Large eddy simulations: modelling and application

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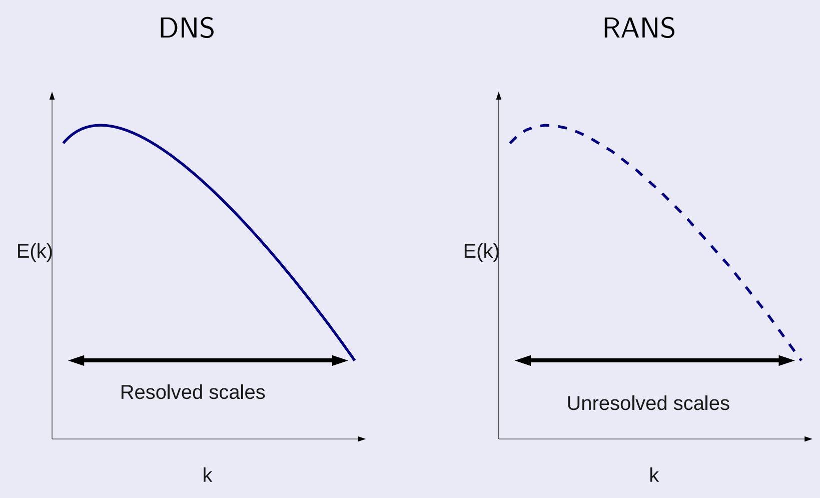
DNS and RANS



Simulations tools

Direct numerical simulations (DNS): all scales of motions are fully resolved

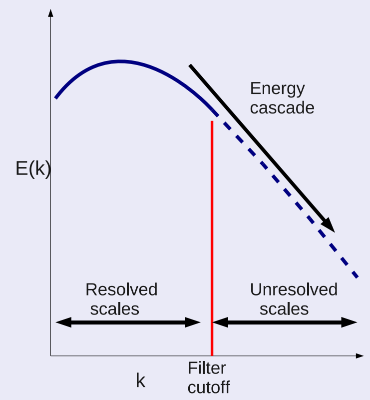
- Accurate but computationally too expensive at practically relevant Reynolds numbers

Reynolds averaged Navier-Stokes (RANS) models: the whole spectrum of turbulent scales are modelled

- Computationally relatively cheap
- Modelling uncertainties and inaccuracies
- No universally applicable models, each flow cases requires often a new model

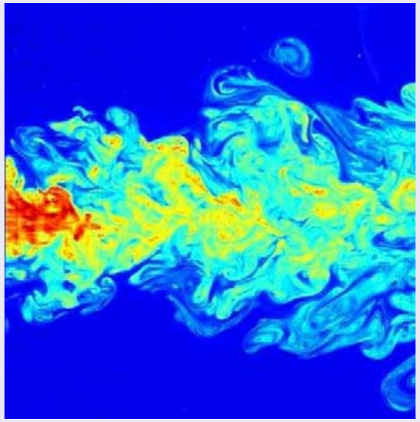
Large eddy simulation (LES)

- Large 3-D unsteady turbulent motions are directly computed
- Effects of small turbulent scales are modelled



Small, unresolved subgrid scales (SGS) dissipate the turbulent kinetic energy and must be accounted for by a model

Large eddy simulation (LES)



Small, unresolved subgrid scales (SGS)

- Less affected by boundary conditions and flow geometry, therefore more universal
- Easier and more accurate to model than the large turbulent scales

History of LES

Originally developed and applied in meteorology (Smagorinsky 1963, Lilly 1967). Still regularly used for simulations of atmospheric boundary layer processes,

First LES of fully developed turbulent channel flows

- Deardorff, J.W. 1970 A numerical study of three-dimensional turbulent channel flow at large Reynolds numbers. *J. Fluid Mech.* **41**, 453–480. Resolution 6720 grid points
- Schumann, U. 1975 Subgrid scale model for finite difference simulations of turbulent flows in plane channels and annuli. *J. Comp. Phys.* **18**, 376–404. Resolution up to 65 536 grid points
- Moin, P. & Kim, J. 1982 Numerical investigation of turbulent channel flows. *J. Fluid Mech.* **118**, 341–377. Resolution up to 516 096 grid points

Advantages/disadvantages LES

Compared to DNS

- Computationally cheaper

Compared to RANS models

- More accurate since large scale motions are directly computed (carry most of the the energy, most affected by boundary conditions and transport most of the momentum)
- Models for the small, subgrid scales are more universal than RANS models
- Suitable for flows with unsteadiness, massive separation, transition etc.
- Computationally more expensive, particularly for wall-bounded flows

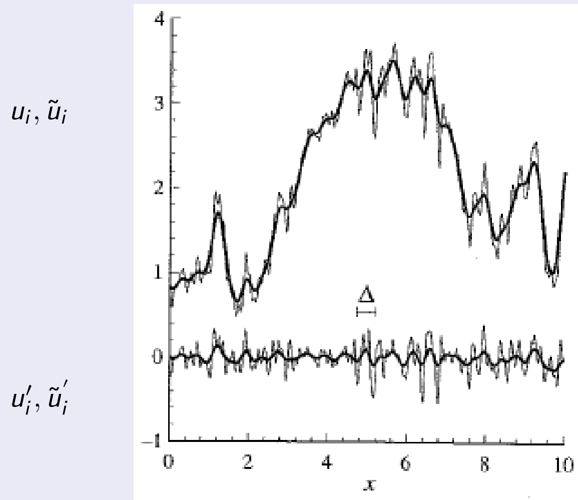
Outline

- Basics of LES (filtering, governing equations)
- Models for the unresolved subgrid scales
- Validation of LES
- Application of LES to wall-bounded flows

LES: basic steps

- Use a filter operation to decompose the velocity into a filtered or resolved and a subgrid-scale (SGS) component
- Derive the governing equations from the filtered Navier-Stokes equations
- Model the unclosed terms in the governing equations
- Simulate the evolution of the large-scale motions by numerically solving the filtered model equations. Typically more than 80% of the turbulent kinetic energy is resolved

Filtering



Filtering the velocity

Filter operation (low-pass)

$$\tilde{u}_i(\mathbf{x}, t) = \int G(\mathbf{r}, \mathbf{x}) u_i(\mathbf{x} - \mathbf{r}, t) \mathrm{d}\mathbf{r}$$

$$\int G(\mathbf{r}, \mathbf{x}) \mathrm{d}\mathbf{r} = 1$$

Filtering and decomposition of the velocity

$$u_i(\mathbf{x}, t) = \underbrace{\tilde{u}_i(\mathbf{x}, t)}_{filtered} + \underbrace{u_i'(\mathbf{x}, t)}_{residual}$$

