turbulent channel flow ─

Time scale ratio

$$R = \frac{\tau_\theta}{\tau_u} = \frac{k_\theta\overline{\varepsilon}}{\overline{\varepsilon_\theta}k}$$

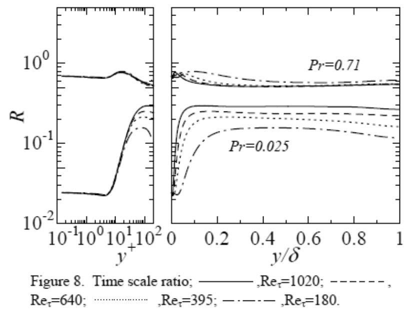


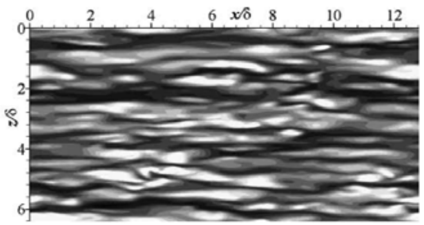
Figure 8. Time scale ratio; ──── ,Re_τ=1020; - - - - ,Re_τ=640; ········ ,Re_τ=395; ─·─·─ ,Re_τ=180.

Kawamura et al. (AFMC, 2004)

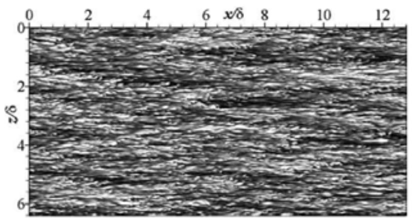
1. At the wall
   $R = Pr$
2. In the outer region
   1) *Pr*=0.71 $R \approx 0.5$
   (consistent with the finding of Béguier et al. 1978)
   2) *Pr*=0.025 Not scaled by either y^+ or y/h

Effects of the Reynolds number (9/11)
─ Turbulent channel flow ─

Instantaneous fields (*Pr*=0.71)
Surface heat-flux fluctuations (temperature fluctuations at y=0)



Re_τ =180



Re_τ =1020

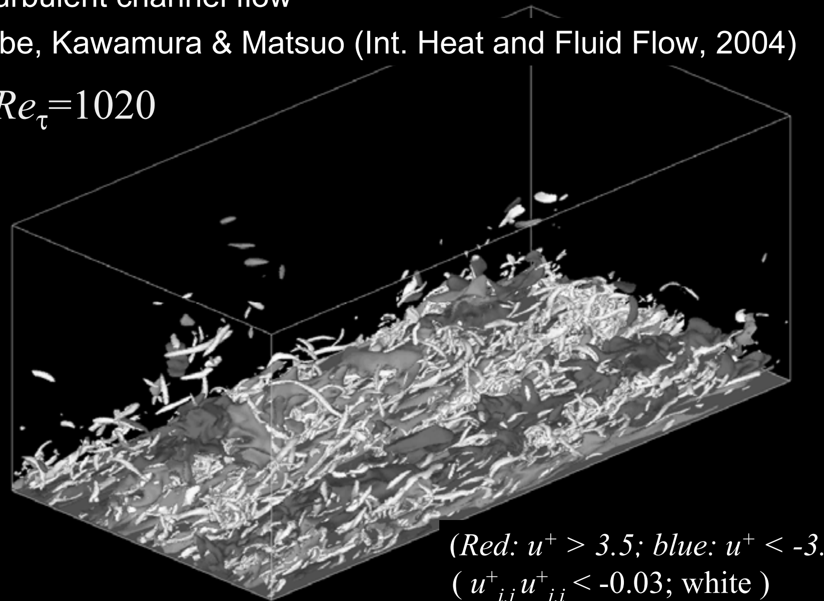
Dense clustering structure

Abe et al. (Int. J. Heat and Fluid Flow, 2004)

Turbulent channel flow

Abe, Kawamura & Matsuo (Int. Heat and Fluid Flow, 2004)

Re_τ=1020



(Red: $u^+ > 3.5$; $blue$: $u^+ < -3.5$)

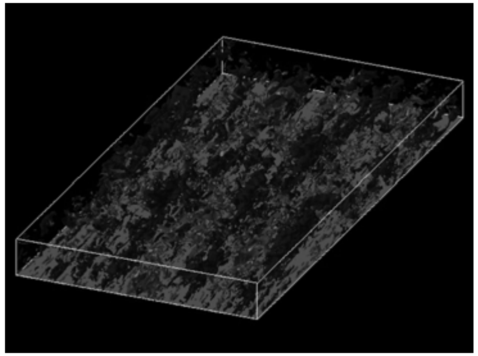
($u^+_{i,j} u^+_{j,i} < -0.03$; white)

The box visualized here is 2550×1020×1275 $(\nu/u_\tau)^3$ in the x, y and z directions, respectively.

