



Stability and transition in wall bounded turbulence

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Lecture 1

1. **Introduction to flow instability**
2. Linear stability theory for boundary layer flows
 - Rayleigh stability theory (inviscid)
 - Orr-Sommerfeld theory (viscous)
3. Wind tunnels for transition research
4. Transition prediction
5. Transient growth
6. Free stream turbulence

Literature

Lin (1955)
The theory of hydrodynamic stability

Chandrasekhar (1961)
Hydrodynamic and hydromagnetic stability

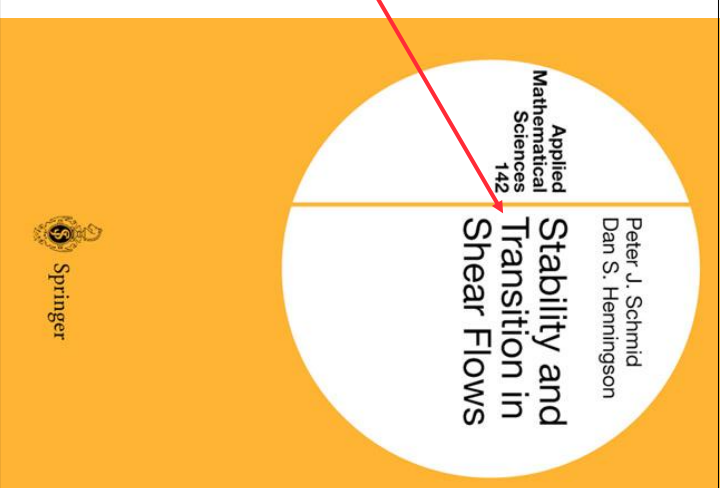
Betchov & Criminale (1967)
Stability of parallel flows

Drazin & Reid (1981)
Hydrodynamic stability

Schmid & Henningson (2001)

Drazin (2002)
Introduction to hydrodynamic stability

Criminale (2003)
Theory and computation in hydrodynamic stability



Water jet from kitchen tap

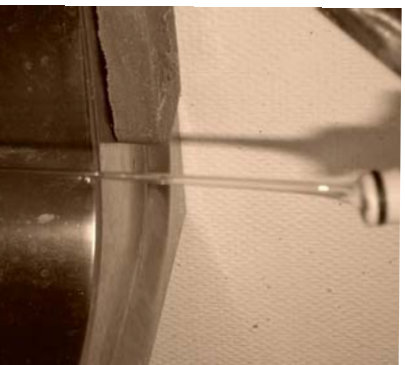


laminar



turbulent

Water jet from kitchen tap



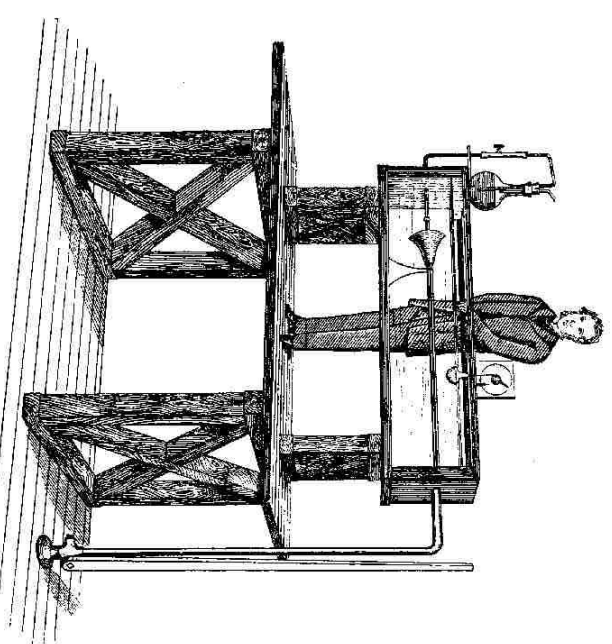
Always unstable to axi-symmetric disturbances with a wavelength larger than the jet circumference, but growth rate $\sim r^{-3/2}$

laminar dripping

Osborne Reynolds (1842-1912)

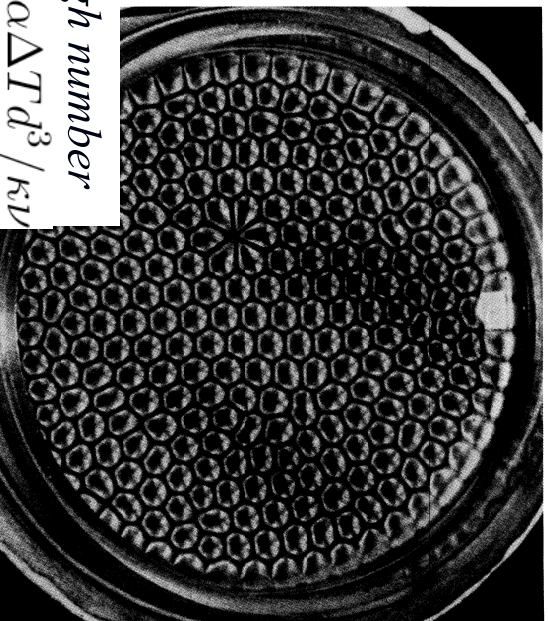


Reynolds experiment (1883)

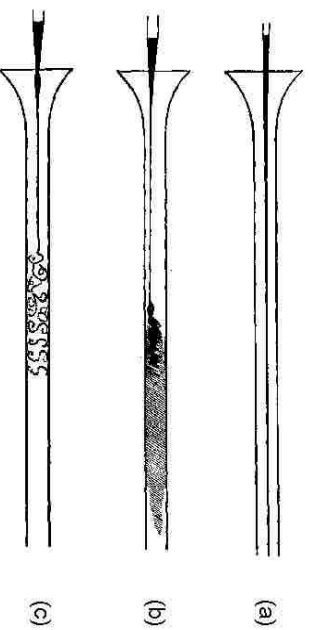


Thermal instability - Bénard convection

Rayleigh number
 $Ra = g\alpha\Delta T d^3 / \kappa\nu$



Reynolds experiment (1883)



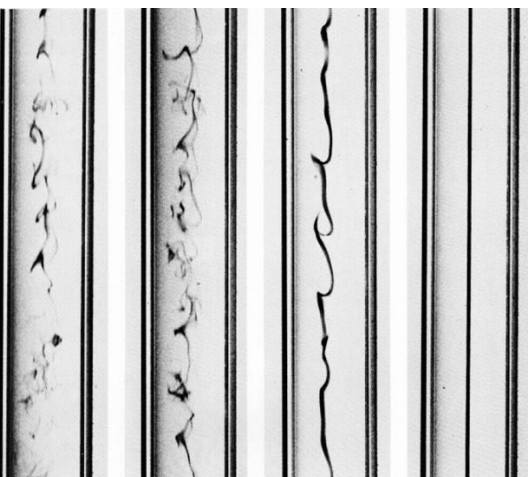
- (a) Laminar flow $Re < Re_{\text{transitional}}$,
- (b) Transition to turbulence $Re > Re_{\text{transitional}}$
- (c) Same as (b) but "short time exposure"

Reynolds experiment (1983)

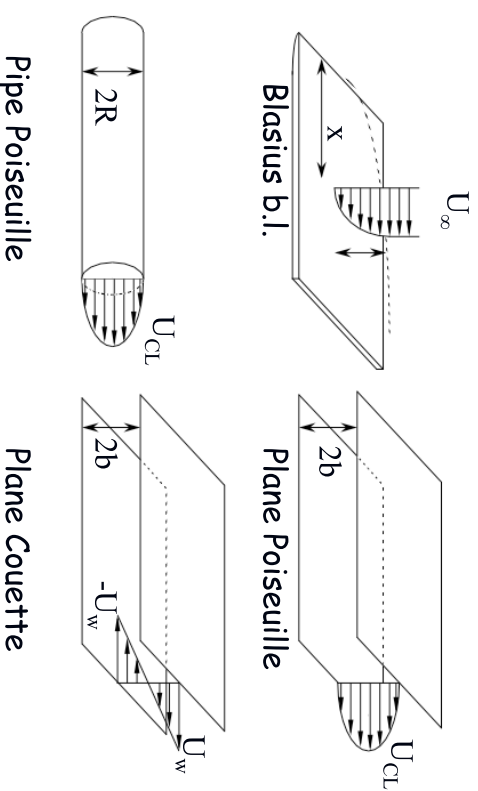
Same apparatus today!

1883 transitional Re
=13000

Lower Re today due to traffic, i.e. external disturbances has increased in amplitude



Canonical wall bounded flows



Transitional Reynolds numbers for the canonical cases

Flow case	Re	Re_{tr}
Blasius	$U_{\infty} \delta^* / \nu$	490
Channel	$U_{cl} b / \nu$	1000
Pipe	$U_{cl} R / \nu$	1760
Couette	$U_w b / \nu$	360

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Hydrodynamic stability theory

Idea: Introduce a controlled disturbance and investigate whether it grows or decays (in time or space) for the given flow situation

Control parameter: Typically a Reynolds number (and possible some other non-dimensional variable depending on the flow situation)

$$\frac{\text{inertia}}{\text{viscous damping}} \sim \frac{\rho u \cdot \nabla u}{\mu \nabla^2 u} \sim \frac{\rho U^2 / L}{\mu U / L^2} = \frac{\rho U L}{\mu} = Re$$

Other variables: Amplitude, wave number and frequency of the disturbance

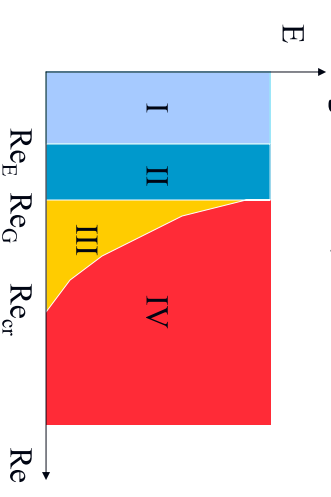
Linear theory: independent of amplitude => modal analysis

Stability definitions

(from Schmid & Henningson 2001)

Region:

- I: Monotonic stability
- II: Global stability
- III: Conditional stability
- IV: Possible instability



Flow case	Re _G	Re _G	Re _{cr}	Re _{tr}
Pipe	81.5	-	∞	1760
Channel	49.6	-	5772	1000
Couette	20.7	125	∞	360

Some landmarks of classical stability theory

Rayleigh (1880) - Inviscid theory

Reynolds (1883) - Pipe flow experiments - the importance of the Reynolds number

Orr (1907), Sommerfeld (1908) Formulation of viscous disturbance eq.

Heisenberg (1924) First eigenvalues found (Dr-thesis)

Tollmien (1928), Schlichting (1933) Solutions to OS-eq.