Note that in general

$$\tilde{u}_i'(\mathbf{x}, t) \neq 0$$

Filter properties

Spectral space: transfer function

$$\hat{G}(k) = \int G(r) e^{-ikr} \mathrm{d}r \quad , \quad \hat{u}(k) = \frac{1}{2\pi} \int u(r) e^{-ikr} \mathrm{d}r$$

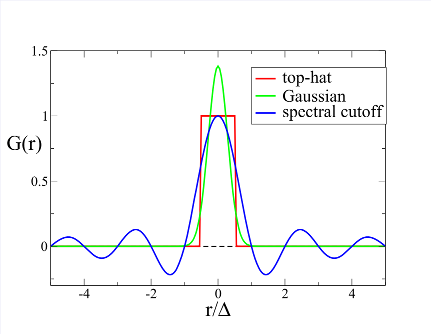
$$\Rightarrow \hat{\tilde{u}}(k) = \hat{G}(k) \hat{u}(k)$$

Some common filters (Δ is the filter width)

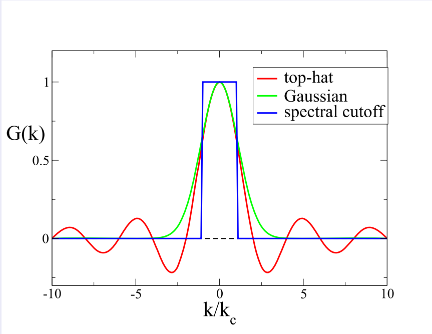
name	filter function $G(r)$	transfer function $\hat{G}(k)$
Top-hat	$\frac{1}{\Delta} H(\frac{1}{2}\Delta - \lvert r \rvert)$	$\frac{\sin(\frac{1}{2} k\Delta)}{\frac{1}{2} k\Delta}$
Gaussian	$\left(\frac{6}{\pi\Delta^2}\right)^{1/2} \exp\left(-\frac{6r^2}{\Delta^2}\right)$	$\exp\left(-\frac{k^2\Delta^2}{24}\right)$
Spectral cutoff	$\frac{\sin(\pi r/\Delta)}{\pi r}$	$H(k_c - \lvert k \rvert) \ , \ k_c = \pi/\Delta$

Filter properties

physical space



spectral space



Filtered governing equations

We assume that filtering and differentiation commute

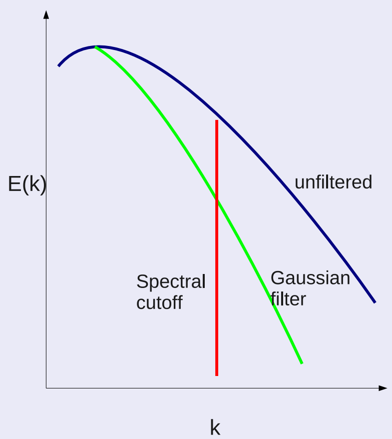
Filtered continuity equation

$$\widetilde{\left(\frac{\partial u_i}{\partial x_i}\right)} = \frac{\partial \tilde{u}_i}{\partial x_i} = 0$$

Then also

$$\frac{\partial u_i'}{\partial x_i} = 0$$

Filtered energy spectrum



Filtered governing equations

Filtered momentum equation

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \widetilde{u_i u_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j^2}$$

Residual or subgrid-scale (SGS) stress tensor

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

SGS kinetic energy

$$K_{SGS} = \tfrac{1}{2}\tau_{ii}$$

Anisotropic SGS stress tensor

$$\tau_{ij}^a = \tau_{ij} - \tfrac{2}{3}K_{SGS}\delta_{ij}$$

Modified pressure

$$\tilde{p} \equiv \tilde{p} + \tfrac{2}{3}K_{SGS}$$

Filtered governing equations

Filtered continuity equation

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0$$

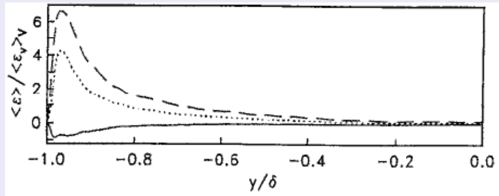
Filtered momentum equation

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial \tau^a_{ij}}{\partial x_j} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j^2}$$

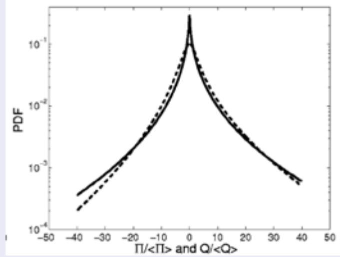
$\tilde{u}_i(\mathbf{x}, t)$, $\tilde{p}(\mathbf{x}, t)$ and $\tau^a_{ij}(\mathbf{x}, t)$ are random, three-dimensional and unsteady, even in homogeneous stationary turbulent flows

SGS dissipation $\varepsilon_{SGS} = -\tau^a_{ij}\tilde{S}_{ij}$ Backscatter

channel flow



homogeneous shear flow



—— plane average ε_{SGS}

..... plane average backscatter

On average the SGS dissipation ε_{SGS} is positive but locally ε_{SGS} is regularly negative implying that energy is transferred from the subgrid scales to the resolved motions

Physical meaning of the SGS stresses

Kinetic energy

$$\tilde{K} = \tfrac{1}{2}\widetilde{u_i u_i} = \tfrac{1}{2}\tilde{u}_i\tilde{u}_i + \tfrac{1}{2}\tau_{ij} = K_R + K_{SGS}$$

Conservation equation for $K_R = \frac{1}{2}\tilde{u}_i\tilde{u}_i$

$$\frac{\partial K_R}{\partial t} + \tilde{u}_j \frac{\partial K_R}{\partial x_j} - \frac{\partial}{\partial x_j}\left[\tilde{u}_i\left(2\nu\tilde{S}_{ij} - \tau^a_{ij} - \frac{\tilde{p}}{\rho}\delta_{ij}\right)\right] = -\varepsilon_R - \varepsilon_{SGS}$$

where $\varepsilon_R = 2\nu\tilde{S}_{ij}\tilde{S}_{ij}$ (visc. diss. by resolved field) and $\varepsilon_{SGS} = -\tau^a_{ij}\tilde{S}_{ij}$ (transfer rate of energy from resolved to SGS scales or SGS dissipation) and $\tilde{S}_{ij} = \frac{1}{2}(\partial\tilde{u}_i/\partial x_j + \partial\tilde{u}_j/\partial x_i)$

If $\Delta \to \eta$ then $\langle\varepsilon_R\rangle \to \langle\varepsilon\rangle$ and $\varepsilon_{SGS} \to 0$

If $\Delta \gg \eta$ and high Re then $\varepsilon_R \approx 0$ and $\langle\varepsilon_{SGS}\rangle \approx \langle\varepsilon\rangle$

LES modelling approaches

Explicit LES approach

- A model for the SGS stress tensor τ_{ij} is introduced in the governing equations.