Effects of the Reynolds number (10/11)

− Turbulent channel flow −

Very large-scale structures of u



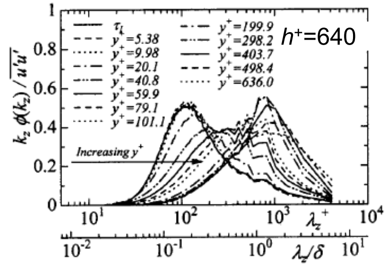
Spanwise spacing: 1.3 – 1.6δ

Red – high speed ($u'/u'_{rms} > 1.75$)

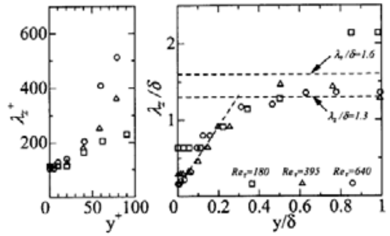
Blue – low speed ($u'/u'_{rms} < -1.75$)

Abe et al. (ASME J. Fluid Eng. 2004)

Spanwise pre-multiplied spectra



Spanwise most energetic wavelengths

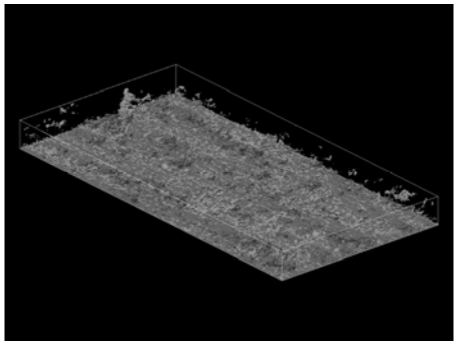


Turbulent channel flow

Abe, Kawamura & Matsuo (Int. Heat and Fluid Flow, 2004)

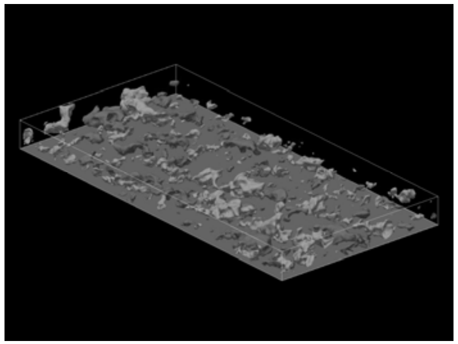
Re_τ=1020

Scalar field



θ^+ (Pr =0.71)

(Red: $\theta^+ > 3.5$; $blue$: $\theta^+ < -3.5$)



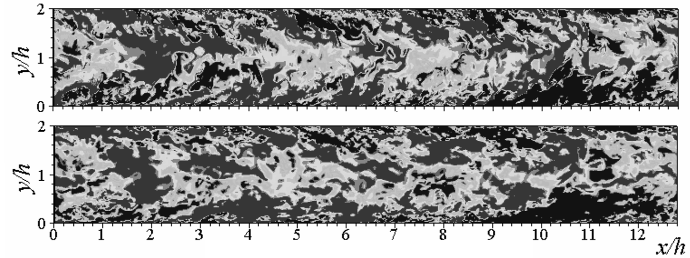
θ^+ (Pr =0.025)

(Red: $\theta^+ > 1.2$; $blue$: $\theta^+ < -1.2$)

Visualized domain is bottom half of the computational domain.