Schubauer & Skramstad (1948) First exp. Verification of TS-waves

Klebanoff et al (1962) Exp. on TS-wave breakdown

Orzag (1970) Numerical solution for OS-eq. in channel flow

Fasel & Konzelmann (1990) Numerical work on OS-eq. in Blasius boundary layer flow

Klingmann et al (1993) New experimental results on TS-waves in boundary layer flow

Blasius boundary layer flow

Equations of motions

Continuity equation (incompressible flow)

$$\nabla \cdot \bar{u} = 0$$

Navier-Stokes equations

$$\rho \left[\frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} \right] = -\nabla p + \mu \nabla^2 \bar{u}$$

In non-dimensional form

$$\nabla \cdot \bar{u} = 0 \quad \text{and} \quad \frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} = -\nabla p + Re^{-1} \nabla^2 \bar{u}$$

where

$$Re = \rho U L / \mu$$

Disturbance decomposition

Assume linear disturbances (capital letters are mean values and prime fluctuations around the mean)

$$u = U + u'$$

$$v = V + v'$$

$$p = P + p'$$

u', v', p' are small disturbances \Rightarrow quadratic terms may be neglected.

We now assume parallel flow, i.e. $V = 0$. Exact for channel flows, good approximation for many boundary layer flows!

Equations of motions (contd)

Assume that the base flow is 2D and that disturbances are 2D.
 $\Rightarrow \bar{u} = (u, v) = u(x, y) \bar{e}_x + v(x, y) \bar{e}_y$

Momentum eqs. in x and y -directions are obtained through scalar multiplication with \bar{e}_x and \bar{e}_y

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Re^{-1} \nabla^2 u \quad (1)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + Re^{-1} \nabla^2 v \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

Disturbance decomposition (contd.)

$$\frac{\partial(U + u')}{\partial t} + (U + u') \frac{\partial(U + u')}{\partial x} + v' \frac{\partial(U + u')}{\partial y} =$$

$$-\frac{\partial(P + p')}{\partial x} + Re^{-1} \nabla^2 (U + u')$$

Write out all the different terms in the equation above, noting that the mean flow is independent of time

quadratic in the disturbance

$$\frac{\partial u'}{\partial t} + U \frac{\partial U}{\partial x} + U \frac{\partial u'}{\partial x} + u' \frac{\partial U}{\partial x} + u' \frac{\partial u'}{\partial x} + v' \frac{\partial U}{\partial y} + v' \frac{\partial u'}{\partial y} =$$

$$-\frac{\partial P}{\partial x} - \frac{\partial p'}{\partial x} + Re^{-1} \nabla^2 U + Re^{-1} \nabla^2 u' \quad (4)$$

(contd.)

Assume that all disturbances are zero!

$$U \frac{\partial U}{\partial x} = -\frac{\partial P}{\partial x} + Re^{-1} \nabla^2 U$$

Subtract that from equation (4) \Rightarrow

$$\frac{\partial u'}{\partial t} + U \frac{\partial u'}{\partial x} + u' \frac{\partial U}{\partial x} + v' \frac{\partial U}{\partial y} = -\frac{\partial p'}{\partial x} + Re^{-1} \nabla^2 u' \quad (5)$$

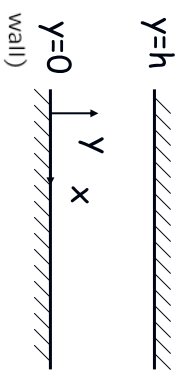
Same operation for eqs. (2) and (3) \Rightarrow

$$\frac{\partial v'}{\partial t} + U \frac{\partial v'}{\partial x} = -\frac{\partial p'}{\partial y} + Re^{-1} \nabla^2 v' \quad (6)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 \quad (7)$$

Boundary conditions for channel flow

At $y = 0$ and $y = h$ we get



$$v' = 0 \quad (\text{impermeable wall})$$

$$u' = 0 \quad (\text{no slip}) \Rightarrow \frac{\partial v'}{\partial y} = 0$$

The latter is given from

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$$

and since $u' = 0$ at the wall $\Rightarrow \partial u' / \partial x = 0$ at the wall as well.

v -disturbance equation

3 equations (5), (6) and (7), 3 unknowns u' , v' , p' !

Eliminate p' by taking $\frac{\partial}{\partial y}$ (5) - $\frac{\partial}{\partial x}$ (6)

By eliminating u' using eq. (7) we obtain an equation with only v' unknown

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \nabla^2 v' - U'' \frac{\partial v}{\partial x} = Re^{-1} \nabla^4 v \quad (8)$$

Equation (8) needs boundary conditions, 4th order equation \Rightarrow 4 BCs needed.

Normal mode analysis

Analyze eq. (8) in terms of normal modes

$$v' = \hat{v}(y) e^{i(\alpha x - \omega t)}$$

where i is the imaginary number and

$\hat{v}(y)$ is a complex amplitude function

α is the wave number ($= 2\pi/\lambda$ where λ is the wave length)

ω is the angular frequency

We can also define the phase speed of the wave as $c = \omega/\alpha$

Normal mode analysis - spatial growth

Chose spatial or temporal approach, i.e. a disturbance that is growing in space (x) or time (t).

Spatial growth $\Rightarrow \alpha = \alpha_r + i\alpha_i$ and ω is real

Temporal growth $\Rightarrow \omega = \omega_r + i\omega_i$ and α is real

or $c = c_r + ic_i$

Why?

$$e^{i(\alpha x - \omega t)} = e^{i[(\alpha_r + i\alpha_i)x - \omega t]} = e^{i(\alpha_r x - \omega t)} e^{-\alpha_i x}$$

$\Rightarrow \alpha_i < 0$ exponentially growing disturbance

$\alpha_i > 0$ exponentially decaying disturbance

The Orr-Sommerfeld equation

Put the normal mode assumption $[e^{i\alpha(x-ct)}]$ into eq. (8)

$$\begin{array}{ccc} \frac{\partial}{\partial t} & \rightarrow & -i\alpha c \\ \frac{\partial}{\partial x} & \rightarrow & i\alpha \\ \frac{\partial}{\partial y} & \rightarrow & \frac{d}{dy} = D \end{array}$$

for instance $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = -\alpha^2 + D^2$

This gives us the so called Orr-Sommerfeld (OS) equation

$$(U - c)(D^2 - \alpha^2)\hat{v} - U''\hat{v} = \frac{1}{i\alpha Re}(D^2 - \alpha^2)^2\hat{v}$$

Normal mode analysis - temporal growth

For the temporal approach, i.e. a disturbance that is growing time (t).

$$e^{i(\alpha x - \omega t)} = e^{i\alpha(x-ct)} = e^{i\alpha(x-c_r t)} e^{\alpha c_i t}$$

$\Rightarrow c_i > 0$ exponentially growing disturbance

$c_i < 0$ exponentially decaying disturbance

The Rayleigh equation

Let us investigate the OS-eq. when $Re \rightarrow \infty \Rightarrow$ Rayleigh eq.

$$(U - c)(D^2 - \alpha^2)\hat{v} - U''\hat{v} = 0$$

BC ? Only two BCs can be retained! $\hat{v}(0) = \hat{v}(h) = 0$

Rewrite :

$$(D^2 - \alpha^2)\hat{v} - \frac{U''}{(U - c)}\hat{v} = 0$$

Multiply with \hat{v}^* , the complex conjugate of \hat{v} and integrate the eq. from $y = 0$ to $y = h$

$$\int_0^h \left[\hat{v}^*(D^2\hat{v} - \alpha^2\hat{v}) - \frac{U''}{(U - c)}\hat{v}\hat{v}^* \right] dy = 0$$

The Rayleigh equation (contd)

By using partial integration we get

$$\int_0^h \hat{v}^* D^2 \hat{v} dy = \underbrace{\left\{ \hat{v}^* D \hat{v} \right\}_0^h}_{=0} - \int_0^h (D \hat{v}^*) D \hat{v} dy = - \int_0^h |D \hat{v}|^2 dy$$

Using $c = c_r + i c_i$

$$\frac{1}{U - c} = \frac{1}{U - c_r - i c_i} = \frac{U - c_r + i c_i}{(U - c_r - i c_i)(U - c_r + i c_i)} = \frac{U - c_r + i c_i}{(U - c_r)^2 + c_i^2}$$

we then can write

$$\int_0^h \left[|D \hat{v}|^2 + \alpha^2 |\hat{v}|^2 + \frac{U''(U - c_r + i c_i)}{(U - c_r)^2 + c_i^2} |\hat{v}|^2 \right] dy = 0$$

The Rayleigh equation (contd.)

Taking the imaginary part of the equation

$$\int_0^h \left[\frac{U'' c_i}{(U - c_r)^2 + c_i^2} |\hat{v}|^2 \right] dy = 0$$

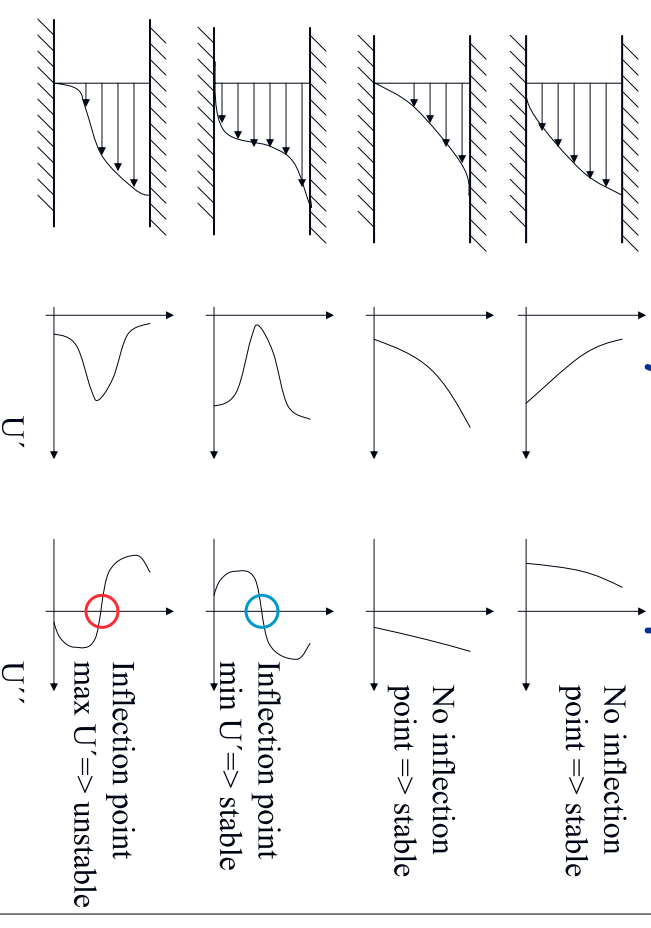
or

$$c_i \int_0^h \left[\frac{U''}{(U - c_r)^2 + c_i^2} |\hat{v}|^2 \right] dy = 0$$

This means that U'' must change sign in the interval if $c_i \neq 0$, i.e. U has to have an inflection point in the interval. This is **Rayleighs inflection point criterion** !

