Introduction to wall-bounded turbulence

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April 6, 2010

NORDITA Spring School 2010

Outline: Turbulence

1. Overview
   - landmark work
   - governing equations / RANS
2. Turbulent wall-bounded flow
   - viscous scaling → law of the wall
   - budget of kinetic energy
3. Dynamics of wall-bounded turbulence
   - flow structures: Streaks etc.
   - near-wall mechanisms
   - visualisations
4. Homework problem

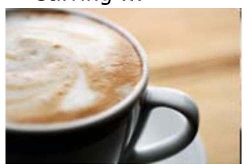
1. Overview

Turbulence

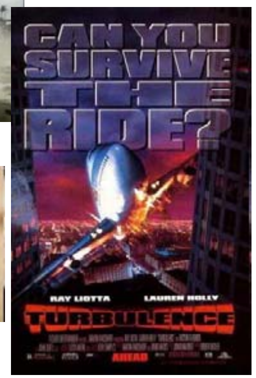
"Fasten your seat-belt"...

storms ...

stirring ...



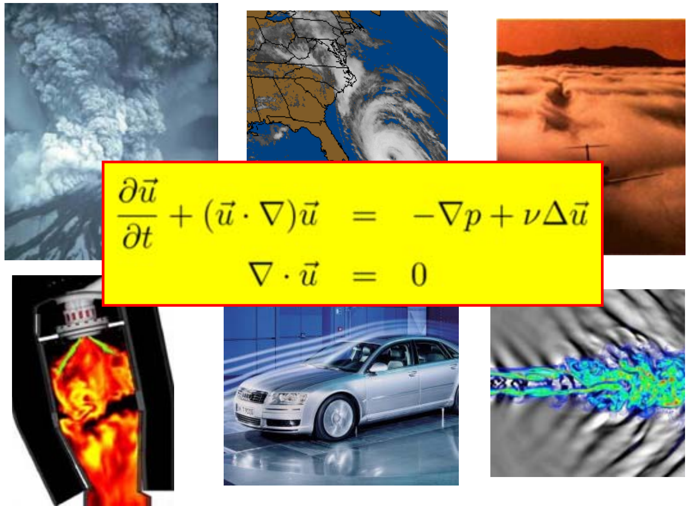
movies ...



→ very chaotic, unpredictable, usually "bad", to be avoided

- always time-dependent
- always three-dimensional
- always statistical/chaotic
- always dissipative

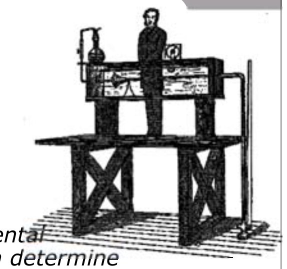
Turbulent flows around us...



$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\nabla p + \nu\Delta\vec{u}$$
$$\nabla\cdot\vec{u} = 0$$

source: www.efluids.com

Flows...



Main influences

- Reynolds number: $Re = \frac{U \cdot L}{\nu}$

  Osborne Reynolds (1883): *"An experimental investigation of the circumstances which determine whether the motion of water in parallel channels shall be direct or sinuous."*

  → **Laminar or turbulent**

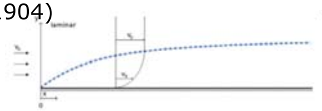
- Mach number: $Ma = \frac{U}{c}$

  → **Influence of compressibility**

Prandtl-Glauert effect

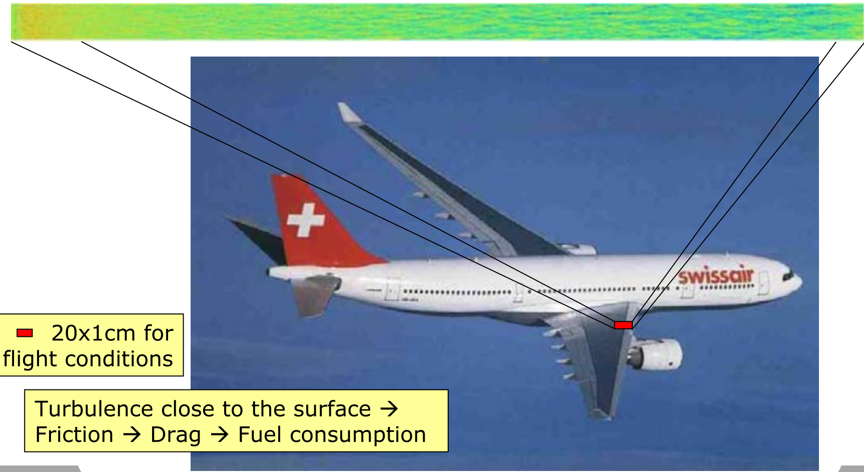
- Boundary layer: Ludwig Prandtl (1904)

  → **Influence of viscosity**



- other physical effects: *e.g.* heat transfer, fluid-structure interaction (FSI), Acustics, combustion, *etc.*

Turbulent flow close to solid walls...

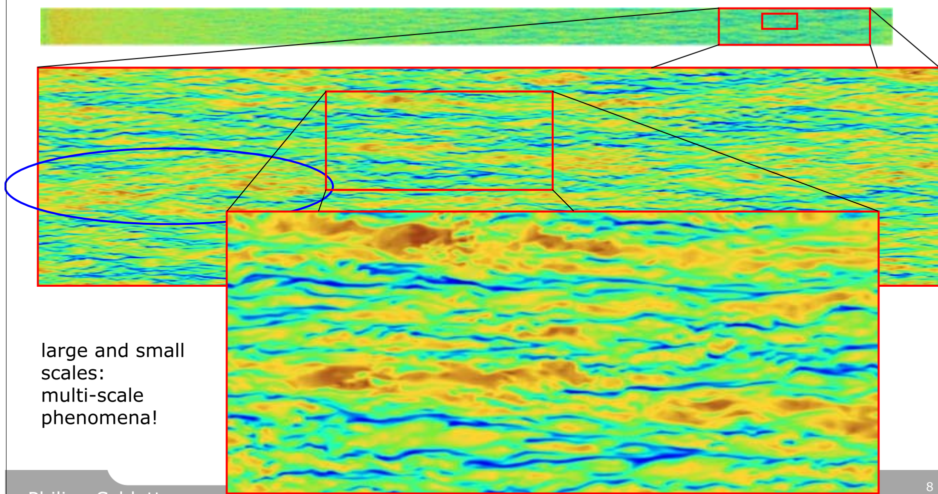


20x1cm for flight conditions

Turbulence close to the surface → Friction → Drag → Fuel consumption

Turbulent flow close to solid walls...