Effects of the Reynolds number (11/11)

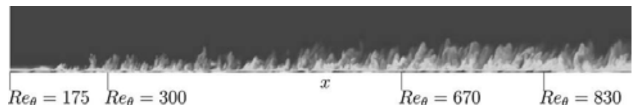
Instantaneous fields (h^+=1020 and Pr=0.71, Channel)



Steeper interfaces for θ than for u

Antonia et al. (2009)

Instantaneous fields (Pr=0.71, TBL)



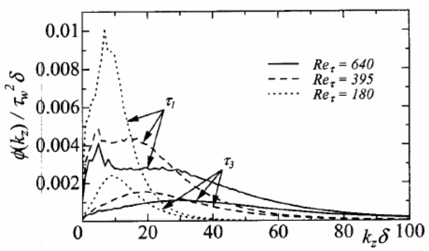
Li et al. (2009)

Inner and outer interactions (1/4)
─ Turbulent channel flow ─

Wall shear-stress fluctuations

1) Streamwise: $\tau_1 = \mu\left(\partial u / \partial y\,|_w\right)$
2) Spanwise: $\tau_3 = \mu\left(\partial w / \partial y\,|_w\right)$

Spanwise spectra in linear scales



Wall shear-stress spectra

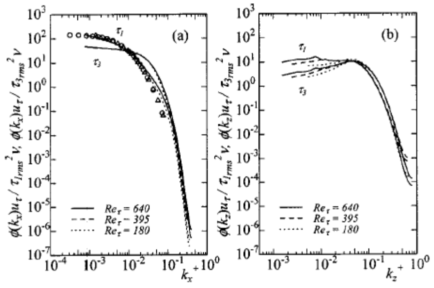


Fig. 10 One-dimensional wave number power spectra of τ_1 and τ_3 normalized by the mean-square values: (a) streamwise wave number; (b) spanwise wave number. (○), Wietrzak and Lueptow [37] (Re_τ=896); (△), Nepomuceno and Lueptow [38] (Re_τ=751).

Local peaks appear in the spanwise spectra at low wavenumbers.

H. Abe, H. Kawamura & H. Choi (ASME J. Fluid Eng. 2004)

Inner and outer interactions (2/4)
─ Turbulent channel flow ─

Spanwise two-point correlations near the wall

Streamwise velocity fluctuation

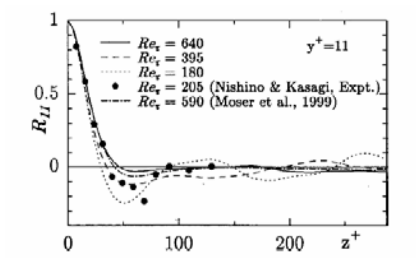
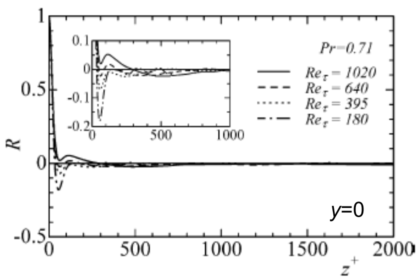


Fig. 8 Spanwise two-point correlation coefficient R_{11} at y^+ =11

Temperature fluctuation



Abe et al. (ASME J. Fluid Eng., 2001)

Abe et al. (Int. J. Heat and Fluid Flow, 2004)

The negative peaks become less pronounced with h^+.

Inner and outer interactions (3/4)
─ Turbulent boundary layer ─

Spanwise two-point correlations of wall shear-stress fluctuation

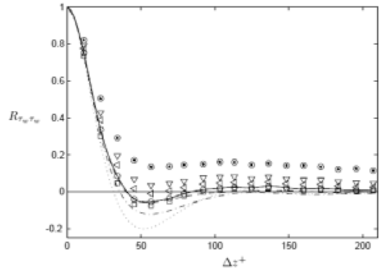


FIGURE 3. Spanwise correlation coefficient $R_{\tau_w\tau_w}$ as a function of Δz^+, Re_θ = 9500 (Re_τ = 2300). Present data, ⊙: unfiltered, ▽: $f_c^+ = 1.3\times10^{-3}$, ◁: $f_c^+ = 2.6\times10^{-3}$, ○: $f_c^+ = 5.3\times10^{-3}$, □: $f_c^+ = 7.9\times10^{-3}$, —: spline fit to ○. DNS of channel flow, correlations of u at $y^+ = 5$, – –: Re_τ = 590, – · –: Re_τ = 395, ···: Re_τ = 180 (Kim *et al.* 1987; Moser *et al.* 1999).