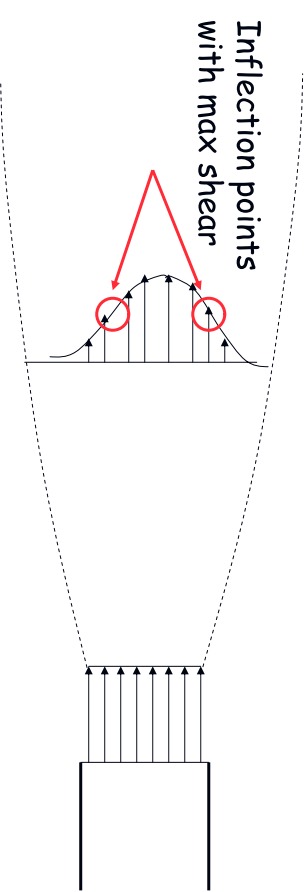
Fjørtofts criterion says that for the inflection point to be unstable, the shear, i.e. U' has to have a maximum at the inflections point! (see Schmid & Henningson, page 21).

The stability of various profiles



Inflectional instability of jet flows

Rayleighs stability criterion (1880) explains why jets and wakes are very unstable, they have inflectional mean velocity profiles!



Inflectional instability of jet

QuickTime™ and a
decompressor
are needed to see this picture.

How to solve the OS-equation ?

The solution to the OS-equation can be seen as an eigenvalue problem.

Input data: Mean velocity profile, BC, Reynolds number, α (ω).

Output: Eigenfunction $\hat{v}(y)$, eigenvalue ω (α).

Solution method: Shooting method
Spectral method

Example: channel flow

Boundary conditions at both channel walls:

$$\hat{v} = 0 \quad D\hat{v} = 0$$

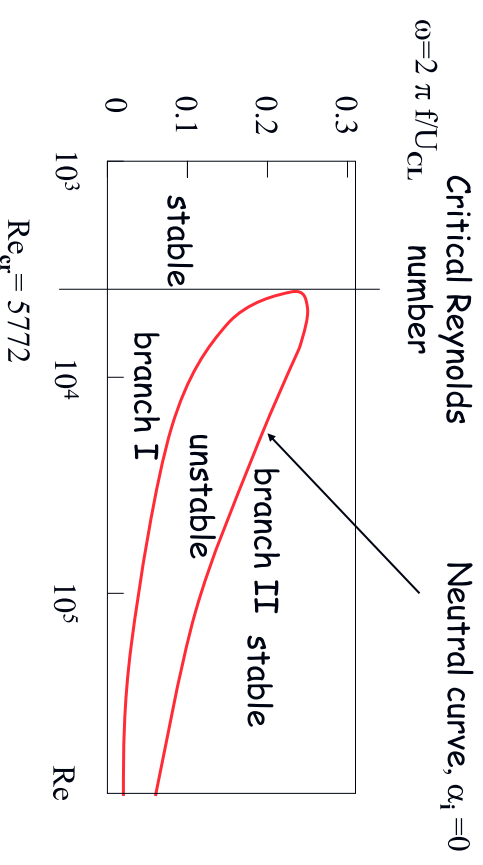
\Rightarrow Trivial solution $\hat{v} = 0$

Other boundary conditions needed!

symmetric mode $\Rightarrow D\hat{v} = D^3\hat{v} = 0$ on CL

anti-symmetric mode $\Rightarrow \hat{v} = D^2\hat{v} = 0$ on CL

Example: channel flow - symmetric mode



What happens at high Re ?

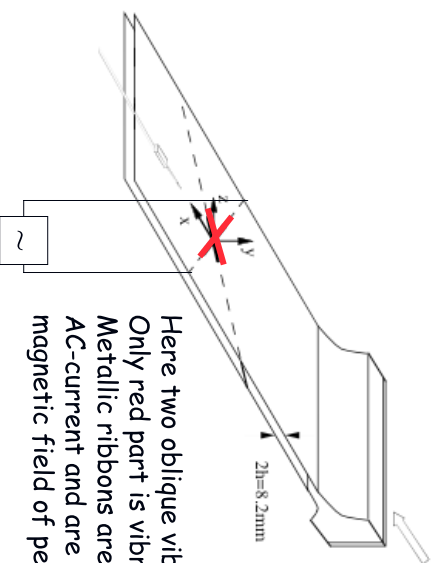
How do we verify the theory ?

Introduce a single frequency 2D wave with a vibrating ribbon (old method) or through a thin, long spanwise slit connected to a loudspeaker (current method).

Measure the development of the wave amplitude in the downstream direction. Check also the eigenfunction (phase and amplitude)!

Experimental channel set-up

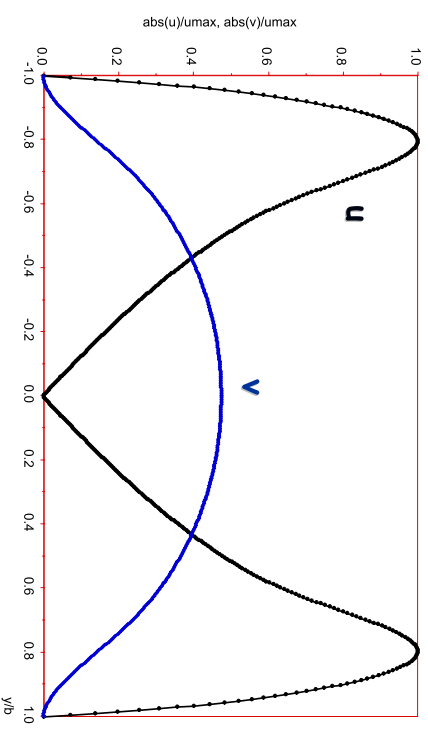
(P. Elofsson, PhD thesis 1998, KTH)



Here two oblique vibrating ribbons. Only red part is vibrating. Metallic ribbons are fed with AC-current and are located in the magnetic field of permanent magnets

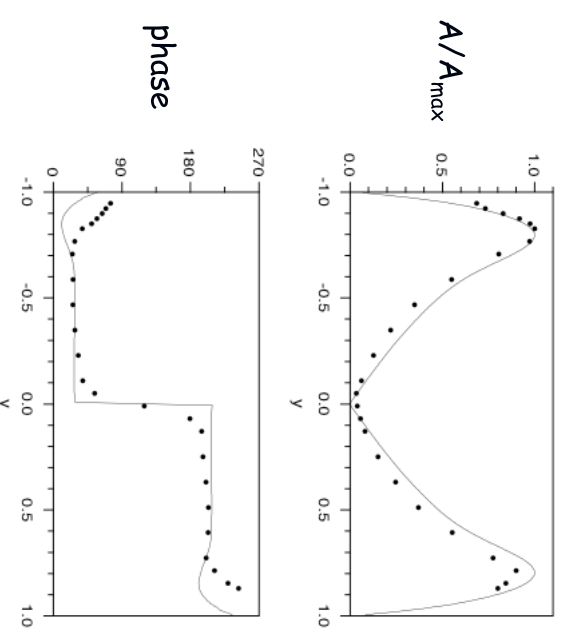
Amplitude functions - channel flow

$Re=1600$, $\omega=0.40$



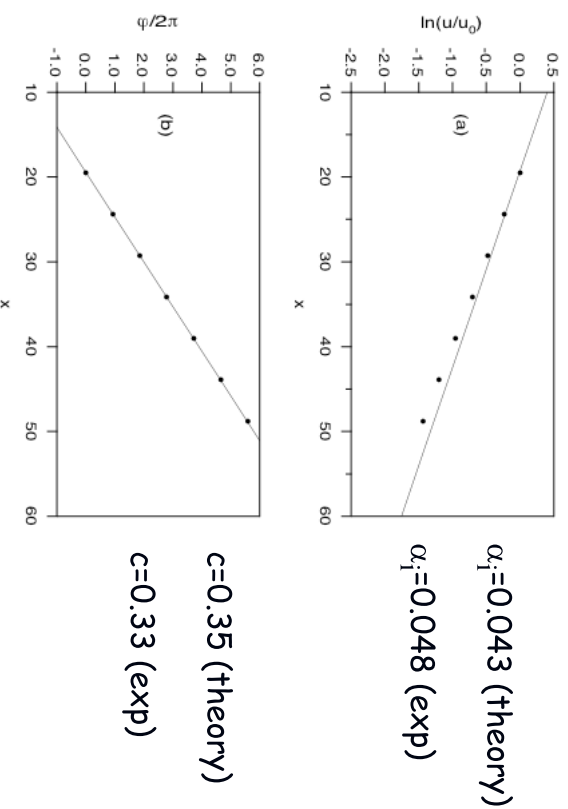
Amplitude function for channel flow

($Re=1600$, $\omega=0.42$, $x/b=40$)



Amplitude and phase development

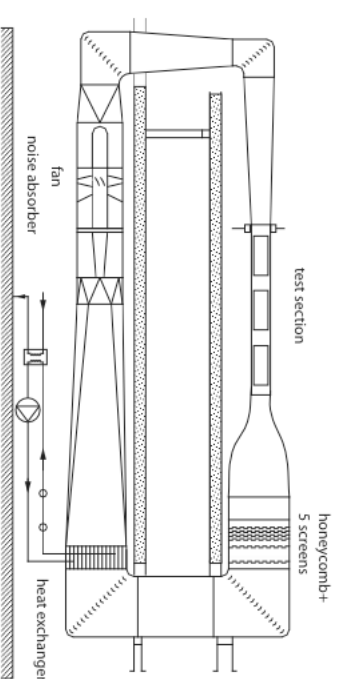
($Re=1600$, $\omega=0.42$)



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MTL wind tunnel for stability and transition research



- Low turbulence (<0.03%) and sound noise level
- Low vibration level
- Velocity and temperature stability at low velocities

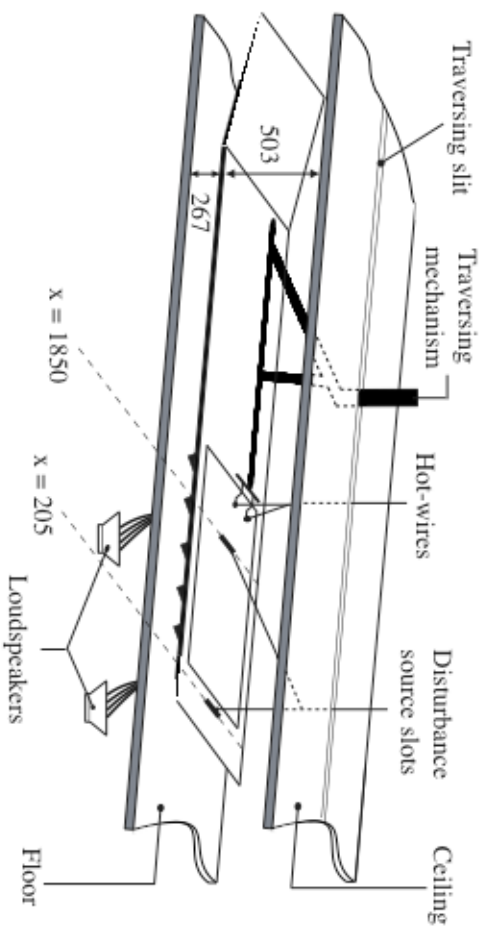
MTL test section looking upstream



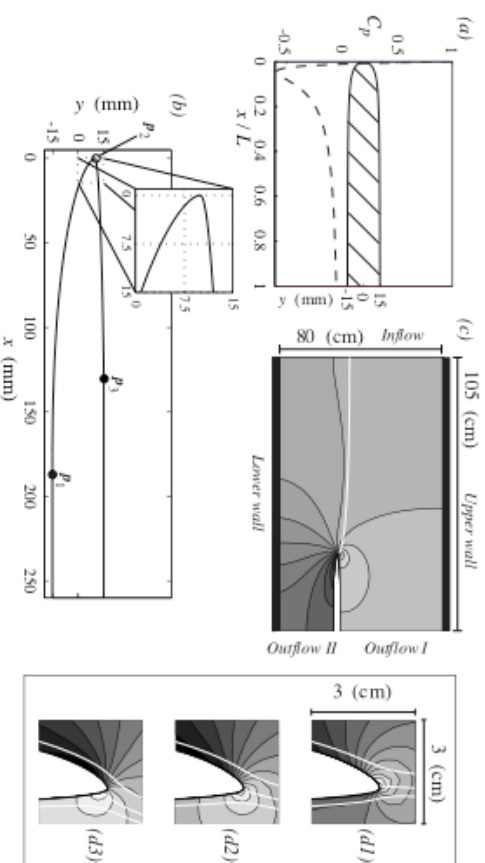
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Experimental set-up