Implicit LES approach

- No SGS model is used. Instead, the numerical scheme takes care of the SGS influence on the resolved motions.

Eddy viscosity assumption

Many SGS models use the eddy-viscosity assumption

$$\tau^a_{ij} = -2\nu_T \tilde{S}_{ij} \qquad , \qquad \tilde{S}_{ij} = \tfrac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

where $\nu_T(\mathbf{x}, t)$ is the SGS eddy viscosity, varying in space and time.
Mixing-length hypothesis using dimensional analysis

$$\nu_T \propto lv$$

where l is a length scale and v a velocity scale.

The most active SGS scales are close to the filter cutoff $\Rightarrow$ Δ is a natural length scale and $K_{SGS}^{1/2}$ is natural velocity scale

Two possibilities to compute K_{SGS}

- Solve transport equations for K_{SGS}
- Algebraic model for K_{SGS} using the equilibrium assumption $\Rightarrow$ the subgrid scales respond instantaneously to the resolved scales since they have a shorter time scale

Smagorinsky model: application to viscous Burgers equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

Filtered equations

$$\frac{\partial \tilde{u}}{\partial t} + \tilde{u}\frac{\partial \tilde{u}}{\partial x} = \nu\frac{\partial^2 \tilde{u}}{\partial x^2} - \tfrac{1}{2}\frac{\partial \tau}{\partial x}$$

where $\tau = \widetilde{uu} - \tilde{u}\tilde{u}$.

The Smagorinsky SGS model reads

$$\tau = -2(C_s\Delta)^2\tilde{\mathcal{S}}\tilde{S}_{ij} = -2(C_s\Delta)^2\left|\frac{\partial \tilde{u}}{\partial x}\right|\frac{\partial \tilde{u}}{\partial x}$$

The modelled viscous Burgers equations becomes then

$$\frac{\partial \tilde{u}}{\partial t} + \tilde{u}\frac{\partial \tilde{u}}{\partial x} = \left[\nu + (C_s\Delta)^2\left|\frac{\partial \tilde{u}}{\partial x}\right|\right]\frac{\partial^2 \tilde{u}}{\partial x^2}$$

Thus the Smagorinsky model acts as an additional positive viscous term which is large at steep gradients

Smagorinsky model (Smagorinsky 1963)

Eddy-viscosity assumption

$$\tau^a_{ij} = -2\nu_T \tilde{S}_{ij} \qquad , \qquad \tilde{S}_{ij} = \tfrac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

Mixing-length hypothesis

$$\nu_T = lK_{SGS}^{1/2}$$

Assume $l = C_s\Delta$ and use the equilibrium assumption

$$K_{SGS}^{1/2} = l\tilde{\mathcal{S}} \qquad , \qquad \tilde{\mathcal{S}} = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}$$

Then we get the Smagorinsky model

$$\nu_T = (C_s\Delta)^2\tilde{\mathcal{S}}$$

The SGS dissipation is always positive since

$$\varepsilon_{SGS} = -\tau^a_{ij}\tilde{S}_{ij} = (C_s\Delta)^2\tilde{\mathcal{S}}^3$$

Smagorinsky model: estimating the Smagorinsky coefficient C_s

Assume high Reynolds number turbulence and Δ in the inertial subrange, then

$$\varepsilon = \langle\varepsilon_{SGS}\rangle = l^2\langle\tilde{\mathcal{S}}^3\rangle \simeq l^2\langle\tilde{\mathcal{S}}^2\rangle^{3/2}$$

and with the Kolmogorov spectrum $E(k) = c_K\varepsilon^{2/3}k^{-5/3}$ we get

$$\langle\tilde{\mathcal{S}}^2\rangle = 2\langle\tilde{S}_{ij}\tilde{S}_{ij}\rangle = 2\int_0^{k_c=\pi/\Delta} k^2E(k)dk = \tfrac{3}{2}c_K\varepsilon^{2/3}(\pi/\Delta)^{4/3}$$

This gives with $l = (C_s\Delta)^2$ and $c_K = 1.6$

$$C_s = \frac{1}{\pi}\left(\frac{2}{3c_K}\right)^{3/4} = 0.17$$

But in practice often $C_s = 0.1$ is used because the SGS model is too dissipative with $C_s = 0.17$.

Smagorinsky model: application to wall-bounded flows

The Smagorinsky model does not behave correctly near the wall since

$$\nu_T = (C_s\Delta)^2\tilde{\mathcal{S}} \simeq (C_s\Delta)^2 \left|\frac{\partial\langle u\rangle}{\partial y}\right|$$

if $y \rightarrow 0$.
Therefore, often the van Driest damping is used near the wall

$$l = C_s\Delta[1 - \exp(y^+/A^+)]$$

with $A^+ = 25$.

Advantages and disadvantages of the Smagorinsky model

- Simple
- Numerically robust and stable
- C_s is not flow and situation independent
- There is no backscatter of energy since $\varepsilon_{SGS} > 0$

Dynamic Smagorinsky model

Germano identity

$$L_{ij} \equiv T_{ij} - \hat{\tau}_{ij} = \widehat{\tilde{u}_i\tilde{u}_j} - \hat{\tilde{u}}_i\hat{\tilde{u}}_j$$

L_{ij} is known in terms of $\tilde{u}_i$, while T_{ij} and τ_{ij} are not

Using the Smagorinsky model (and $c \equiv C_s^2$) gives

$$\tau_{ij}^a = \tau_{ij} - \tfrac{1}{3}\tau_{kk}\delta_{ij} = -2c\Delta^2\tilde{\mathcal{S}}\tilde{S}_{ij}$$

$$T_{ij}^a = T_{ij} - \tfrac{1}{3}T_{kk}\delta_{ij} = -2c\hat{\Delta}^2\hat{\tilde{\mathcal{S}}}\hat{\tilde{S}}_{ij}$$

Assuming c is the same on both grid and $M_{ij} = 2\Delta^2\widehat{\tilde{\mathcal{S}}\tilde{S}_{ij}} - 2\hat{\Delta}^2\hat{\tilde{\mathcal{S}}}\hat{\tilde{S}}_{ij}$ gives

$$L_{ij} - \tfrac{1}{3}L_{kk}\delta_{ij} = cM_{ij}$$

This gives five independent equations at every point and time (M_{ij} is trace free) and is thus an over-determined system. A common solution is to minimize the mean-square error leading to

$$c = M_{ij}L_{ij}/M_{kl}M_{kl}$$

Dynamic Smagorinsky model (Germano et al. 1991)