(simulation result: $Re_\theta = 4300$)

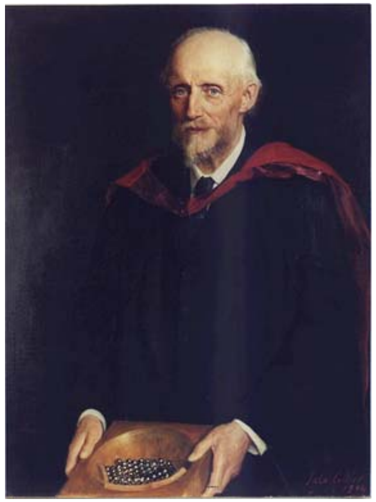


large and small scales: multi-scale phenomena!

Literature (samples)

- **Batchelor** (1953): The theory of homogeneous turbulence
- **Hinze** (1959, 1975): Turbulence
- **Lumley** (1970): Stochastic tools in turbulence
- **Monin & Yaglom** (1971, 1975): Statistical fluid mechanics: Mechanics of turbulence
- **Tennekes & Lumley** (1972): A first course in turbulence
- **Schlichting** (1979): Boundary layer theory
- **Lesieur** (1990): Turbulence in fluids
- **Frisch** (1995): Turbulence
- **Pope** (2000): Turbulent flows

Osborne Reynolds (1842-1912)



Short biography:

- Born in Belfast (1842)
- Cambridge University
- Professor of engineering in Manchester (1868)
- Started with fluids 1870
- Retired 1905

- Transition to turbulence
- Reynolds number
- Reynolds decomposition → RANS equation

Two recommended historical papers:

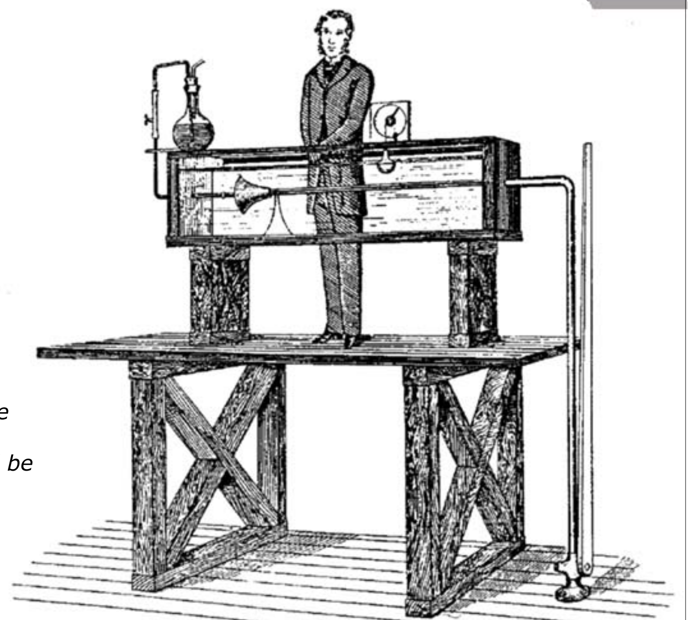
- **Rott, N.**, "Note on the history of the Reynolds number," Annu. Rev. Fluid Mech. 22, 1990, pp. 1-11.
- **Jackson, D. and Launder, B.**, "Osborne Reynolds and the Publication of His Papers on Turbulent Flow," Annu. Rev. Fluid Mech. 39, 2006, pp. 19-35.

Some landmarks in turbulence

- **Euler (1757), Navier (1822)** and **Stokes (1845)** - Equations of motion
- **Boussinesq** (1877) - Eddy viscosity assumption
- **O. Reynolds** (1883) - Pipe flow experiments - the importance of the - Reynolds number
- **O. Reynolds** (1895) - Reynolds decomposition and Reynolds averaging
- **Prandtl** (1904) - Boundary-layer theory
- **Prandtl** (1925) - Mixing length theory
- **Nikuradse** (1929,1930) Pipe and channel flow experiments
- **von Kármán** (1930) - Logarithmic velocity law
- **Kolmogorov** (1941) -5/3 scaling of spectra, smallest scale of turbulence
- **Kline & W.C. Reynolds** (1963) - Flow visualization of coherent turbulent structures
- **Kim, Moin & Moser** (1987) - DNS of turbulent channel flow
- **Spalart** (1988) DNS of turbulent boundary layer

Osborne Reynolds experiment in Manchester (1883):

"An experimental investigation of the circumstances which determine whether the motion of water in parallel channels shall be direct or sinuous."



Equations of motion

- Continuity equation (**incompressible flow**)

  $\nabla \cdot \underline{u} = 0$

- Navier-Stokes equations

μ	kinematic viscosity
$\nu = \mu/\rho$	dynamic viscosity

$$\rho \left[\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right] = -\nabla p^\circ + \mu \nabla^2 \underline{u}$$

- Non-dimensionalisation with reference scales U^* and L^*

$$\nabla \cdot \underline{u} = 0 \quad \text{and} \quad \frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} = -\nabla p + \frac{1}{Re} \nabla^2 \underline{u}$$

with

$$Re = \rho U^* L^* / \mu = \frac{U^* L^*}{\nu} \quad \text{and} \quad p = \frac{p^\circ}{\rho}$$

→ Equations valid for laminar, transitional and turbulent flow!