Österlund et al. (2003)

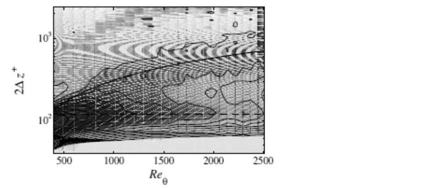
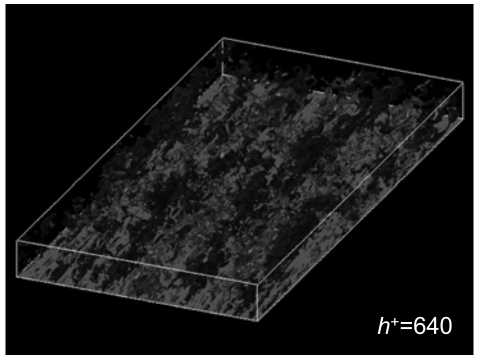


FIG. 4. Spanwise two-point correlation $R_{\tau\tau}$ of the wall-shear stress τ_w computed from DNS. The spanwise axis is scaled by the displacement $2\Delta z^+$ to show the spanwise pattern spacing. (—) corresponds to $0.85\delta_{99}$, (– – –) to $120\ell_*$. Shades range from dark ($R_{\tau\tau}\leq -0.06$) to light ($R_{\tau\tau}\geq 0.06$); contour lines go from −0.15 to 0.15 with spacing 0.02.

Schlatter et al. (2009)

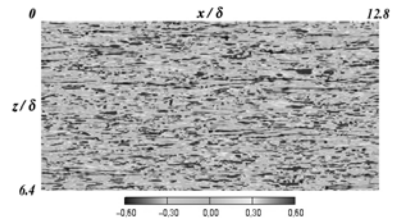
Inner and outer interactions (4/4)
─ Turbulent channel flow ─

Instantaneous fields

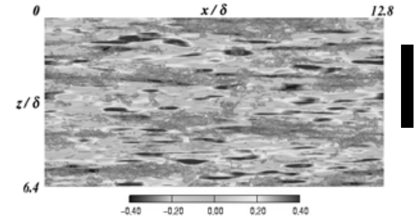


Spanwise spacing: 1.3 – 1.6δ

Red – high speed (u'/u'_{rms} > 1.75)
Blue – low speed (u'/u'_{rms} < -1.75)



Instantaneous τ_1



Filtered τ_1 overshadowed with streamwise low velocity regions (u'/u'_{rms}< -1.75)

Abe et al. (ASME J. Fluid Eng. 2004)

DNS on small-scale scalar mixing

DNS on small-scale scalar mixing (1/12)

There are only a few DNS works in wall turbulence.

- DNSs of turbulent channel flow

1) Antonia and Kim (1994) examined the isotropy of the small scales for Re_τ =180 ,392 and *Pr*=0.71.

2) Abe, Antonia and Kawamura (2009) investigated the spatial structures associated with these scales across the channel for Re_τ =180 ,395, 640 and *Pr*=0.71.

Here, we focus on the latter DNS (Abe et al. 2009).

(Abe et al. 2009)

DNS on small-scale scalar mixing (2/12)

Approach

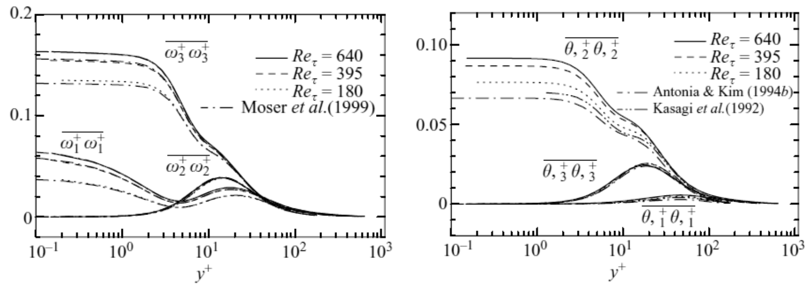
The focus is on the relationship between the three components of the fluctuating vorticity vector ω_i (i=1,2,3 represent the streamwise, wall-normal and spanwise directions, respectively) and those of the fluctuating scalar derivative vector $\theta,_i (\equiv \partial\theta/\partial x_i)$.