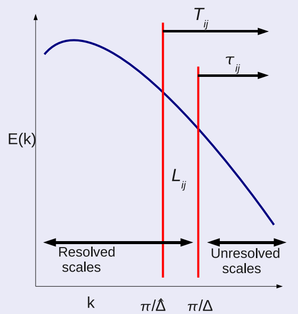
Basic concept: determining an appropriate value for the Smagorinsky coefficient C_s from the smallest resolved scales

Apply a test filter with width $\hat{\Delta}$ (typically $\hat{\Delta} = 2\Delta$)

$$\hat{u}_i(\mathbf{x},t) = \int \hat{G}(\mathbf{r},\mathbf{x})u_i(\mathbf{x}-\mathbf{r},t)\mathrm{d}\mathbf{r} \quad , \quad \hat{\tilde{u}}_i(\mathbf{x},t) = \int \hat{G}(\mathbf{r},\mathbf{x})\tilde{u}_i(\mathbf{x}-\mathbf{r},t)\mathrm{d}\mathbf{r}$$

Subgrid scale stresses (Δ level): $\tau_{ij} = \widetilde{u_iu_j} - \tilde{u}_i\tilde{u}_j \approx \tau_{ij}^{mod}(C_s,\Delta,\tilde{u}_i)$

Sub-test scale stresses ($\hat{\Delta}$ level): $T_{ij} = \widehat{\widetilde{u_iu_j}} - \hat{\tilde{u}}_i\hat{\tilde{u}}_j \approx \tau_{ij}^{mod}(C_s,\hat{\Delta},\hat{\tilde{u}}_i)$



Dynamic Smagorinsky model

- The coefficient c is determined by the model itself and needs not to be specified.
- The coefficient c can be negative, therefore the SGS model can provide backscatter of energy
- In practice c given by $c = M_{ij}L_{ij}/M_{kl}M_{kl}$ shows large fluctuations, causing numerical problems. A common approach is to average c along a homogeneous direction, for example, along the spanwise direction in boundary layer and channel flow, but there are also other approaches
- In order to avoid numerical instabilities $\nu + \nu_T \geq 0$ is often imposed (no backscatter)
- In wall bounded flows $c \rightarrow 0$ if $y \rightarrow 0$. This results in the correct near-wall behaviour of ν_T and thus no wall-damping is needed, in contrast to the normal Smagorinsky model.

Mixed models (Bardina et al. 1980)

The modelled SGS stress often correlates poorly with the observed one, for example, the eddy viscosity assumption gives $\tau^a_{ij} \propto \tilde{S}_{ij}$ which is often a poor approximation according to experiments and DNS.

The scale similarity model assumes that the SGS stress and the resolved stress at scales just above Δ are similar, i.e.

$$\tau^{sim}_{ij} = C_{sim}(\overline{\tilde{u}_i \tilde{u}_j} - \overline{\tilde{u}}_i \overline{\tilde{u}}_j)$$

where the overbar denotes a second filter with width $\alpha\Delta$ ($\alpha > 1$).

However, this model is not dissipative enough, therefor it is often combined with a (dynamic) Smagorinsky model. This gives the mixed model

$$\tau^{mixed}_{ij} = C_{sim}(\overline{\tilde{u}_i \tilde{u}_j} - \overline{\tilde{u}}_i \overline{\tilde{u}}_j) - 2(C_s\Delta)^2 \tilde{S}\tilde{S}_{ij}$$

This model correlates much better with the observed SGS stresses.

Approximate deconvolution models

The nonlinear term can now be computed by using the approximate velocity and gives

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \widetilde{u^*_i u^*_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial \tau^a_{ij}}{\partial x_j} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j^2}$$

- The deconvolution steps can basically be used to reduce aliasing/numerical errors. u^*_i givis only a better approximation of the unfiltered velocity up to the cutoff wave number k_c of the filter
- In spectral simulations the deconvolution procedure is in fact not necessary but with other numerical methods it gives better results
- A model is still needed for the energy transfer from the resolved to the subgrid scales

Approximate deconvolution models (Stolz et al. 2001)

Basic concept: reconstructing the unfiltered velocity field from the filtered field by defiltering

If the filter operator G is invertible the unfiltered velocity u_i can be obtained from the filtered velocity $\tilde{u}_i$ through

$$u_i = G^{-1} * \tilde{u}_i$$

Usually G is not invertible but an approximation Q_N can be obtained by

$$Q_N = \sum_{\alpha=0}^{N} (I - G)^\alpha \approx G^{-1}$$

Then an approximation of the unfiltered velocity u^*_i is obtained from

$$u^*_i = Q_N * \tilde{u}_i = 3\tilde{u}_i - 3\tilde{\tilde{u}}_i + \tilde{\tilde{\tilde{u}}}_i + \ldots$$

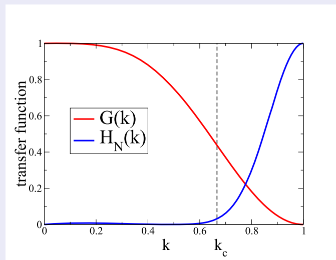
Thus u^*_i is computed by repeated filtering of the resolved velocity $\tilde{u}_i$.

Approximate deconvolution models

To model energy transfer from resolved to subgrid scales a relaxation term is used

$$\chi(I - Q_N * G) * \tilde{u}_i \equiv \chi H_N * \tilde{u}_i$$

where H_N is a high-pass filter. It only affects the smallest resolved scales.



The final modelled momentum equations read

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \widetilde{u^*_i u^*_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j^2} - \chi H_N * \tilde{u}_i$$

Multiscale models (Hughes et al. 2000)

Motivation

- Eddy viscosity SGS models, for example Smagorinsky models, affect a large range of resolved scales. But observations show that unresolved subgrid scales mainly interact with the small resolved scales, the SGS influence on large scales is small

Basic approach of multiscale models

- Split the resolved scales into a large- and small-scale part
- Model the influence of the subgrid scales only in the equations of the small-scale part, i.e. the large-scale motions are not directly affected by the SGS model

Multiscale models using filters

- Scale separation is easily achieved in spectral space
- An analogous multiscale approach can be obtained by using (discrete) filters (Vreman 2003)

Basic approach: use a low-pass filter G to split the resolved velocity $\tilde{u}_i$ into a low-pass filtered (large-scale) part

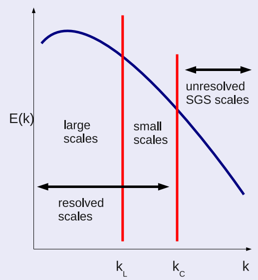
$$\tilde{u}_i^L = G * \tilde{u}_i$$

and a high-pass filtered (small-scale) part by subtracting the low-pass filtered velocity from the resolved one

$$\tilde{u}_i^S = \tilde{u}_i - \tilde{u}_i^L = \tilde{u}_i - G * \tilde{u}_i \equiv H * \tilde{u}_i$$