Einstein Summation Convection

- Simplified way to write sums:

$$a_i b_i = \sum_{i=1}^{3} a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$$

→ Summation over repeated indices

$$a_i b_i = a_j b_j \qquad F_j = \frac{a_i b_j}{c_i} \qquad ~~F_j = d_i \frac{a_i b_j}{c_i}~~$$

→ i is dummy index and can be renamed to e.g. j
→ each index can only appear once or twice

- Examples:

Divergence: $\nabla \cdot \underline{u} = \frac{\partial u_i}{\partial x_i}$

Gradient: $[\nabla f]_j = \frac{\partial f}{\partial x_j}$

Energy: $\frac{1}{2}\rho|\underline{u}|^2 = \frac{1}{2}(u^2 + v^2 + w^3) = \frac{1}{2}\rho u_i u_i$

Equations of motion (contd.)

Assume that the base flow is 2D and that disturbances are 2D.
$\Rightarrow \overline{u} = (u, v) = u(x,y)\overline{e_x} + v(x,y)\overline{e_y}$

Momentum eqs. in x and y-directions are obtained through scalar multiplication with $\overline{e_x}$ and $\overline{e_y}$

Henrik Alfredsson's slides

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Re^{-1}\nabla^2 u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + Re^{-1}\nabla^2 v$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Disturbance decomposition

Assume ~~linear~~ disturbances (capital letters are mean values and prime fluctuations around the mean)

$$u = U + u'$$
$$v = V + v'$$
$$p = P + p'$$
$$w = w'$$

u', v', p' are ~~small~~ disturbances $\Rightarrow$ ~~quadratic terms may be neglected.~~

→ **Reynolds decomposition**
U, V, P are mean values
u', v', p' are turbulent fluctuations

$\langle \cdot \rangle$ averaging operation: $U \equiv \langle u \rangle$

Disturbance decomposition

- Insert Reynolds decomposition into NS equations (2D mean flow U,V)

$$\frac{\partial(U+u')}{\partial t}+(U+u')\frac{\partial(U+u')}{\partial x}+(V+v')\frac{\partial(U+u')}{\partial y}+w'\frac{\partial(U+u')}{\partial z}=$$
$$-\frac{1}{\rho}\frac{\partial(P+p')}{\partial x}+\nu\left[\frac{\partial^2(U+u')}{\partial x^2}+\frac{\partial^2(U+u')}{\partial y^2}+\frac{\partial^2(U+u')}{\partial z^2}\right]$$

- average $\langle\cdot\rangle$, and consider:

$$\langle u'\rangle=\langle v'\rangle=\langle w'\rangle=\langle p'\rangle=\langle u'w'\rangle=0$$

- This yields the **RANS** (Reynolds-Averaged Navier-Stokes equations)

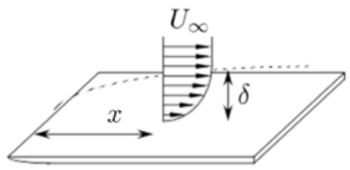
$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}\left[\nu\frac{\partial U}{\partial x}-\langle u'u'\rangle\right]+\frac{\partial}{\partial y}\left[\nu\frac{\partial U}{\partial y}-\langle u'v'\rangle\right]$$

- New (unclosed terms):

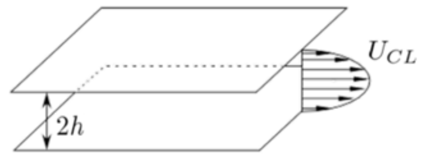
$\langle u'u'\rangle$, $\langle u'v'\rangle$: **Reynolds stresses**

2. Turbulent wall-bounded flow

Canonical wall-bounded flows

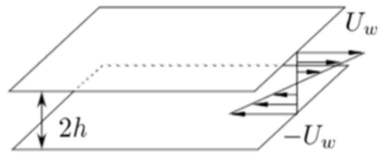


flat-plate boundary layer



plane Poiseuille (channel) flow

Hagen-Poiseuille (pipe) flow



Couette flow

Boundary-Layer Thickness

- 99% boundary-layer thickness

$$\delta_{99}: U(y=\delta_{99})=0.99U_\infty$$

- Displacement thickness

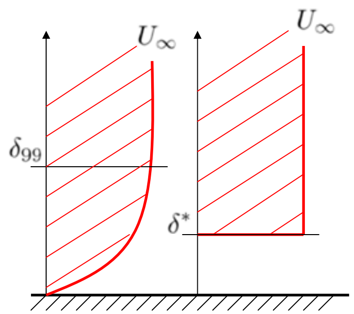
$$\delta^*=\int_0^\infty\left(1-\frac{U}{U_\infty}\right)\mathrm{d}y$$

- Momentum-(loss) thickness

$$\theta=\int_0^\infty\frac{U}{U_\infty}\left(1-\frac{U}{U_\infty}\right)\mathrm{d}y$$