In particular, we compare mean-square values, correlation coefficients and instantaneous fields associated with those two vector fields.

(Abe et al. 2009)

DNS on small-scale scalar mixing (3/12)

Mean-square values normalized by wall variables



1) Good similarity between $\overline{\omega_3\omega_3}$ and $\overline{\theta,_2\theta,_2}$

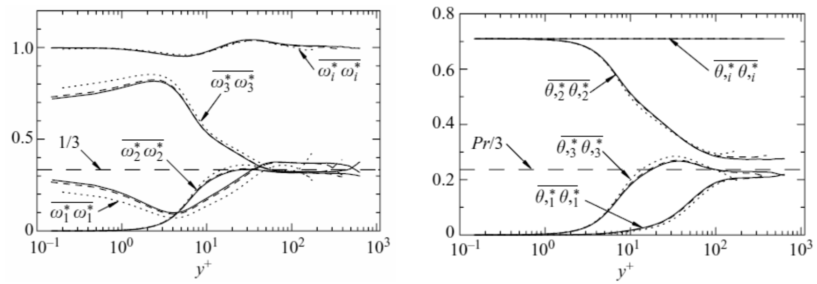
between $\overline{\omega_2\omega_2}$ and $\overline{\theta,_3\theta,_3}$

2) Corrsin (1953) first indicated the similarity when comparing the transport equations of $\overline{\theta,_i\theta,_i}$ and $\overline{\omega_i\omega_i}$.

(Abe et al. 2009)

DNS on small-scale scalar mixing (4/12)

Mean-square values normalized by Kolmogorov and Batchelor scales



1) Larger anisotropy for $\overline{\theta,_\alpha \theta,_\alpha}$ than for $\overline{\omega_\alpha \omega_\alpha}$

2) Local axisymmetry

$\overline{\omega_\alpha \omega_\alpha}$ for y^+>60 $\overline{\theta,_\alpha \theta,_\alpha}$ for y^+>100

(Abe et al. 2009)

DNS on small-scale scalar mixing (5/12)

Corrsin (1953)

- Temperature derivative

$$\Theta,_i = \bar{\Theta},_i + \theta,_i \qquad \left(\Theta,_i \equiv \frac{\partial \Theta}{\partial x_i}\right)$$

- Vorticity

$$\Omega_i = \bar{\Omega}_i + \omega_i$$

$\Theta,_i$ is lamellar $\left(\nabla \times \Theta,_i \equiv 0\right)$

Ω_i is solenoidal $\left(\nabla \cdot \Omega_i \equiv 0\right)$

(Abe et al. 2009)

DNS on small-scale scalar mixing (6/12)

- Transport equation of $\overline{\theta,_i^+ \theta,_i^+}$

$$0 = \underbrace{-2\overline{\theta,_i^+ u_j^+}\frac{\partial \bar{\Theta},_i^+}{\partial x_j^+} + 2\overline{\left(\frac{\partial \langle \bar{T}_w\rangle^+}{\partial x_1^+}\right)\left(\frac{\partial u_1^+}{\partial x_i^+}\right)\left(\frac{\partial \theta^+}{\partial x_i^+}\right)}}_{\text{gradient production}} \underbrace{-\frac{\partial}{\partial x_j^+}\left(\overline{\theta,_i^+ \theta,_i^+ u_j^+}\right)}_{\text{turbulent diffusion}} \underbrace{-2\bar{\Theta},_j^+ \cdot \overline{\theta,_i^+ \frac{\partial u_j^+}{\partial x_i^+}}}_{\text{mean gradient production (1)}} \underbrace{-2\overline{\theta,_i^+ \theta,_j^+}\frac{\partial \bar{U}_j^+}{\partial x_i^+}}_{\text{mean gradient production (2)}}$$

$$\underbrace{-2\overline{\theta,_i^+ \theta,_j^+ \frac{\partial u_j^+}{\partial x_i^+}}}_{\text{turbulent production}} + \underbrace{\frac{1}{\Pr}\frac{\partial^2}{\partial x_j^+ \partial x_j^+}\left(\overline{\theta,_i^+ \theta,_i^+}\right)}_{\text{molecular diffusion}} \underbrace{-\frac{2}{\Pr}\overline{\left(\frac{\partial \theta,_i^+}{\partial x_j^+}\right)\left(\frac{\partial \theta,_i^+}{\partial x_j^+}\right)}}_{\text{dissipation}} \qquad \left[2\overline{\left(\frac{\partial \langle T_w\rangle^+}{\partial x_1^+}\right)\left(\frac{\partial u_1^+}{\partial x_i^+}\right)\left(\frac{\partial \theta^+}{\partial x_i^+}\right)} \approx 0\right]$$