Here G and the high-pass filter H can be discrete filters

Multiscale models



If $\tilde{v}(\mathbf{k})_i \equiv \mathcal{F}\{\tilde{u}_i\}$ is the Fourier transform of $\tilde{u}_i$ then the modelled equations according to the multiscale method in spectral space read

$$\text{large scales: } \frac{\partial \tilde{v}_i}{\partial t} + ik_j\mathcal{F}\{\tilde{u}_i\tilde{u}_j\} = -ik_i\tilde{p} - \nu k_i^2\tilde{v}_i \qquad |\mathbf{k}| < k_L$$

$$\text{small scales: } \frac{\partial \tilde{v}_i}{\partial t} + ik_j\mathcal{F}\{\tilde{u}_i\tilde{u}_j\} = -ik_i\tilde{p} - \nu k_i^2\tilde{v}_i - ik_j\mathcal{F}\{\tau_{ij}\} \qquad k_L < |\mathbf{k}| < k_C$$

These two equations are coupled through the nonlinear term, but the SGS model is only included in the equation for the small-scale motions

Multiscale models using (discrete) filters

The modelled equations for the resolved scales read

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j\frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial \tau_{ij}^a}{\partial x_j} + \nu\frac{\partial^2 \tilde{u}_i}{\partial x_j^2}$$

According to the multiscale approach τ_{ij}^a is now modelled in terms of $\tilde{u}_i^S$.
Example: the multiscale variant of the Smagorinsky model reads

$$\tau_{ij}^a = -2\nu_T\tilde{S}_{ij}^S = -\nu_T\left(\frac{\partial \tilde{u}_i^S}{\partial x_j} + \frac{\partial \tilde{u}_j^S}{\partial x_i}\right)$$

with two options for ν_T

$$\nu_T = (C_s\Delta)^2(2\tilde{S}_{ij}\tilde{S}_{ij})^{1/2}$$

$$\nu_T = (C_s\Delta)^2(2\tilde{S}_{ij}^S\tilde{S}_{ij}^S)^{1/2}$$

These multiscale models act mainly on the small-scale (high wavenumber) part of the resolved field. The SGS stress contribution goes to zero for $y \rightarrow 0$ in wall-bounded flows, but some use still a damping for the coefficient C_s in order to get the correct near-wall behaviour.

Explicit algebraic SGS models (Marstorp et al. 2009)

Basic approach and idea

- Avoid the eddy viscosity assumption
- Develop instead subgrid scale stress models based on the SGS stress transport equations using the same method as for explicit algebraic Reynolds stress models

Explicit algebraic SGS stress model

$$\tau_{ij} = K_{SGS}\left(\frac{2}{3}\delta_{ij} + \beta_1\tau^*\tilde{S}_{ij} + \beta_4\tau^{*2}(\tilde{S}_{ik}\tilde{\Omega}_{kj} - \tilde{\Omega}_{ik}\tilde{S}_{kj})\right)$$

isotropic - eddy visc. - nonlinear parts

where $\tilde{\Omega}_{kj}$ is the rotation rate tensor

This model gives an improved prediction of the subgrid scale anisotropy

Validating LES: a priori and a posteriori tests

A posteriori tests of a SGS model τ_{ij}^{model}

Compare results of a large eddy simulation with that SGS model with experimental or DNS data

- LES results of for example the mean velocity, Reynolds stress profiles, spectra can be compared with DNS or experimental data
- The DNS or experimental data need to be filtered for a fair comparison
- A posteriori tests do not give much insight into the detailed performance of a model and the reasons that they fail or work

Note that that LES never can reproduce completely a filtered DNS, realization by realization. Only conditional/mean statistics of a DNS can be reproduced (Pope 2000)

Implicit large-eddy simulation

- Problem: in finite volume and finite difference codes the numerical error can be of the order as the SGS stress, therefore the combination of SGS stress models and numerical errors can lead to poor predictions
- Solution: instead of applying a SGS stress model use the truncation errors of the numerical scheme to model the SGS stress contribution

Possible ILES approaches

- Use monotonic (MILES) or non-oscillatory schemes to compute the nonlinear terms in the momentum equations. But this approach seems not well founded
- Employ the approximate deconvolution method to compute a truncation error that acts as a SGS model

Validating LES: a priori and a posteriori tests

A priori tests of a SGS model τ_{ij}^{model}

Compare directly $\tau_{ij}^{model}(\mathbf{x}, t)$ with the actual $\tau_{ij}(\mathbf{x}, t)$ using DNS or experimental data

- The actual or real $\tau_{ij}(\mathbf{x}, t)$ is computed using its definition $\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j$
- The modelled SGS stress $\tau_{ij}^{model}(\mathbf{x}, t)$ is computed by using the filtered data
- Requires high resolution data (for example, from DNS and 2-D or 3-D PIV measurements) since the subgrid scales must be resolved
- Also for instance the modelled SGS dissipation can be compared to the real one
- Gives more detailed information on the validity and accuracy of a SGS model

But a good agreement between $\tau_{ij}^{model}(\mathbf{x}, t)$ and the actual $\tau_{ij}(\mathbf{x}, t)$ does not guarantee that a LES with that model gives accurate a posteriori predictions of the mean velocity and Reynolds stresses

$\Rightarrow$ An posteriori test are the ultimate required test of a SGS model

Application of LES to wall-bounded flows

Previous applications of LES

- Fully developed turbulent channel, pipe and duct flow
- Spatially developing turbulent boundary layer flow
- Atmospheric boundary layers
- Transition in wall-bounded flows
- Ekman layers
- Separation in wall-bounded flows
- Flows over rough walls

Wall-resolved LES

Resolution requirements

Most of the energy of the near-wall structures and the near-wall streaks must be resolved, no wall-model is used

No clear scale separation in the near-wall region and near-wall structures scale in terms of wall units (streak spacing is about $\lambda_z = 100$ wall units)
$\Rightarrow$ wall-resolved LES is very demanding

- A resolution of $\Delta z^+ \approx 15 - 30$ in the spanwise direction and $\Delta x^+ \approx 40 - 80$ in the streamwise direction is required in order to resolve the near-wall structures (Δx^+ and Δz^+ are the grid spacings in terms of wall units)
- The shear layer must be resolved which requires that the first grid point is located at $y^+ < 1$

The computational costs scales approximately as $Re^{2.4}$

$\Rightarrow$ A wall-resolved LES of a high-Reynolds number wall-bounded flow is very expensive