- Shape factor

$$H_{12}=\delta^*/\theta$$

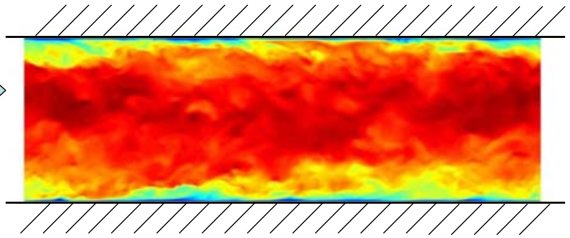


laminar Blasius b.l. H_{12}=2.59, turbulent ZPG b.l. $H_{12}\approx$ 1.3

Turbulent wall-bounded flow

- Consider turbulent channel flow

pressure gradient

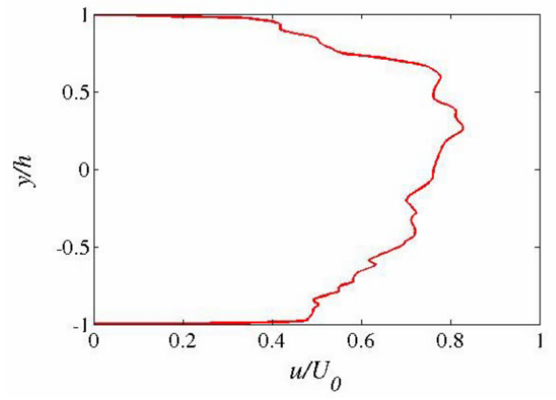


colours of streamwise velocity u (high and low)

DNS Re_τ=590

Turbulent wall-bounded flow

- Instantaneous profiles as function of time

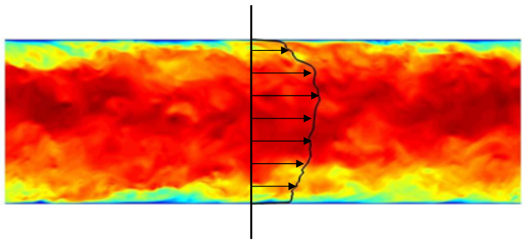


DNS Re_τ=590

Turbulent wall-bounded flow

- Consider turbulent channel flow

pressure gradient



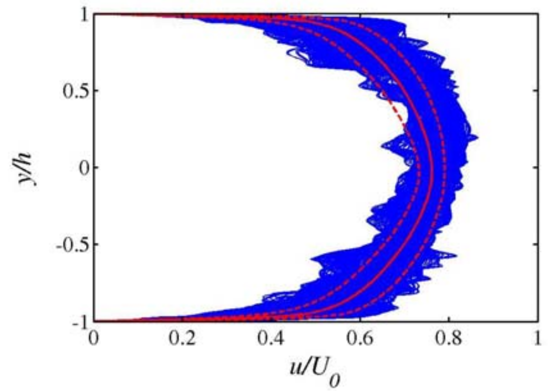
colours of streamwise velocity u (high and low)

instantaneous velocity profile

DNS Re_τ=590

Turbulent wall-bounded flow

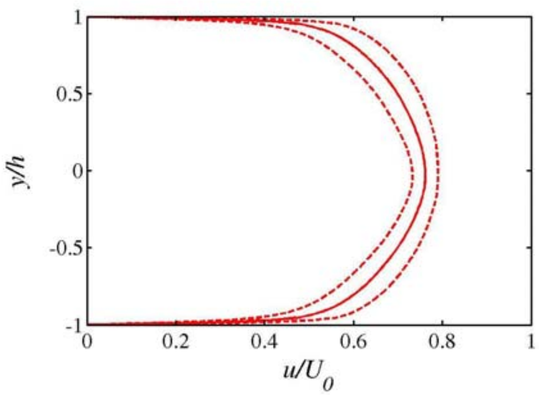
- Instantaneous and averaged profiles



DNS Re_τ=590

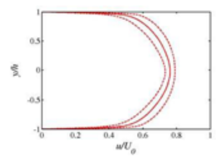
Turbulent wall-bounded flow

- Instantaneous and averaged profiles



DNS Re_τ=590

Turbulent wall-bounded flow



Statistical description of turbulence:

- **Reynolds decomposition**

$$\underbrace{u}_{\text{total}} = \underbrace{U}_{\text{mean}} + \underbrace{u'}_{\text{fluctuations}}$$

$$U = \langle u \rangle \ , \quad \langle U \rangle = U \ , \quad \langle u' \rangle = 0$$

→ **Fluctation amplitude** via the variance

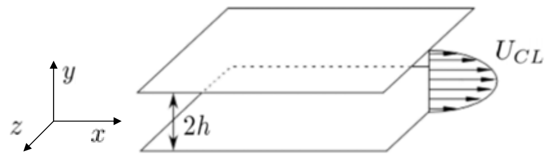
$$\langle u'^2 \rangle = \langle u^2 \rangle - U^2$$

$$u_{\text{rms}} = \sqrt{\langle u'^2 \rangle} \quad \text{(rms=root-mean-square)}$$

- Higher-order moments (skewness, flatness), probability density functions *etc*.

Turbulent channel flow

- Channel flow driven by streamwise pressure gradient $-\frac{dP}{dx}$
- 2D geometry $\frac{\partial}{\partial z} = W = 0$
- Fully developed turbulence: $\frac{\partial}{\partial x} = \frac{\partial}{\partial t} = 0$



- Continuity gives: $\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad \Rightarrow \quad V = 0$

Turbulent channel flow (contd.)

- 2D RANS Equations:

x: $$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left[\nu\frac{\partial U}{\partial x} - \langle u'u' \rangle\right] + \frac{\partial}{\partial y}\left[\nu\frac{\partial U}{\partial y} - \langle u'v' \rangle\right]$$

y: $$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left[\nu\frac{\partial V}{\partial x} - \langle u'v' \rangle\right] + \frac{\partial}{\partial y}\left[\nu\frac{\partial V}{\partial y} - \langle v'v' \rangle\right]$$

- Simplification:

y: $$0 = \frac{\partial}{\partial y}\left[P + \langle v'v' \rangle\right] \quad \Rightarrow P_w(x) = P + \langle v'v' \rangle$$

x: $$0 = \underbrace{-\frac{1}{\rho}\frac{\partial P}{\partial x}}_{-\frac{1}{\rho}\frac{\mathrm{d}P_w}{\mathrm{d}x}} + \frac{\mathrm{d}}{\mathrm{d}y}\left[\nu\frac{\mathrm{d}U}{\mathrm{d}y} - \langle u'v' \rangle\right]$$

$$\boxed{\frac{1}{\rho}\frac{\mathrm{d}P_w}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}y}\left[\nu\frac{\mathrm{d}U}{\mathrm{d}y} - \langle u'v' \rangle\right]}$$

Turbulent channel flow (contd.)