- Transport equation of $\overline{\omega_i^+ \omega_i^+}$

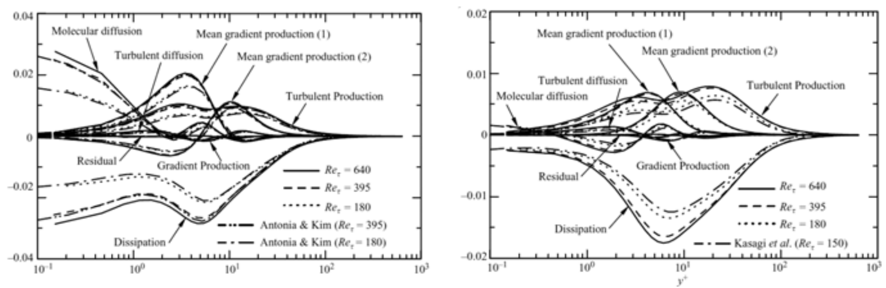
$$0 = \underbrace{-2\overline{\omega_i^+ u_j^+}\frac{\partial \bar{\Omega}_i^+}{\partial x_j^+}}_{\text{gradient production}} \underbrace{-\frac{\partial}{\partial x_j^+}\left(\overline{\omega_i^+ \omega_i^+ u_j^+}\right)}_{\text{turbulent diffusion}} + \underbrace{\bar{\Omega}_j^+ \cdot \overline{\omega_i^+\left(\frac{\partial u_j^+}{\partial x_i^+} + \frac{\partial u_i^+}{\partial x_j^+}\right)}}_{\text{mean gradient production (1)}} + \underbrace{\overline{\omega_i^+ \omega_j^+} \cdot \left(\frac{\partial \bar{U}_i^+}{\partial x_j^+} + \frac{\partial \bar{U}_j^+}{\partial x_i^+}\right)}_{\text{mean gradient production (2)}}$$

$$\underbrace{+\overline{\omega_i^+ \omega_j^+\left(\frac{\partial u_i^+}{\partial x_j^+} + \frac{\partial u_j^+}{\partial x_i^+}\right)}}_{\text{turbulent production}} + \underbrace{\frac{\partial^2}{\partial x_j^+ \partial x_j^+}\left(\overline{\omega_i^+ \omega_i^+}\right)}_{\text{molecular diffusion}} \underbrace{-2\overline{\left(\frac{\partial \omega_i^+}{\partial x_j^+}\right)\left(\frac{\partial \omega_i^+}{\partial x_j^+}\right)}}_{\text{dissipation}}$$

(Abe et al. 2009)

DNS on small-scale scalar mixing (7/12)

Budget of Budget of



(Abe et al. 2009)

1) Good similarity except close to the wall

2) Noticeable differences close to the wall due to differences in BCs

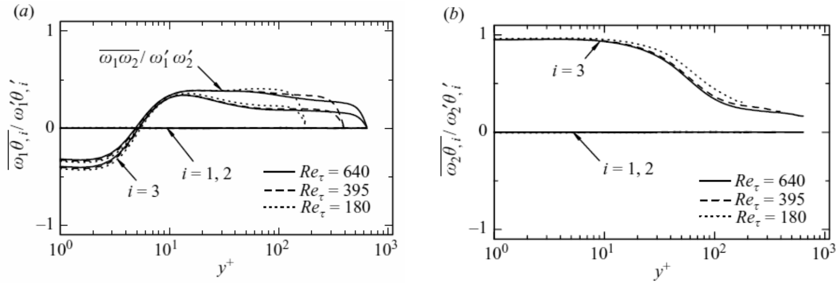
$$\left(\begin{aligned} \omega_1^+ &= b_3 + 2c_3 y^+ + O\left(y^{+2}\right) \\ \omega_3^+ &= -b_1 - 2c_1 y^+ + O\left(y^{+2}\right) \\ \theta,_2^+ &= b_\theta + O\left(y^{+2}\right) \end{aligned}\right)$$