Application of LES to wall-bounded flows

Variants of LES of turbulent wall-bounded flows

- Wall-resolved LES: the near-wall eddies in the inner layer and the large-scale motions in the outer layer are both resolved
- Wall-modelled LES: only the large-scale motions in the outer layer are resolved, the inner wall layer is modelled

Wall-modelled LES

Approaches

- **Approximate boundary conditions or wall functions**: equilibrium laws for the stresses and velocity based on the logarithmic velocity profile typically of the form (but there are also alternative methods)

$$\tau_w = \frac{\langle \tau_w \rangle}{\langle \tilde{u}(y_1) \rangle} \tilde{u}(y_1)$$

  where $\tilde{u}(y_1)$ is the streamwise velocity at the first grid point and $\langle \tilde{u}(y_1) \rangle$ obeys the logarithmic law

- **Zonal models**: solving simplified turbulent boundary layer equations on a fine grid in the inner layer assuming a weak coupling with the outer layer LES
- **Hybrid RANS-LES methods**: use a RANS model for the inner layer and LES for the outer layer

Wall-modelled LES

Advantages/disadvantages

- Much cheaper than wall-resolved LES ($\Delta x^+ \approx 100 - 600$ and $\Delta z^+ \approx 100 - 300$ required)
- Problems with non-equilibrium flows and separation if approximate boundary conditions or wall functions are used
- Problems with the coupling between the wall-model (RANS) and LES in the outer layer since there are no resolved eddies in the interface between the model and LES. The inner (RANS) layer and the outer (LES) layer have very different time and length scales

Wall-resolved LES of turbulent open channel flow

Example computation

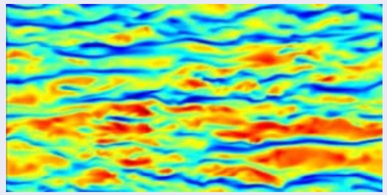
- $Re_\tau = 380$
- Pseudospectral code
- DNS resolution $160 \times 65 \times 160$ ($\Delta x^+ = 12$ and $\Delta z^+ = 6$)
- LES resolution $48 \times 65 \times 48$ ($\Delta x^+ = 40$ and $\Delta z^+ = 20$)
- Dynamic Smagorinsky SGS stress model
- Test filter applied only in planes parallel to the wall

Some practical issues in LES

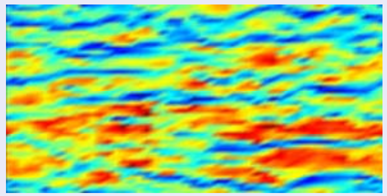
- Most of the turbulent kinetic energy (typically $\gtrsim 80\%$) must be resolved
- It is common to use a DNS code supplemented with a SGS model for LES
- When the filter is non-homogeneous, an average filter width $\Delta = (\Delta_x \Delta_y \Delta_z)^{1/3}$ can be used in the SGS models
- Usually no explicit filter is used. The grid spacing h is the filter implying a spectral cutoff filter in pseudospectral codes and a top hat filter in finite difference/volume codes
  $\Rightarrow$ Only test filters (dynamic Smagorinsky) or high-pass filter operations (multiscale models, ADM) are then needed.
- The test filter is sometimes (often) applied only in planes parallel to the wall

Wall-resolved LES of turbulent open channel flow

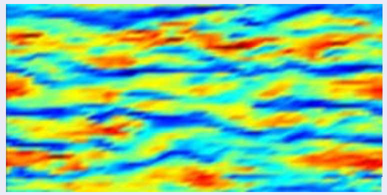
DNS



filtered
DNS



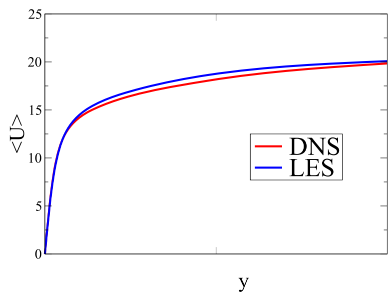
LES

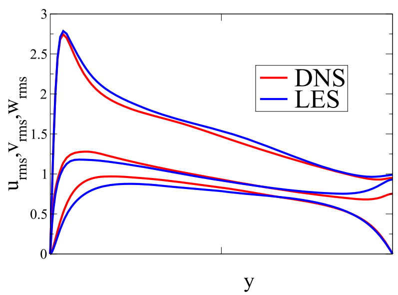


Wall-resolved LES of turbulent open channel flow

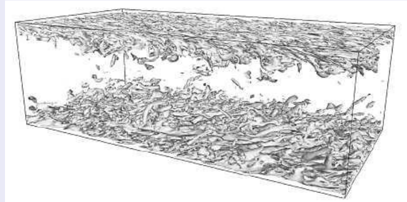
mean streamwise velocity

rms of velocity fluctuations

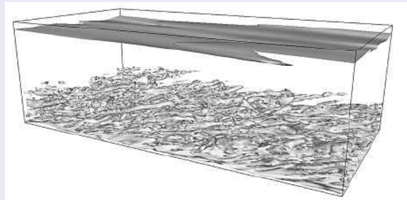




LES of channel flow



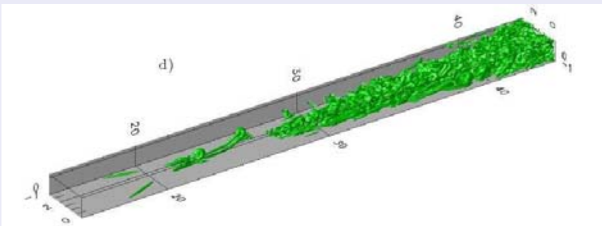
Non-rotating channel flow (Lamballais *et al.* 1998)



Rotating channel flow (Lamballais *et al.* 1998)

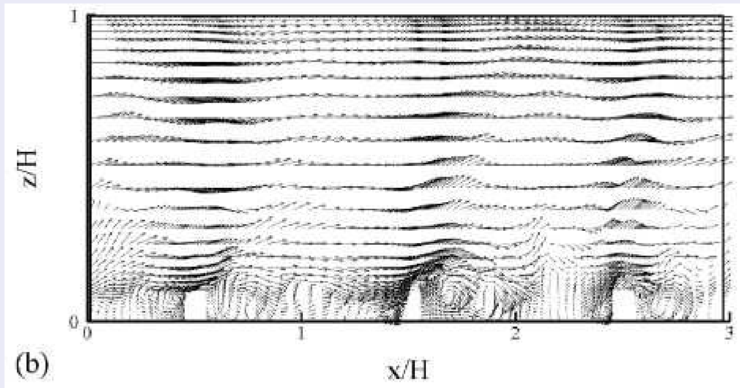
LES of boundary layers

Boundary layer with transverse shear (Kannepalli & Piomelli 2000)