- 2D RANS Equations:

$$\frac{1}{\rho}\frac{\mathrm{d}P_w}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}y}\left[\nu\frac{\mathrm{d}U}{\mathrm{d}y} - \langle u'v'\rangle\right]$$

- Integrate in y, use boundary condition at y=0 (wall), and define the wall shear stress τ_w:

$$\tau_w := \mu\left.\frac{\mathrm{d}U}{\mathrm{d}y}\right|_w = -h\frac{\mathrm{d}P_w}{\mathrm{d}x}$$

integral force balance

$$\frac{\tau_w}{\rho}\left(1-\frac{y}{h}\right) = \nu\frac{\mathrm{d}U}{\mathrm{d}y} - \langle u'v'\rangle$$

pressure gradient — viscosity — turbulence

Turbulent channel flow (contd.)

$$\frac{\tau_w}{\rho}\left(1-\frac{y}{h}\right) = \nu\frac{\mathrm{d}U}{\mathrm{d}y} - \langle u'v'\rangle$$

- Rescale this equation based on ”viscous units”
- Two dimensional groups → define:

Friction velocity: $u_\tau = \sqrt{\tau_w/\rho}$

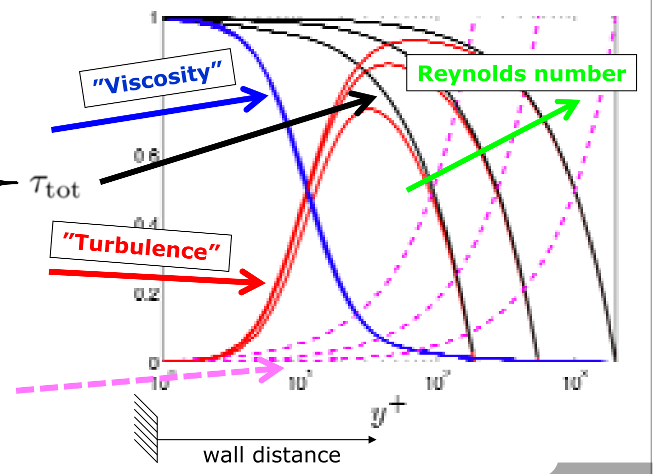
Viscous length scale: $\ell_* = \nu/u_\tau$

- Inner/wall/viscous scaling: ('+' units) $U^+ = U/u_\tau$, $y^+ = y/\ell_*$ $\Big\}$ $Re^+ = u_\tau l_*/\nu \equiv 1$
- Outer length scale: $\delta\,(R, H, \delta_{99}) \Rightarrow \delta^+ = \delta u_\tau/\nu \equiv Re_\tau$ (Reynolds number)

$$1 - \frac{y^+}{h^+} = \frac{\mathrm{d}U^+}{\mathrm{d}y^+} - \langle u'v'\rangle^+$$

Streamwise momentum conservation
for fully-developed internal flows

$$1 = \left.\begin{array}{l} +\dfrac{\mathrm{d}U^+}{\mathrm{d}y^+} \\ -\langle u'v'\rangle^+ \end{array}\right\}\tau_{\mathrm{tot}} \quad +\frac{y^+}{h^+}$$

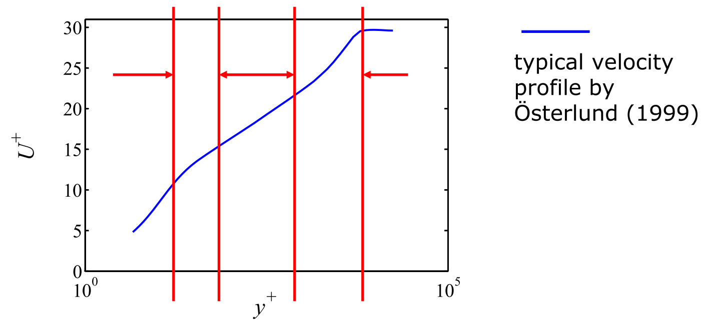


Data from channel-flow DNS for Re$_\tau$=180, 550, 2000 by Jiménez Group

Slide by Ramis Örlü

Mean Velocity: Law of the wall

- Measured profile of the mean velocity in plus units



- usually plotted with logarithmic abscissa
- various regions with different scalings

Mean Velocity: Log Law (contd.)

- Region very close to the wall:

  Viscosity more important than Reynolds stresses

$$1 - \frac{y^+}{h^+} = \frac{dU^+}{dy^+} - \langle u'v' \rangle^+$$

- Linear profile:

$$\boxed{U^+ = y^+}$$

viscous sublayer,
valid up to about $y^+ < 5$

Mean Velocity: Log Law (contd.)

- In high Reynolds number wall bounded flows $\delta^+ \gg 1$ we will have an inner viscous region $y \ll \delta$ where the mean velocity is a function of

$$U_{\text{inner}} = f(y, u_\tau, \nu)$$

- In the outer region $y \gg \ell_\star$ the mean velocity will be independent of viscosity and can be described as function of the following quantities

$$U_{\text{outer}} = F(y, u_\tau, U_\infty, \delta)$$

Mean Velocity: Log Law (contd.)