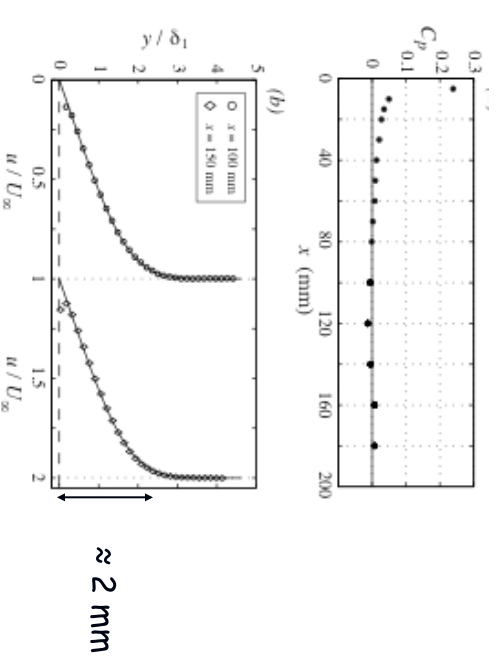
(J. Fransson Ph.D. Thesis 2003, KTH)



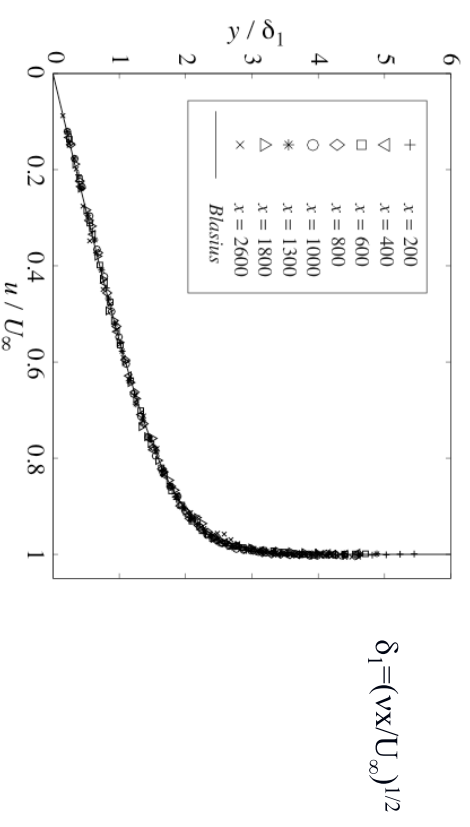
Leading edge geometry



Pressure coefficient and velocity profiles near leading edge



Experimental boundary layer profiles

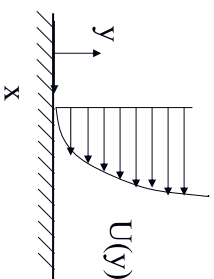


Boundary conditions for the Blasius boundary layer

Boundary conditions at the wall

$$v' = 0 \text{ at } y = 0$$

$$\frac{\partial v'}{\partial y} = 0 \text{ at } y = 0$$

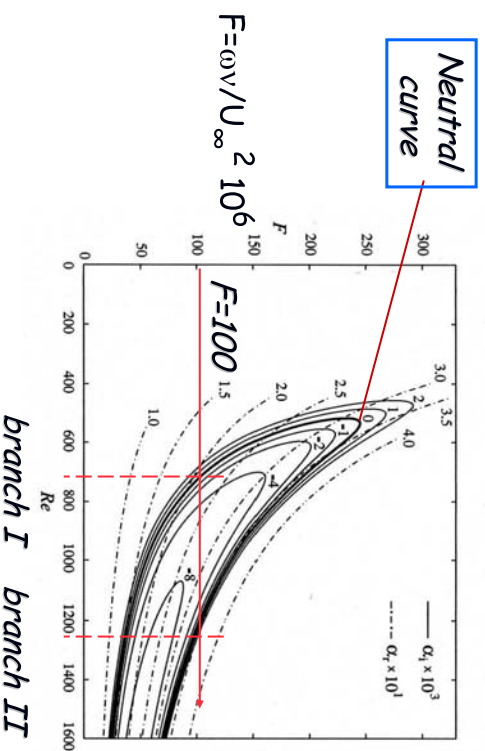


and boundary conditions in the free stream

$$v' \rightarrow 0 \text{ as } y \rightarrow \infty$$

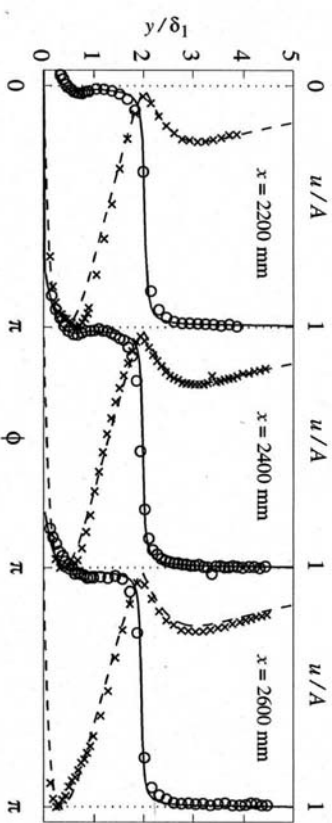
$$\frac{\partial v'}{\partial y} \rightarrow 0 \text{ as } y \rightarrow \infty$$

Stability diagram for Blasius b.l.



Example of disturbance profiles

x: exp amplitude, - - - theory
o: exp phase, — theory



Behaviour of disturbance profiles as $y \rightarrow \infty$

$$(U - c)(D^2 - \alpha^2)\hat{v} - U''\hat{v} = \frac{1}{i\alpha Re}(D^2 - \alpha^2)^2\hat{v}$$

Solve the OS-equation in the free stream where $U = 1$ (i.e. U_∞)
 $\Rightarrow U'' = 0$

This gives the following form of the OS-equation:

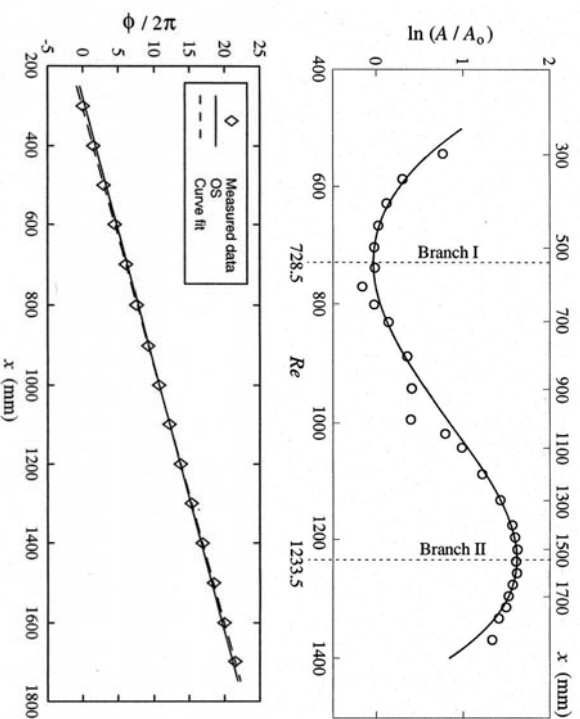
$$\{D^2 - [\alpha^2 + i\alpha Re(1 - c)]\}(D^2 - \alpha^2)\hat{v} = 0$$

Solution:

$$\hat{v} = \underbrace{Ae^{\alpha y}}_{y \rightarrow \infty \Rightarrow A=0} + Be^{-\alpha y} + C \underbrace{e^{\sqrt{\alpha^2 + i\alpha Re(1-c)}y}}_{y \rightarrow \infty \Rightarrow C=0} + \underbrace{De^{-\sqrt{\alpha^2 + i\alpha Re(1-c)}y}}_{\text{small if } Re \text{ is large}}$$

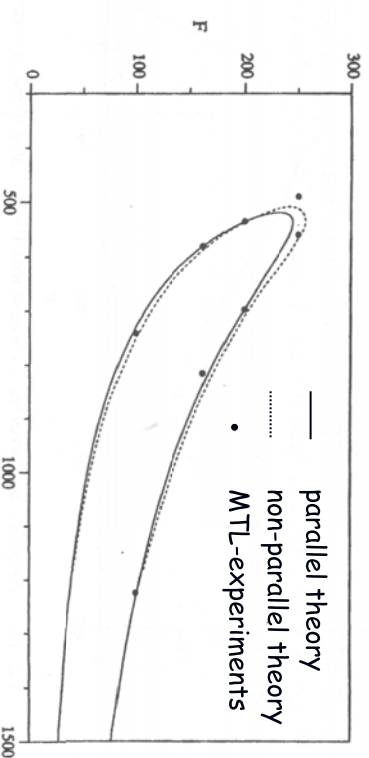
As $y \rightarrow \infty$ $\hat{v} \sim e^{-\alpha y}$ and $D\hat{v} \sim -\alpha e^{-\alpha y}$ which gives $|\hat{u}| = |\hat{v}| \sim e^{-\alpha y}$