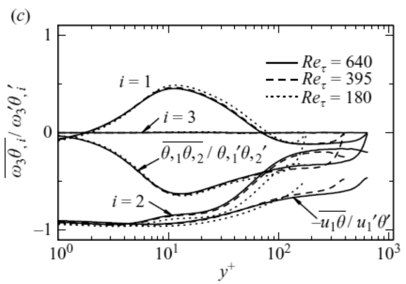
3) Significant *Re* effects for the production terms (except for the gradient production term) and the dissipation rate term

(Abe et al. 2009)

DNS on small-scale scalar mixing (8/12)

Correlation coefficients between ω_i and $\theta_{,i}$





Four non-zero correlations: $\overline{\omega_{,1}\theta_{,3}}$; $\overline{\omega_{,2}\theta_{,3}}$; $\overline{\omega_{,3}\theta_{,1}}$; $\overline{\omega_{,3}\theta_{,2}}$.

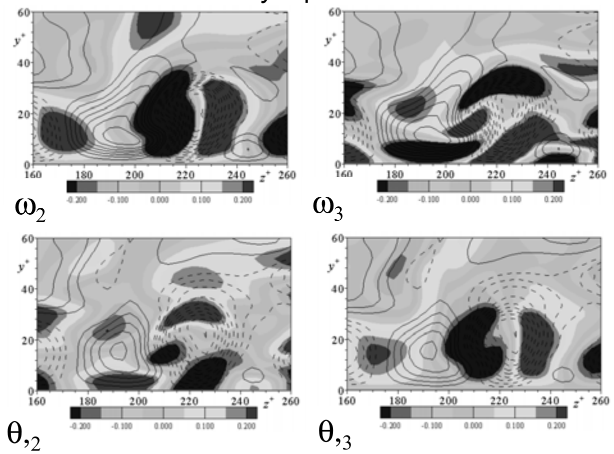
Other five correlations are zero due to the symmetry.

(Abe et al. 2009)

DNS on small-scale scalar mixing (9/12)

Instantaneous fields (h^+=180 and Pr=0.71)

y-z plane



lines are *u*

Solid; positive
Dashed; negative

lines are θ

Solid; positive
Dashed; negative

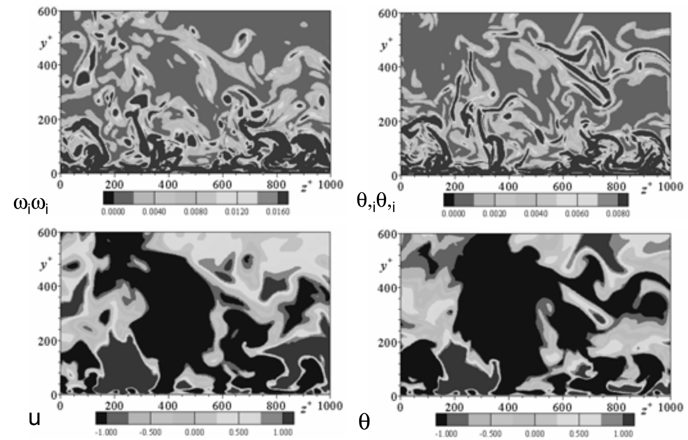
Good similarity between ω_2 and $\theta_{,3}$ and between ω_3 and $\theta_{,2}$

(Abe et al. 2009)

DNS on small-scale scalar mixing (10/12)