- There should exist an overlap region where both descriptions should be valid ("matched asymptotics").
- Write the functional dependence in non-dimensional form

$$U_{\text{inner}} = f(y, u_\tau, \nu) \quad \Rightarrow \frac{U_{\text{inner}}}{u_\tau} = f(yu_\tau/\nu)$$

$$U_{\text{outer}} = F(y, u_\tau, U_\infty, \delta) \quad \Rightarrow \frac{U_\infty - U_{\text{outer}}}{u_\tau} = F(y/\delta)$$

Mean Velocity: Log Law (contd.)

- Or we can write it in the following form $(\eta = y/\delta)$

$$\frac{U_{\text{inner}}}{u_\tau} = U^+_{\text{inner}} = f(yu_\tau/\nu) = f(y^+)$$

$$\frac{U_\infty - U_{\text{outer}}}{u_\tau} = U^+_\infty - U^+_{\text{outer}} = F(y/\delta) = F(\eta)$$

- Now we should match these two descriptions in the overlap region. Comparing velocities does not give us any new information, however we can compare the derivative (slope) of the distribution, which has to be equal for the two descriptions.

Mean Velocity: Log Law (contd.)

$$\frac{\mathrm{d}U_{\text{inner}}}{\mathrm{d}y} = \frac{\mathrm{d}U_{\text{outer}}}{\mathrm{d}y}$$

- If we take the derivatives we get

$$\frac{\mathrm{d}U_{\text{inner}}}{\mathrm{d}y} = f' \frac{u_\tau}{\nu} u_\tau$$

$$\frac{\mathrm{d}U_{\text{outer}}}{\mathrm{d}y} = -F' \frac{1}{\delta} u_\tau$$

Mean Velocity: Log Law (contd.)

- Putting them equal we get

$$f' \frac{u_\tau}{\nu} u_\tau = -F' \frac{1}{\delta} u_\tau$$

- Multiply by y, and we get in non-dimensional form

$$y^+ f'(y^+) = -\eta F'(\eta)$$

- The LHS is now a function only of y^+ and the RHS of η. If both should be valid the only possibility is that both are equal to a constant.

$$y^+ f'(y^+) = -\eta F'(\eta) = C$$

Mean Velocity: Log Law (contd.)

- Thus we obtain for the two regions independently:

$$f'(y^+) = \frac{C}{y^+} \quad \text{and} \quad F'(\eta) = -\frac{C}{\eta}$$

- Integration gives:

$$f(y^+) = C \ln y^+ + B$$

$$F(\eta) = -C \ln \eta + D$$

Mean Velocity: Log Law (contd.)

- Finally we get the **"Logarithmic Region (log law)":**

$$\frac{U}{u_\tau} = U^+ = \frac{1}{\kappa} \ln y^+ + B$$

$$\frac{U_\infty - U}{u_\tau} = U_\infty^+ - U^+ = -\frac{1}{\kappa} \ln(y/\delta) + D$$

- The coefficient $\kappa = 1/C$ is called von Kármán constant.
- Valid for about: $30 < y^+$ and $y/\delta < 0.3$
- Low-Re values: $\kappa = 0.41\ ,\quad B = 5.2$ (Pope 2000)
  High-Re values: $\kappa = 0.38\ ,\quad B = 4.1$ (Österlund et al. 2000)

Classical two-layer approach: Law of the wall

Zero pressure-gradient (ZPG) turbulent boundary layer (TBL): Re_τ=1000

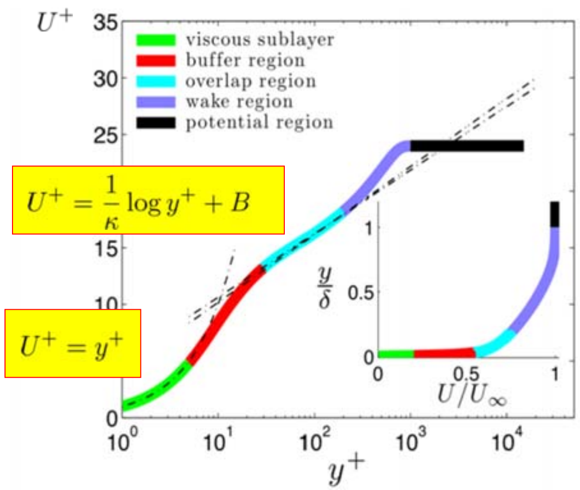


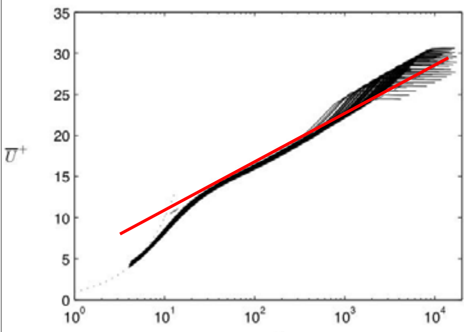
Figure by Örlü (2009)

Mean Velocity: Log Law (contd.)