Growth and phase development



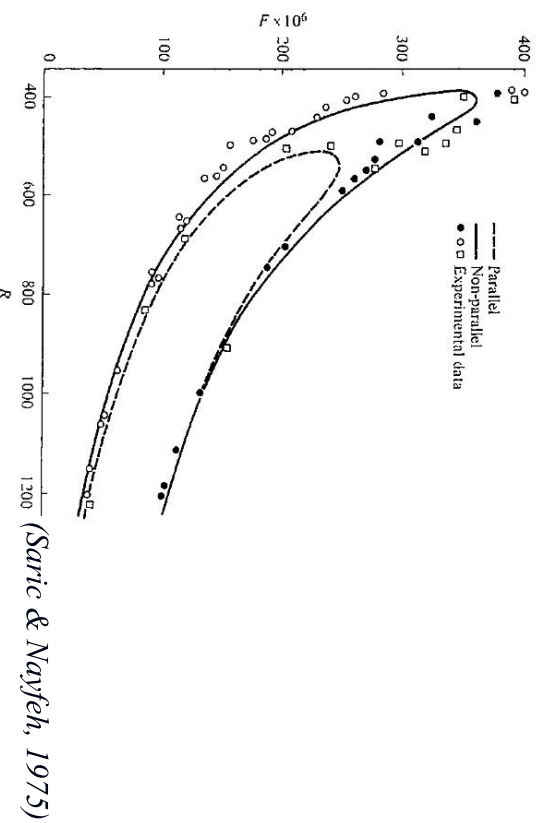
Neutral stability curve for Blasius b.l. with experimental points from MTL

(Klingmann et al. Eur. J. Mech/Fluids 1993)

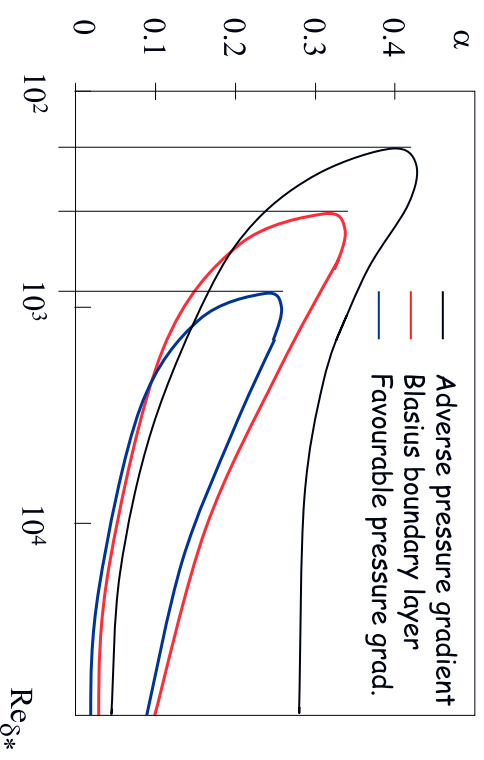


The influence of non-parallel effects ($V \neq 0$) are small !
Local analysis sufficient for Blasius b.l.

Early experimental data and theory

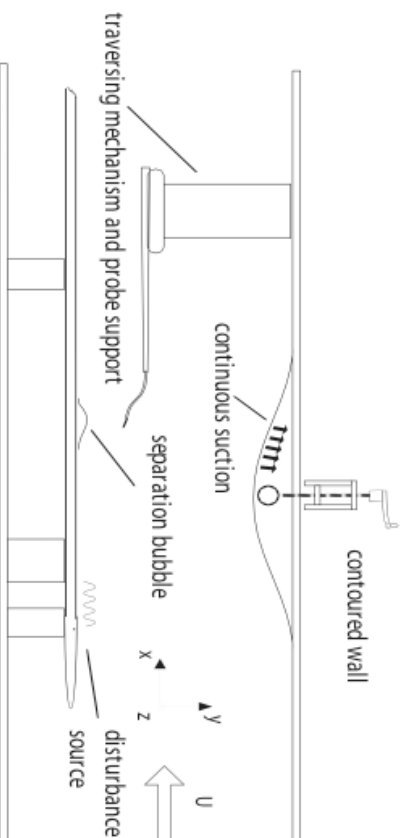


Neutral stability curve for flat plate b.l. for various pressure gradients

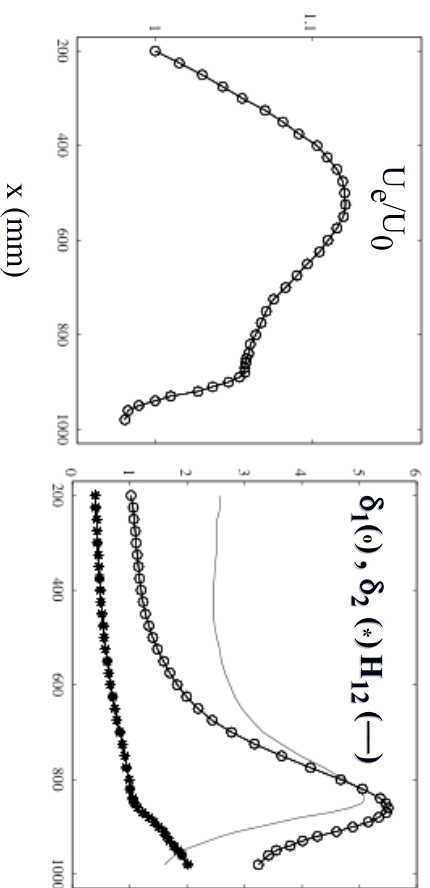


Setup for laminar separation bubble experiment

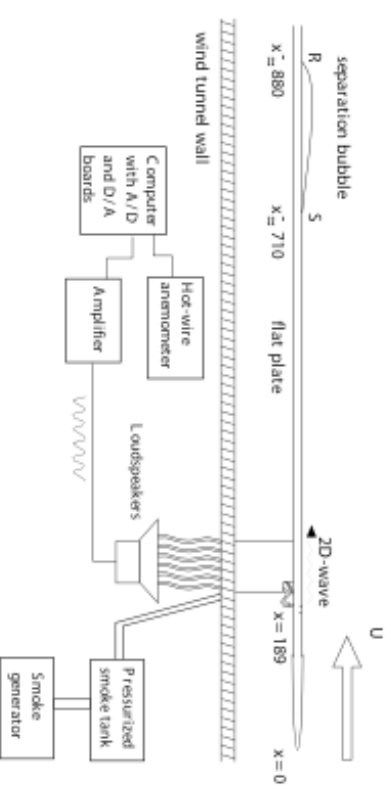
(C. Häggmark, Ph.D. thesis 2000, KTH)



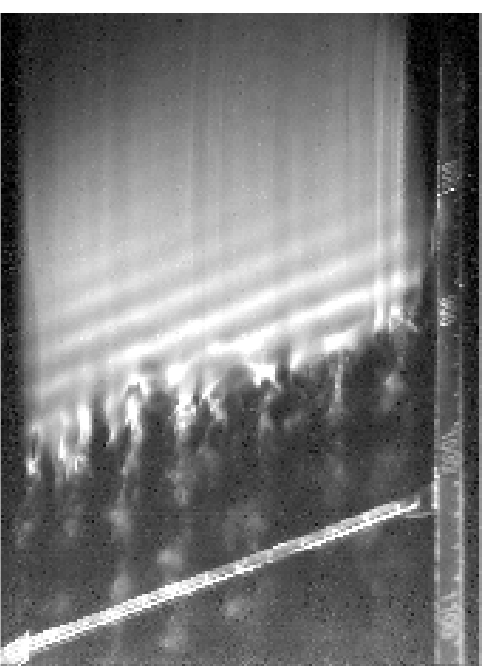
Boundary layer development



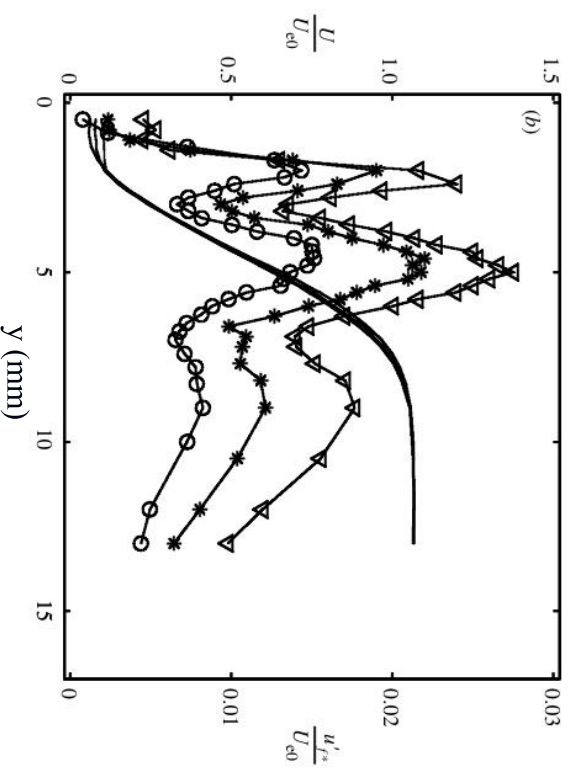
Experimental setup - laminar separation bubble



Flow visualization of forced instability waves



Disturbance velocity profiles



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Disturbance growth

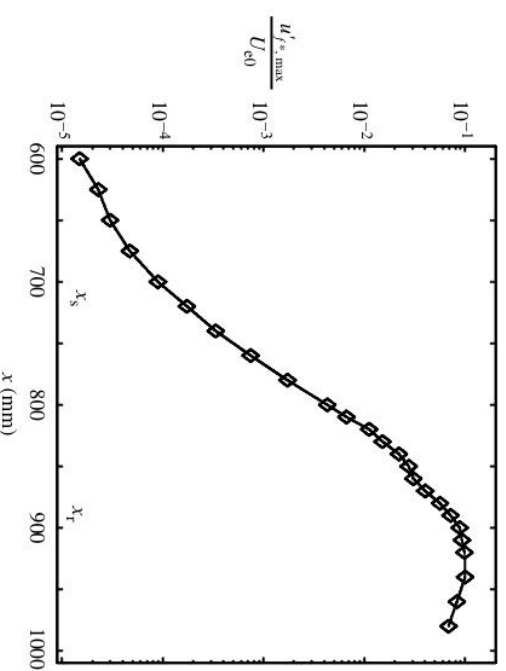
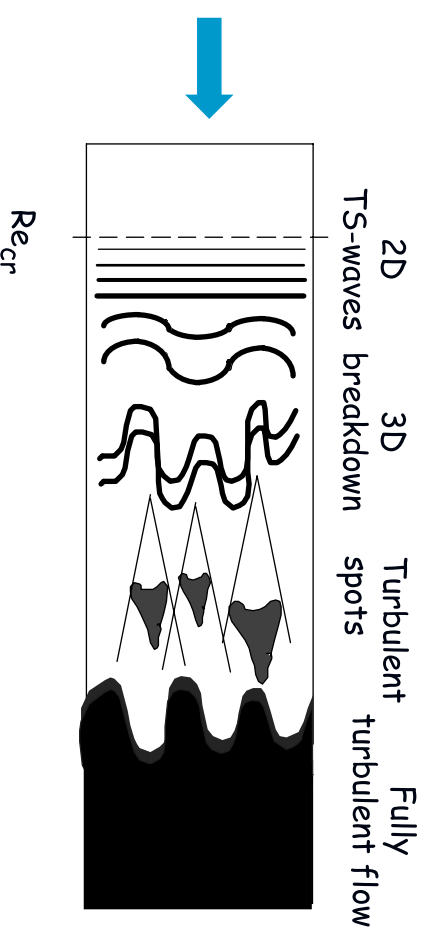
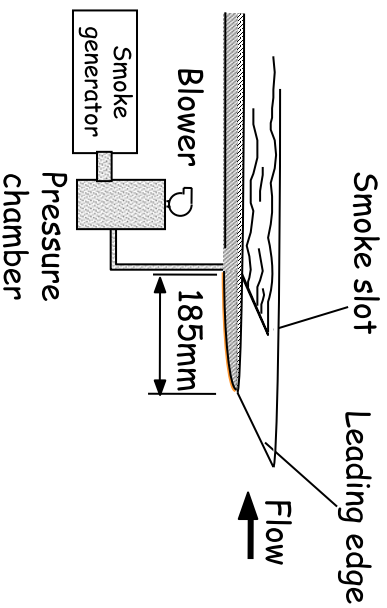


Figure 9. Amplification curve of forced two-dimensional instability wave in the separation bubble with $f^* = 83.3$ Hz.