Instantaneous fields (h^+=640 and Pr=0.71)

y-z plane

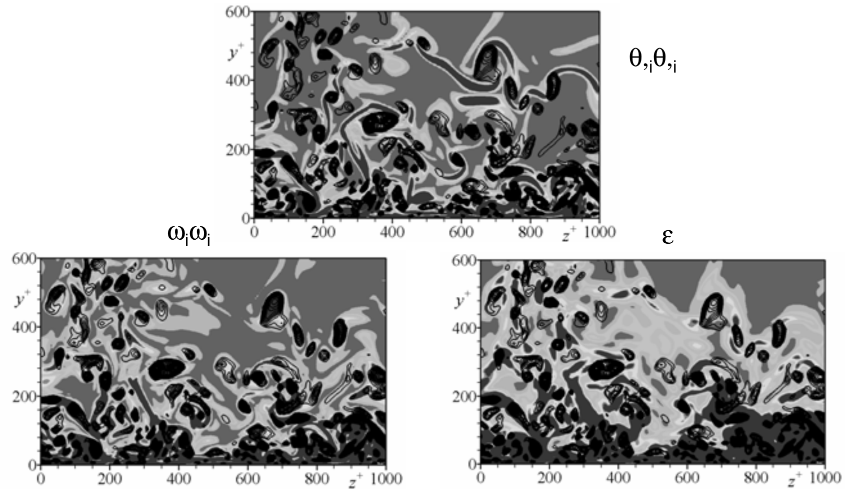


In the outer region, $\theta_{,i}\theta_{,i}$ exhibits sheet-like structures

(Abe et al. 2009)

DNS on small-scale scalar mixing (11/12)

Instantaneous fields (h^+=640 and Pr=0.71) *y-z* plane

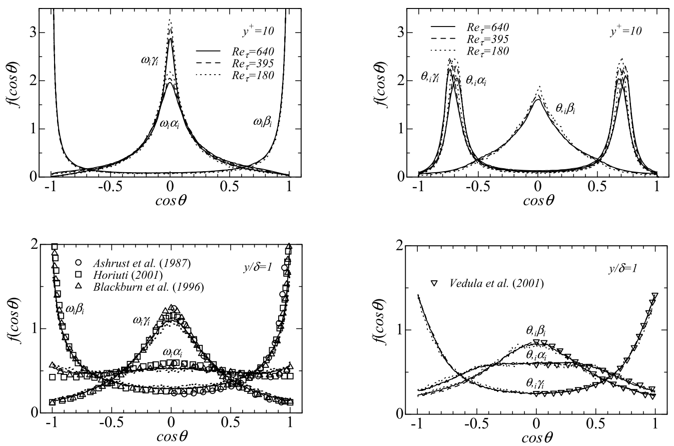


Lines denote positive values of the second invariant of the velocity gradient tensor $Q\left(=\omega_i\omega_i/4 - s_{i,j}s_{j,i}/2 = -u_{i,j}u_{j,i}/2\right)$

(Abe et al. 2009)

DNS on small-scale scalar mixing (12/12)

PDFs for the cosine of the angle between ω_i and the principal strain rate directions and and between $\theta_{,i}$ and the principal strain rate directions



α_i: Extensive eigenvector; β_i: Intermediate eigenvector
γ_i: Compressive eigenvector

(Abe et al. 2009)

Acknowledgements

I am grateful to

Prof. H. Kawamura (Tokyo Univ. Science)

and

Prof. R. A. Antonia (Newcastle Univ.)

for the fruitful collaborations on our DNS works.

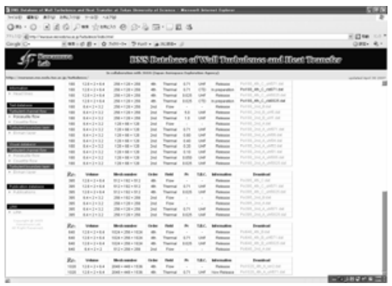
DNS databases in a turbulent channel flow with passive scalar transport

Database sites

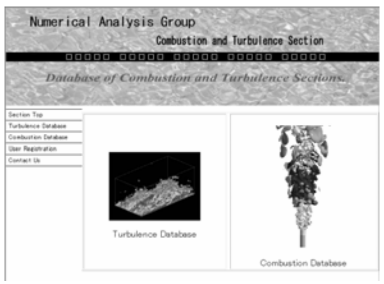
1) URL: http://murasun.me.noda.tus.ac.jp/
2) URL: http://www.iat.jaxa.jp/db/index.html/

(Collaborative work between JAXA and Tokyo Univ. Science)

h^+=180,395,640,1020 and Pr=0.025, 0.71 (11 cases)



Website of Tokyo Univ. Science



Website of JAXA