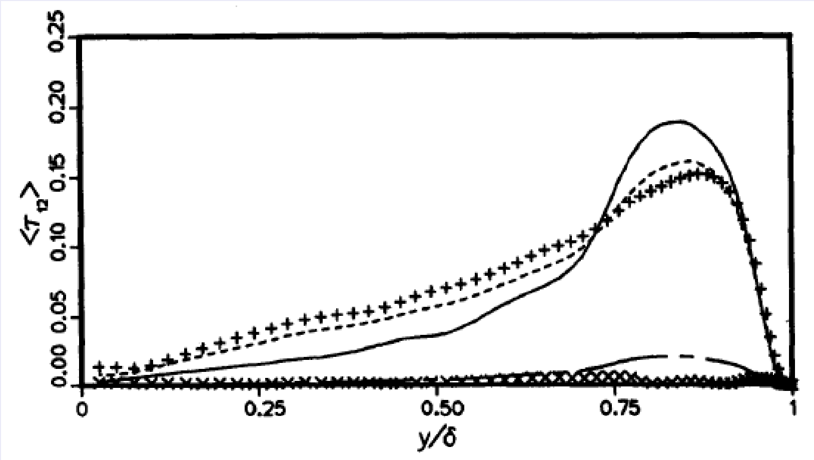
Transition to turbulence (Schlatter 2005)

LES of flow over rough walls



Channel flow with transverse ribs (Cui *et al.* 2003)

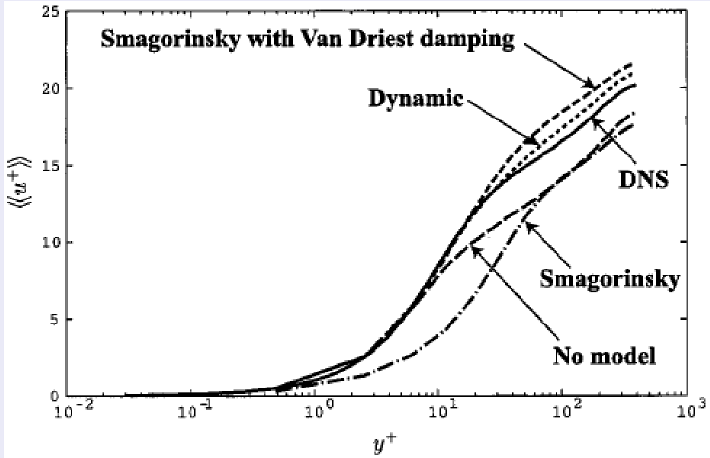
A priori tests in channel flow (Piomelli et al. 1988)



+ + + exact SGS stress, – – – mixed model, —— Smagorinsky model

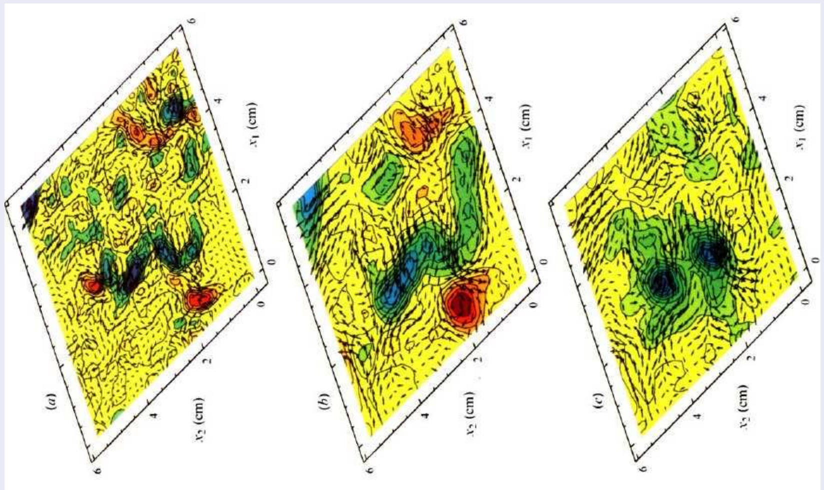
LES of turbulent channel flow

Mean streamwise velocity profiles (a posteriori tests)



Channel flow at $Re_\tau = 395$ (Hughes *et al.* 2001) resolution $32 \times 32 \times 32$

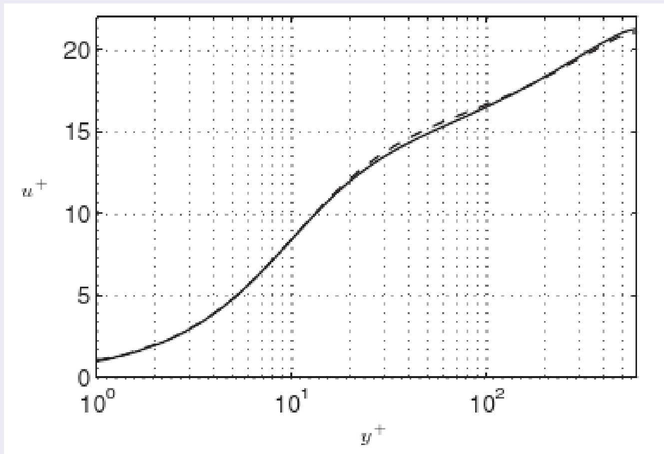
A priori tests in a turbulent jet (Liu et al. 1994)



exact SGS stress   mixed model   Smagorinsky model

LES of turbulent channel flow

Mean streamwise velocity profiles (a posteriori tests)

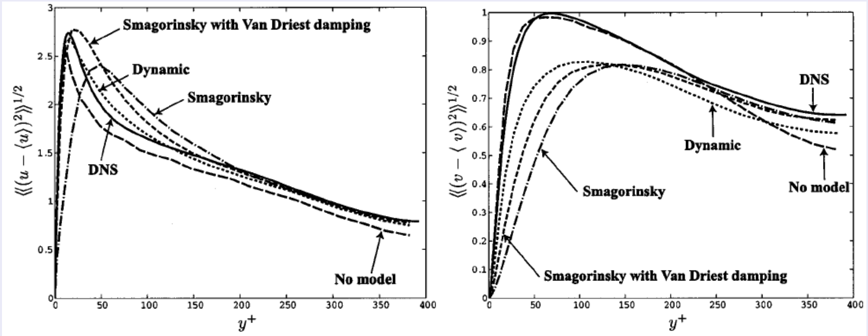


—— DNS, – – – multiscale model

Channel flow at $Re_\tau = 590$ (Bricteux *et al.* 2009) resolution $96 \times 64 \times 96$

LES of turbulent channel flow

Rms profiles of the velocity fluctuations (a posteriori tests)