Transition in boundary layer flows



Set-up for flow visualization



Transition prediction with the e^N -method

Idea: Assume that transition occurs where the linear eigenmodes (waves) have reached a certain amplification from the start of amplification. Describe this amplification as e^N .

Experiment shows that, in low disturbance environments, the onset and end of transition are about amplifications (N) of 8 and 11, respectively. Transition is often said to be at $N=9$.

Smoke visualization of TS-waves and K - breakdown

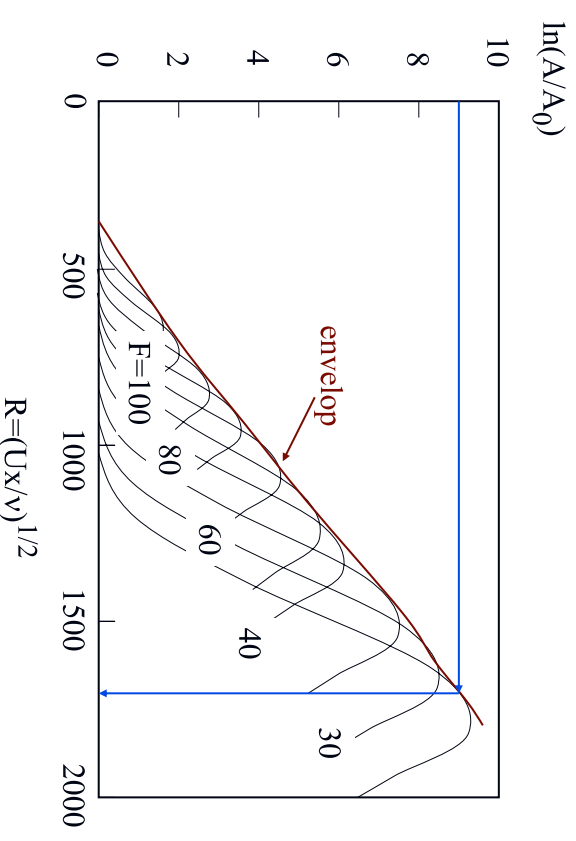
$$F = 168$$

$$U_{\infty} = 7.5 \text{ m/s}$$

$$A_{TN} = 0.8\%$$

QuickTime™ and a DV-FIL decompressor are needed to see this picture.

Illustration of the e^N -method



By pass transition with $Tu = 2.2\%$

(Matsubara & Alfredsson, 2001)



Flow direction

Lecture 1

1. Introduction to flow instability
2. Linear stability theory for boundary layer flows
 - Rayleigh stability theory (inviscid)
 - Orr-Sommerfeld theory (viscous)
3. Wind tunnels for transition research
4. Transition prediction
5. Transient growth
6. Free stream turbulence

Transition prediction at high turbulence levels

Observation: At high turbulence levels (Tu) transition occurs earlier corresponding to a smaller N -factor.

Idea: Correlate the N -factor with Tu !

Experiments show that:

$$N = 8.4 - 2.4 Tu$$

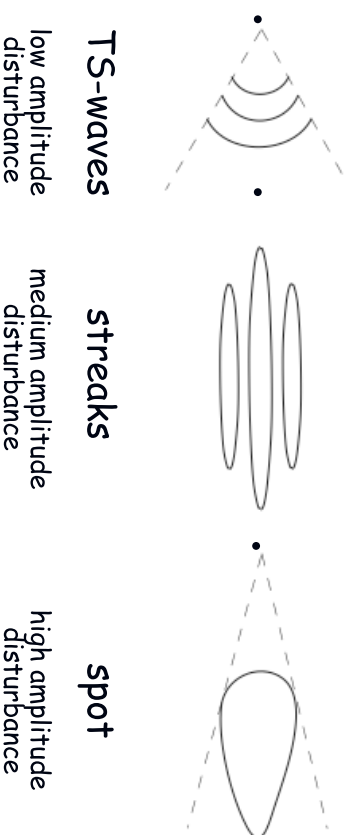
But of course it has nothing to do with the physics!

Failure of modal wave theory to predict the transitional Re

Flow case	Re	Re_{cr}	Re_{tr}
Blasius	$U_{\infty} \delta^* / \nu$	≈ 500	490
Channel	$U_{CL} b / \nu$	5772	1000
Pipe	$U_{CL} R / \nu$	∞	1760
Couette	$U_w b / \nu$	∞	360

Three different disturbances

(Grek, Kozlov & Ramazanov, 1984)



Two different instability mechanisms

Modal growth

- Tollmien-Schlichting waves
- Exponential growth $\sim e^x$
- Amplitude $\approx 1 - 2\%$
- 3D deformation
- Formation of Λ -structures (staggered or aligned)
- Breakdown

Non-modal growth

- Streaky structures
- Algebraic growth $\sim x^5$
- Amplitude $\approx 10\%$
- Localized high frequency wave packet
- Local breakdown to (turbulent spot)

Two different instability mechanisms

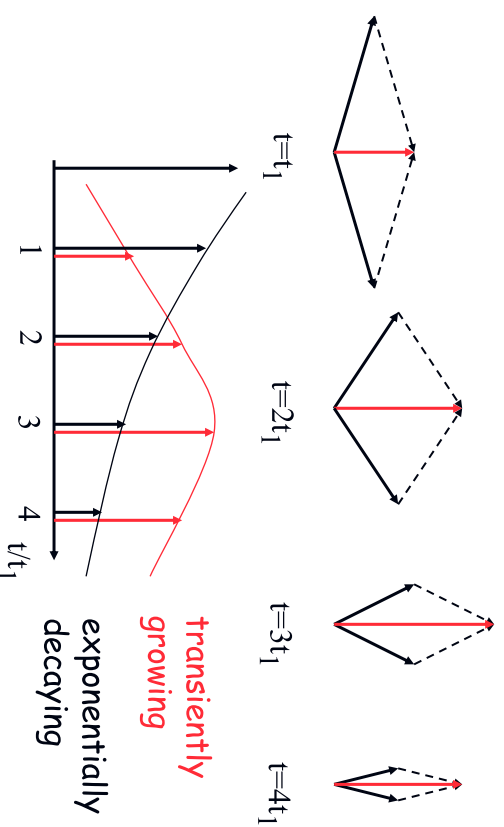
Modal growth

- **Theory - inviscid**
Rayleigh (1880)
- **Theory - viscous**
Orr (1907)
Sommerfeld (1908)
Tollmien (1928)
Schlichting (1933)
Orzag (1970)
Fasel & Konzelmann (1990)
- **Experiments**
Schubauer & Skramstad (1948)
Klebanoff et al. (1962)
Klingmann et al. (1993)

Non-modal growth

- **Theory - inviscid**
Ellingsen & Palm (1975)
Landahl (1980)
- **Theory - viscous**
Gustavsson (1991)
Schmid & Henningson (1992)
Butler & Farrell (1992)
Luchini (1996, 2000)
- **Experiments**
Grek, Kozlov & Ramazanov (1984)
Breuer & Haritonidis (1990)
Klingmann (1991)
Elofsson et al. (1998)

Growth of decaying waves ?!



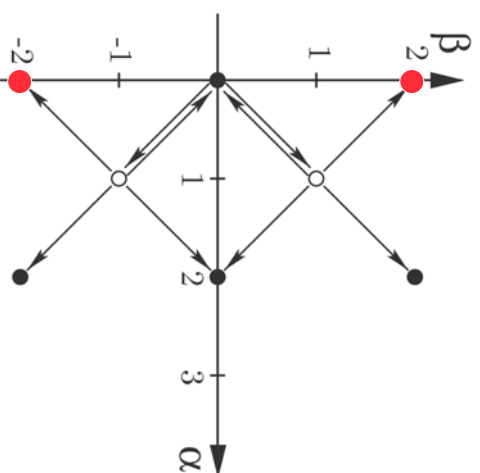
Main results from theoretical work on transient, algebraic or optimal growth

1. Elongated streaks most amplified ($\alpha = 0$)
2. Disturbance energy grows proportional to x
3. Amplitude maximum in centre of boundary layer (compare TS-waves)
4. Spanwise scale close to boundary layer thickness

Principle of oblique transition

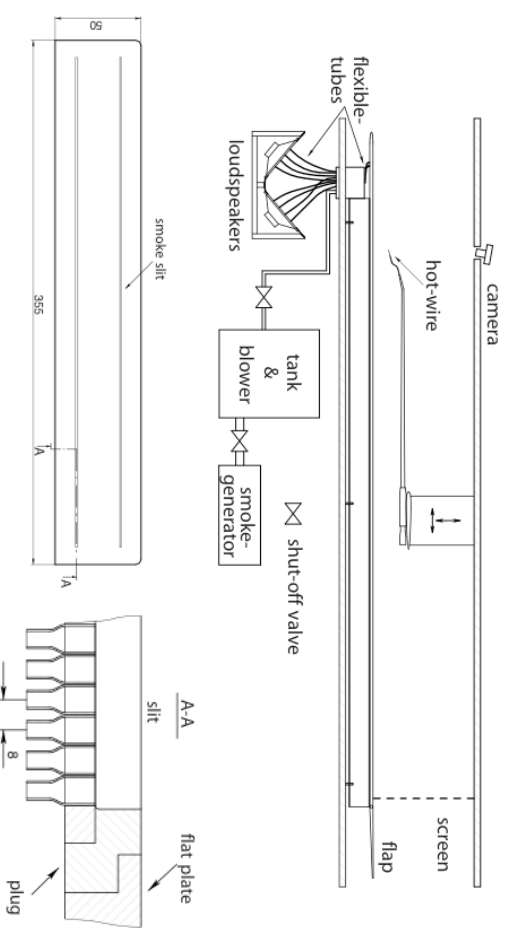
$$\text{Wave}_1 \sim \exp[i(\alpha x + \beta z - \omega t)]$$

$$\text{Wave}_2 \sim \exp[i(\alpha x - \beta z - \omega t)]$$



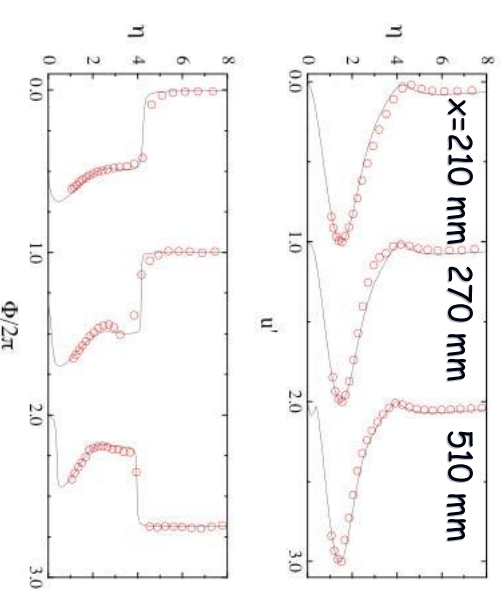
Experimental set-up

(P. Elofsson, PhD thesis 1998, KTH)