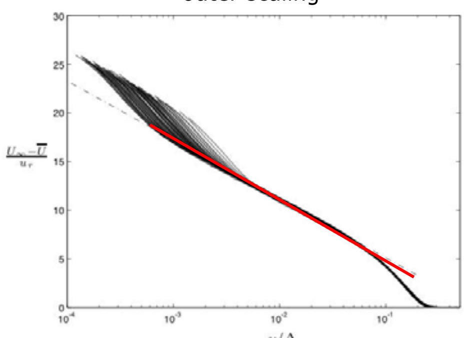
$$U^+ = \frac{1}{\kappa}\ln y^+ + B$$

$$U^+_\infty - U^+ = -\frac{1}{\kappa}\ln(y/\delta) + D$$

viscous/inner scaling

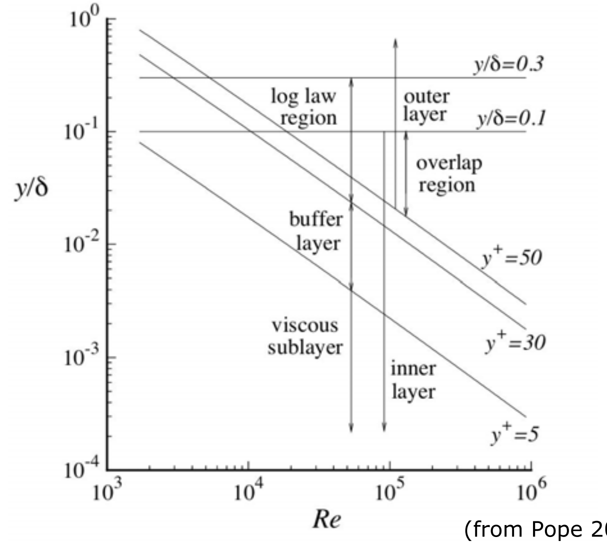


outer scaling



(plots from Nagib et al., 2006)

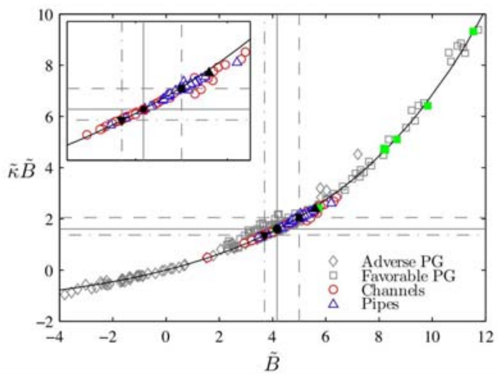
Scales in the boundary layer



(from Pope 2000)

Non-universality of κ and B

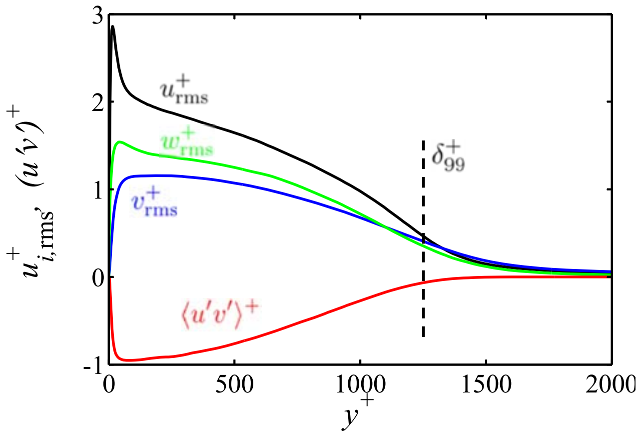
- Recent experimental work has shown that the universality of κ and B is at least doubtful
- In particular pressure gradients and flow case can greatly influence the respective values.



(Nagib et al. 2009)

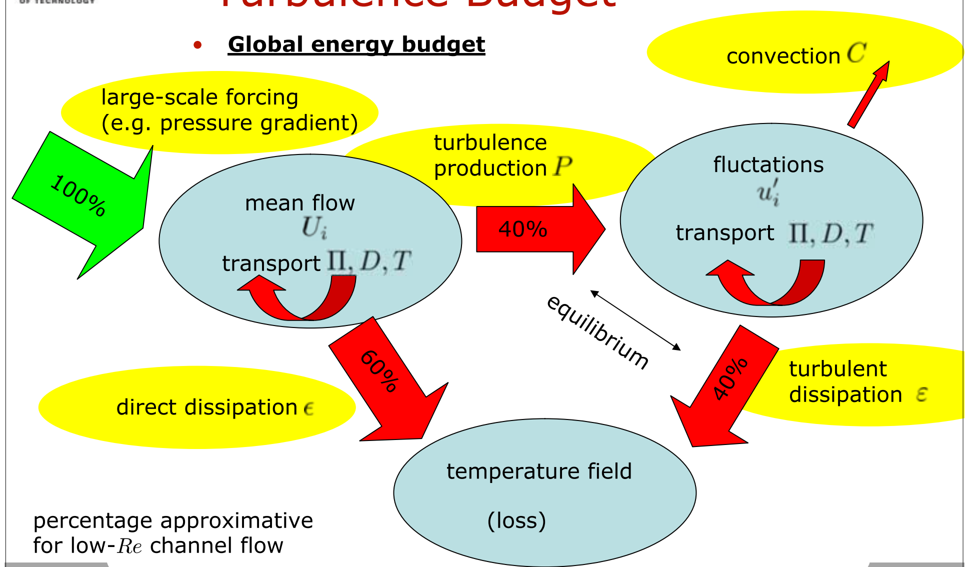
Turbulence Fluctuations

- DNS of a turbulent boundary layer (Schlatter 2010)
  Re_θ=4000, Re_τ=1250



Turbulence Budget

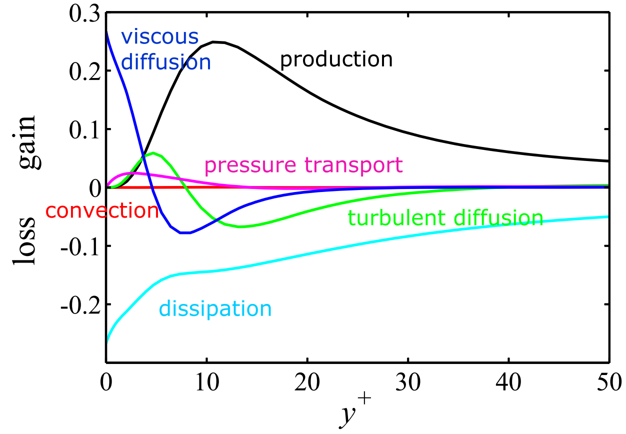
- **Global energy budget**



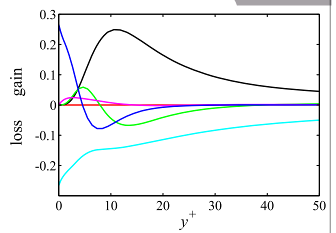
percentage approximative
for low-Re channel flow