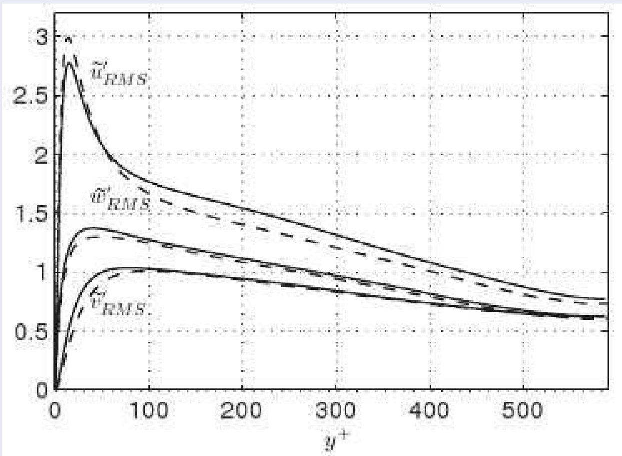


streamwise fluctuations | wall-normal fluctuations

Channel flow at $Re_\tau = 395$ (Hughes *et al.* 2001) resolution $32 \times 32 \times 32$

LES of turbulent channel flow

Rms profiles of the velocity fluctuations (a posteriori tests)

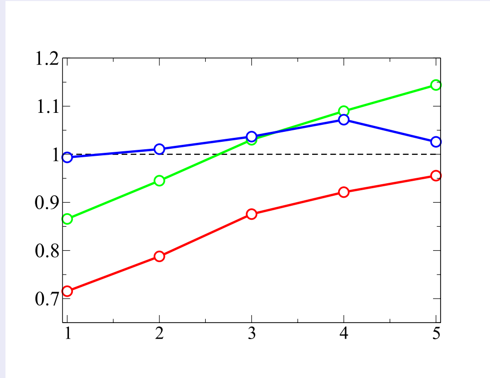


—— DNS, – – – multiscale model

Channel flow at $Re_\tau = 590$ (Bricteux *et al.* 2009) resolution $96 \times 64 \times 96$

LES of turbulent channel flow at $Re_\tau = 950$
Rasam et al. (2010)

Wall shear stress at different resolutions scaled with the exact DNS value from Del Álamo *et al.* (2004)

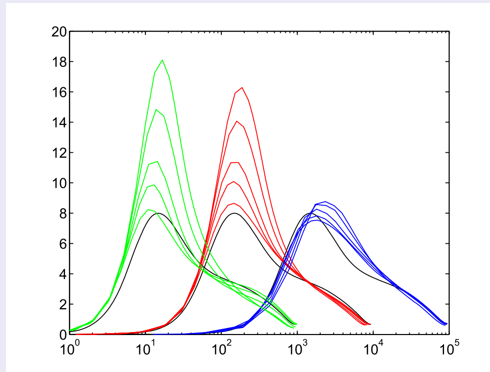


red: dynamic Smagorinsky; green: high-pass filter model; blue: explicit algebraic SGS model.

LES resolution increases from $\Delta x^+ = 240$ and $\Delta z^+ = 120$ (left) to $\Delta x^+ = 70$ and $\Delta z^+ = 28$ (right) with 13 million grid points

LES of turbulent channel flow at $Re_\tau = 950$
Rasam et al. (2010)

Streamwise Reynolds stress at different resolutions compared to the exact DNS value from Del Álamo *et al.* (2004)

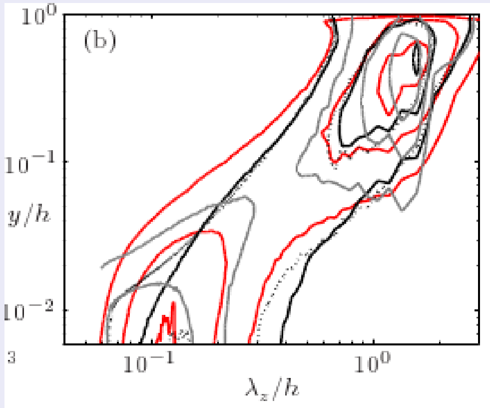


black: DNS, red: dynamic Smagorinsky, green: high-pass filter model, blue: explicit algebraic SGS model.

LES resolution increases from $\Delta x^+ = 240$ and $\Delta z^+ = 120$ (left) to $\Delta x^+ = 70$ and $\Delta z^+ = 28$ (right) with 13 million grid points

LES of turbulent channel flow at $Re_\tau = 950$
Rasam et al. (2010)

Contour plot of the premultiplied 1-D spectrum of the streamwise velocity



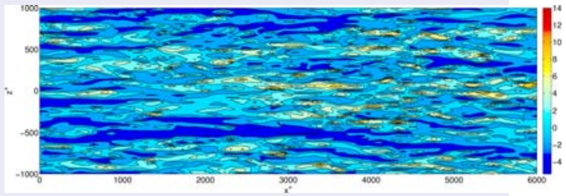
DNS, ... dynamic Smagorinsky, high-pass filter model, explicit algebraic SGS model.
LES resolution is $\Delta x^+ = 70$ and $\Delta z^+ = 28$

Further extensions and applications of LES

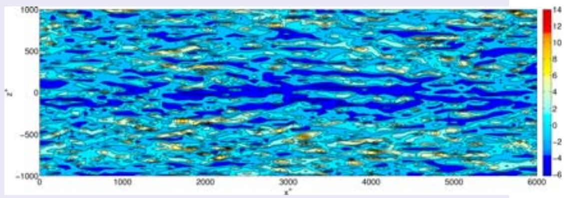
- Scalar mixing
- Stratified and buoyant flows
- Compressible flows
- Aeroacoustics
- (Vehicle) aerodynamics
- Turbulent particle or droplet flows
- Combustion
- Pollutant dispersion in urban areas
- Oceanography
- Interactions windturbines - atmospheric boundary layer

LES of turbulent channel flow at $Re_\tau = 950$
Rasam et al. (2010)

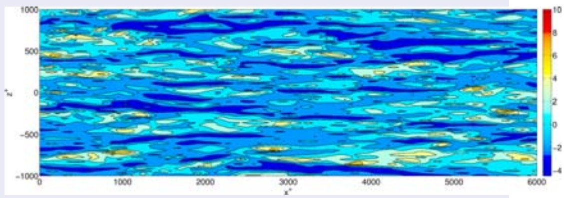
Dyn. Smagorinsky



HPF



EASGS



Remaining issues in LES and conclusions

- LES is a useful tool to investigate turbulence and structures in wall-bounded flows
- Validation of LES of high Reynolds number wall-bounded flows is needed
- There are very few studies on grid or filter width dependence of LES ⇒ more studies of LES at different resolutions are needed
- Resolution requirements must be reduced
- The combination of numerical errors and SGS models in LES of complex flow geometries with (unstructured) finite difference/volume codes is still poorly understood