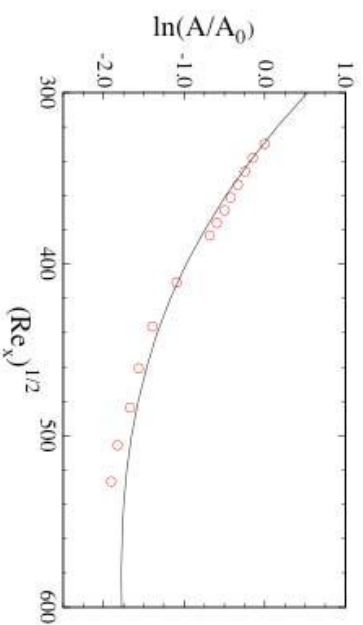
Eigenfunctions for one oblique wave - theory and experiments

$U=8.2 \text{ m/s}$
 $F=106$
 $\alpha/\beta \approx 1$



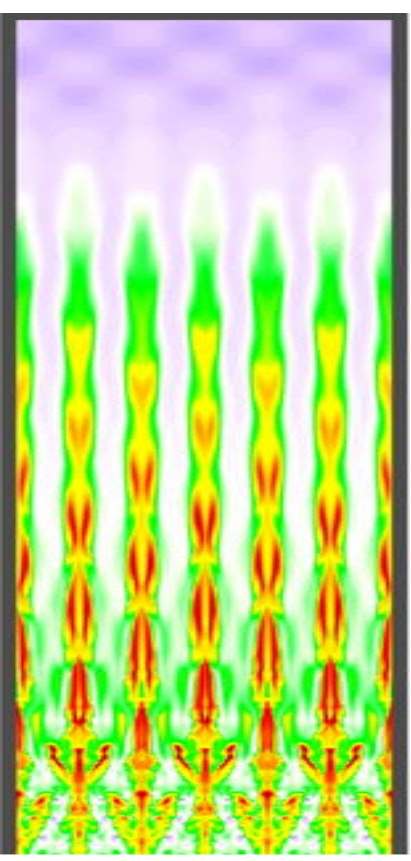
Decay of oblique wave

○ : exp.
— : theory

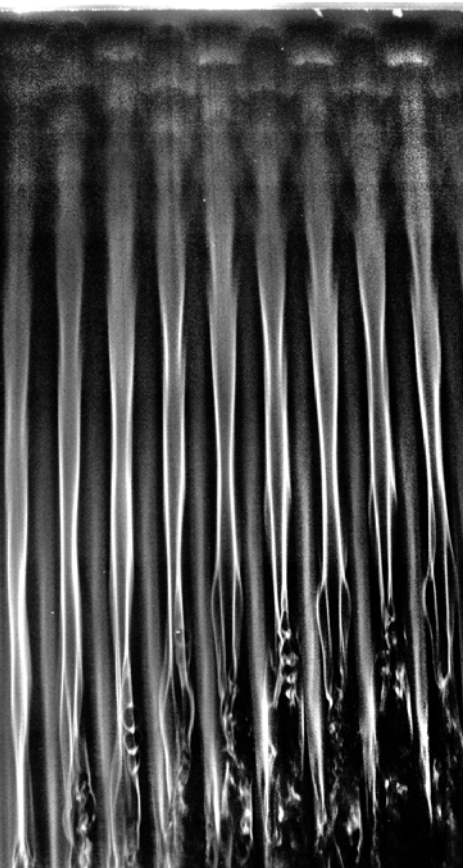


Oblique transition in BBL (DNS)

(S. Berlin, PhD thesis 1998, KTH)



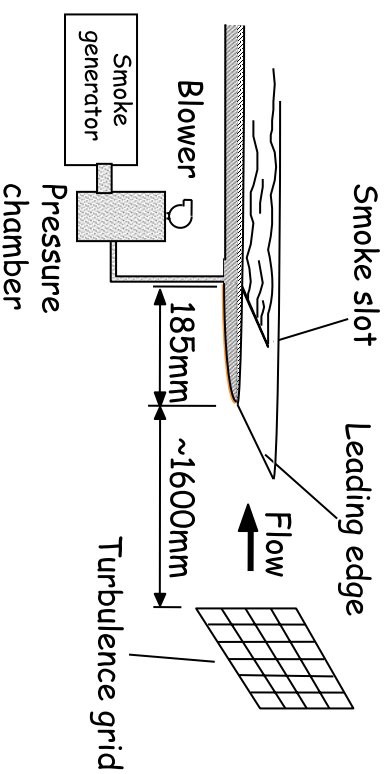
Oblique transition in BBL (experiment)



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Set-up for flow visualization of FST induced by-pass transition



By pass transition - the linear wave mode scenario is by-passed

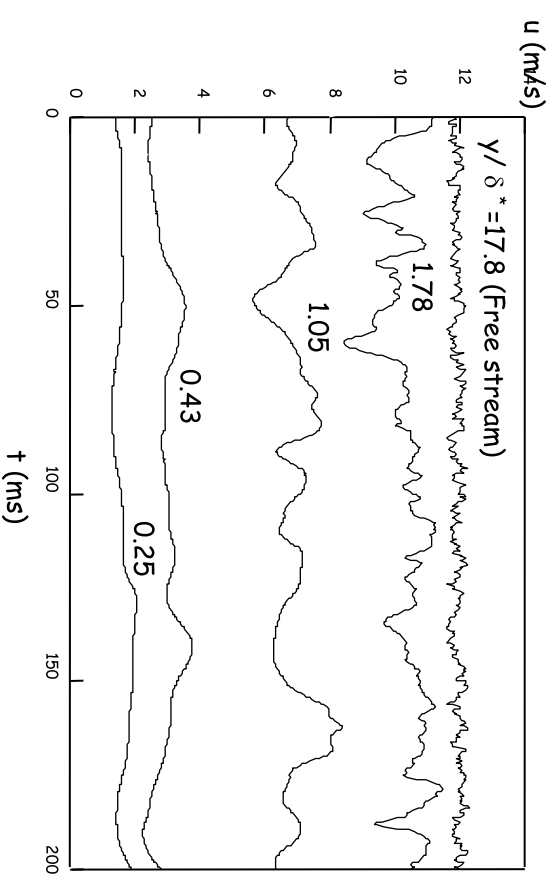
Streaky structures and turbulent spots

(Matsubara & Alfredsson, JFM 2001)



Flow direction

Streamwise velocity for varying y



FST induced transition- various stages

- Receptivity
- Streak formation and algebraic growth
- Secondary instability
- Turbulent spot formation
- Spot growth, merging and formation of a turbulent boundary layer

FST induced transition

Turbulent spot production

Figure 10.10: FST induced transition

$$U_{\infty} = 3 \text{ m/s}$$

$$Tu = 2.2\%$$

FST induced transition

Secondary instability

Figure 10.11: FST induced transition

$$U_{\infty} = 3 \text{ m/s}$$

$$Tu = 2.2\%$$

FST induced transition

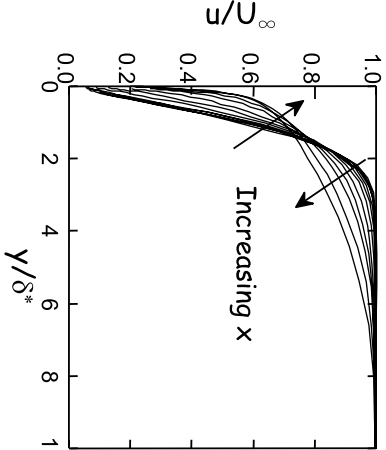
$$Tu = 5.3 \%$$

$$U_{\infty} = 2 \text{ m/s}$$

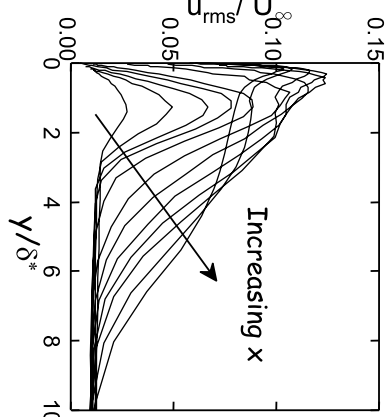
Figure 10.12: FST induced transition

Mean and fluctuating velocity development

Mean velocity



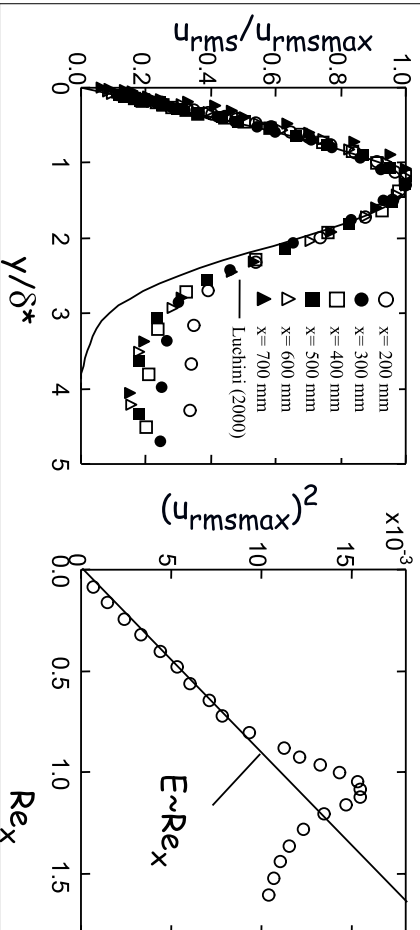
Velocity fluctuations



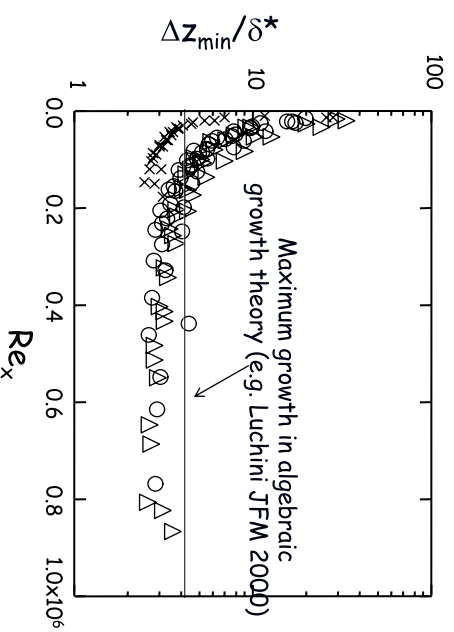
Amplitude development - experiments and spatial non-modal growth theory

Disturbance amplitude

Disturbance energy growth



Spanwise distance to correlation minimum

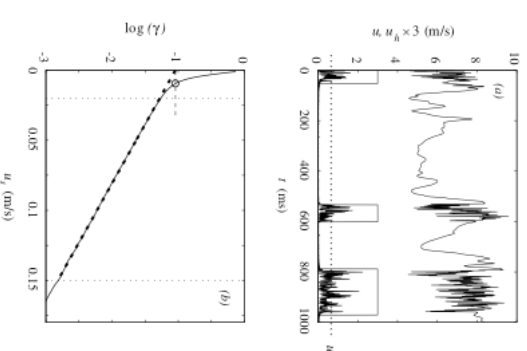


Intermittency detection scheme (Fransson et al, JFM 2004)

Intermittent velocity signal

Detection signal and detector function

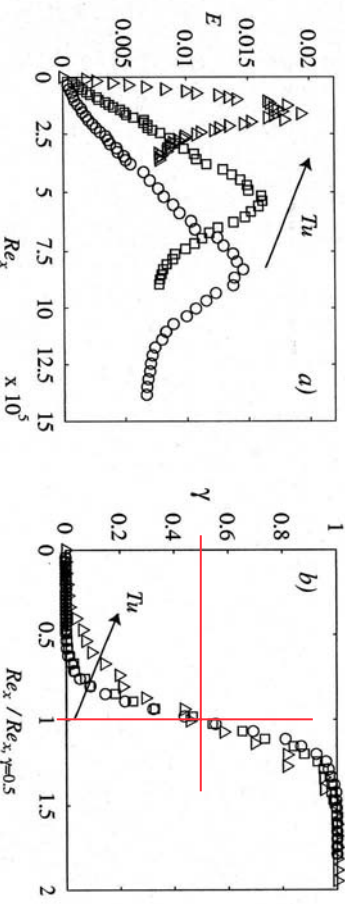
Method to determine threshold value



Energy growth and intermittency

Energy growth

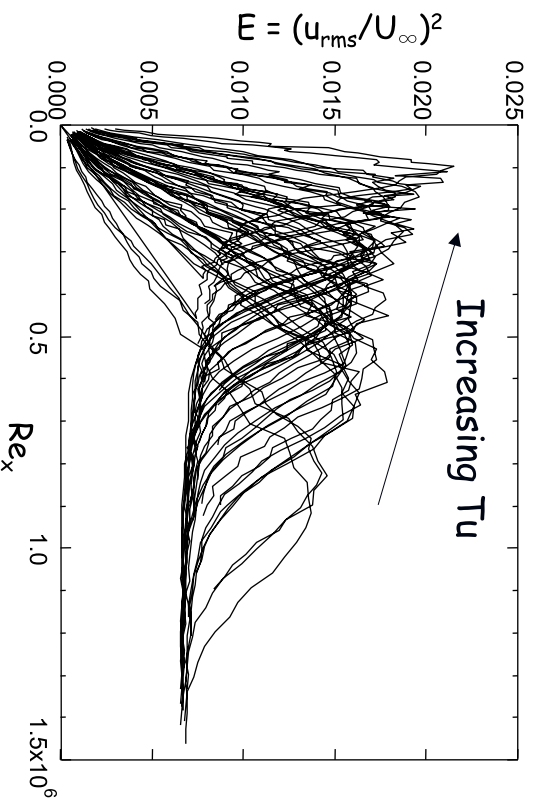
Intermittency function



Transition location where $\gamma=0.5$

Streamwise development of energy

$$y=1.4\delta^*$$



Transition prediction

(Andersson, Berggren & Henningson, Phys Fluids 1999)

Disturbances in boundary layer are assumed to be proportional to input (i.e. linear receptivity) and grow as

$$A(x) = C \cdot Tu \cdot Re_x^{1/2}$$

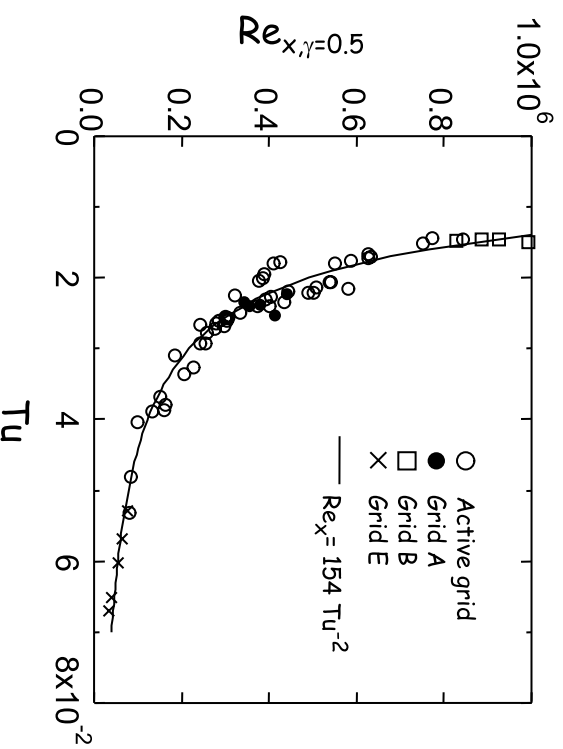
Assume that transition occurs at a certain amplitude (when secondary instability grows exponentially)

$$A_{tr} = C \cdot Tu \cdot Re_{tr}^{1/2}$$

Solve for Re_{tr}

$$Re_{tr} = (A_{tr}/C)^2 \cdot Tu^{-2}$$

Transition Reynolds Number



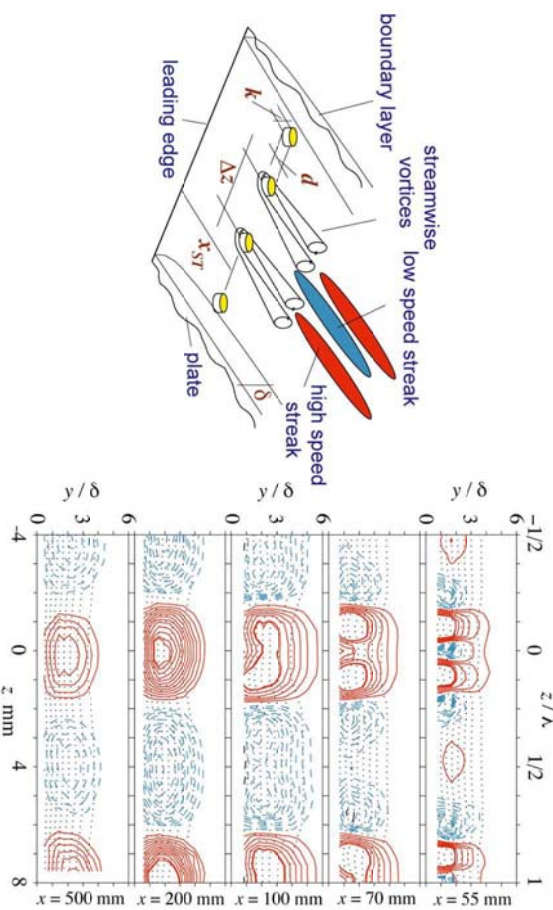
Passive control with roughness elements

Observation: TS-waves grow slower in FST affected boundary layer !?

Use of streaks to control TS-waves ?

Streak generation with roughness

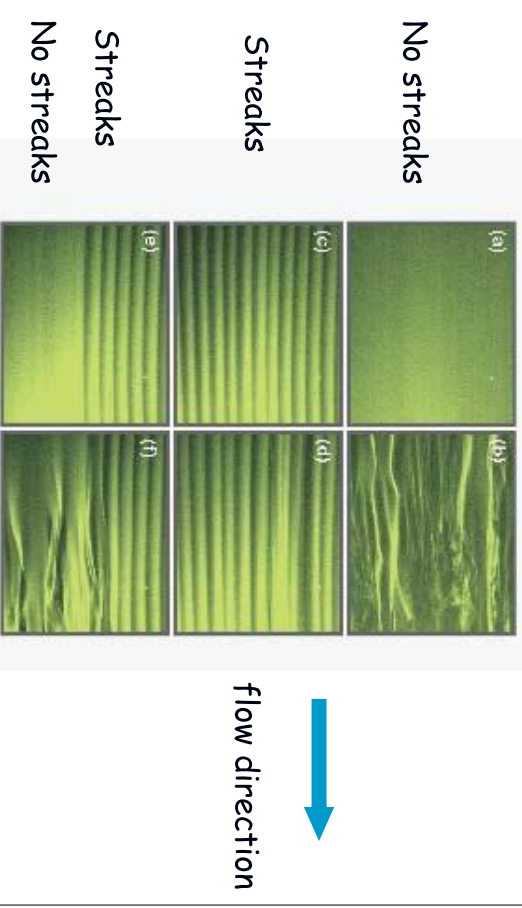
(Fransson et al. Phys Fluids 2004)



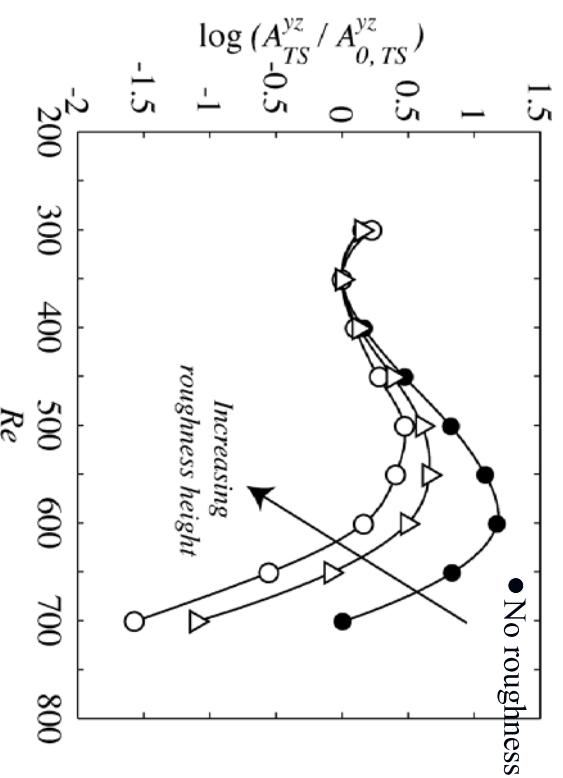
Flow visualization of streak control

(Fransson et al. 2006, Phys Rev Lett.)

without dist. with dist.



Decrease of TS-wave growth



Conclusion and questions

- Transition in boundary layers can follow different routes as we have a fairly good understanding of the routes but still not the transitional Reynolds number !
- We do not know how the transition route affect the turbulent boundary layer ?

This is important for both experiments and simulations of boundary layer flows !