Turbulence Budget

- DNS of a turbulent boundary layer (Schlatter 2010)
  Re_θ=4000, Re_τ=1250
- **Budget for turbulent kinetic energy**



Turbulence Budget



- Transport equation for turbulent kinetic energy

$$C = P + \Pi + D + T - \varepsilon \qquad k = \frac{1}{2}\langle u'_i u'_i \rangle$$

$C = U_j \frac{1}{2}\frac{\partial u'_i u'_i}{\partial x_j}$: convection (=0 in parallel flows)

$P = -\langle u'_i u'_j \rangle \frac{\partial U_i}{\partial x_j}$: Production. Adds energy to fluctations (mainly u)

$\Pi = \frac{\partial \langle u'_j p' \rangle}{\partial x_j}$: Pressure transport. Redistributes between directions

$D = 2\nu \frac{\langle u'_i s'_{ij} \rangle}{\partial x_j}$: Viscous diffusion

$$s'_{ij} = \frac{1}{2}\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right)$$

$T = -\frac{1}{2}\frac{\partial \langle u'_i u'_i u'_j \rangle}{\partial x_j}$: Turbulent diffusion

$\varepsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle$: Dissipation. Destruction of kinetic energy (→ heat)

Turbulent Production: Maximum?

- Production of $k = \langle u_i' u_i' \rangle$: $P = -\langle u'v' \rangle \frac{dU}{dy}$
- Momentum equation: $\nu \frac{dU}{dy} - \langle u'v' \rangle = u_\tau^2 (1 - \frac{y}{h})$

$$\Rightarrow P = \left[u_\tau^2 (1 - \frac{y}{\delta}) - \nu \frac{dU}{dy} \right] \frac{dU}{dy}$$

- Look for a maximum of P: $\frac{dP}{dy} = 0$

$$\frac{dP}{dy} = \left[-\nu \frac{d^2U}{dy^2} \right] \frac{dU}{dy} + \left[u_\tau^2 - \nu \frac{dU}{dy} \right] \frac{d^2U}{dy^2}$$

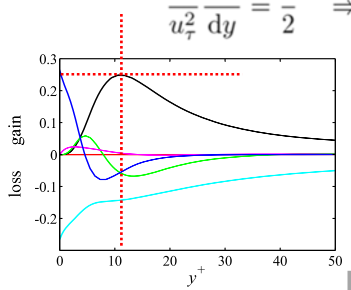
Turbulent Production: Maximum?

- Look for maximum of P: $\frac{dP}{dy} = 0$

$$\frac{dP}{dy} = \left[-\nu \frac{d^2U}{dy^2} \right] \frac{dU}{dy} + \left[u_\tau^2 - \nu \frac{dU}{dy} \right] \frac{d^2U}{dy^2} = 0$$

$$-2\nu \frac{dU}{dy} + u_\tau^2 = 0$$

$$\frac{\nu}{u_\tau^2} \frac{dU}{dy} = \frac{1}{2} \quad \Rightarrow \quad \frac{dU^+}{dy^+} = \frac{1}{2}, \quad P^+_{max} = \frac{1}{4} \quad \Rightarrow y^+ \approx 11$$



Reynolds numbers

- Reynolds number is used as a measure for **"how turbulent"** or **"how downstream"** or **"how thick"** a boundary layer is:
- Friction Reynolds number (Kármán measure)

$$Re_\tau = \delta^+ = \frac{u_\tau \delta}{\nu}$$

- Based on momentum thickness

$$Re_\theta = \frac{U_\infty \theta}{\nu}$$

with momentum-loss thickness

$$\theta = \int_0^\infty \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty} \right) dy$$

- Based on streamwise distance (boundary layers)

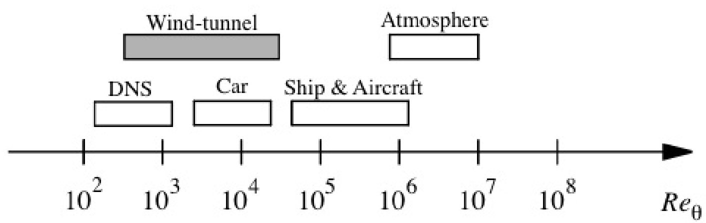
$$Re_x = \frac{U_\infty x}{\nu}$$

- Based on bulk velocity (channels, pipe, ...)

$$Re_b = \frac{U_b h}{\nu} \qquad U_b = \frac{1}{2h} \int_{-h}^{+h} U(y) dy$$

Reynolds number (contd.)

- Attainable Reynolds numbers for boundary layers



Reynolds number based on momentum loss

$$Re_\theta = \frac{U_\infty \theta}{\nu} \qquad \theta = \int_0^\infty \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty} \right) dy$$

(Österlund 1999)

Integral Momentum Equation

- Assume a ZPG boundary layer ($U_\infty = const.$)
- Start with boundary-layer momentum equations to get:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{1}{\rho}\frac{\partial \tau}{\partial y} - \frac{\partial}{\partial x}(\langle u'^2\rangle - \langle v'^2\rangle) \qquad \frac{\tau}{\rho} = \nu\frac{\partial U}{\partial y} - \langle u'v'\rangle$$

- Use continuity, $U_\infty = const.$ and integrate in y,

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^\infty U(U_\infty - U)\mathrm{y} + V(U_\infty - U)|_0^\infty = \frac{1}{\rho}\tau|_0^\infty + \frac{\partial}{\partial x}\int_0^\infty (\langle u'^2\rangle - \langle v'^2\rangle)\mathrm{d}y$$

- With boundary conditions at 0 and ∞, and the definition of

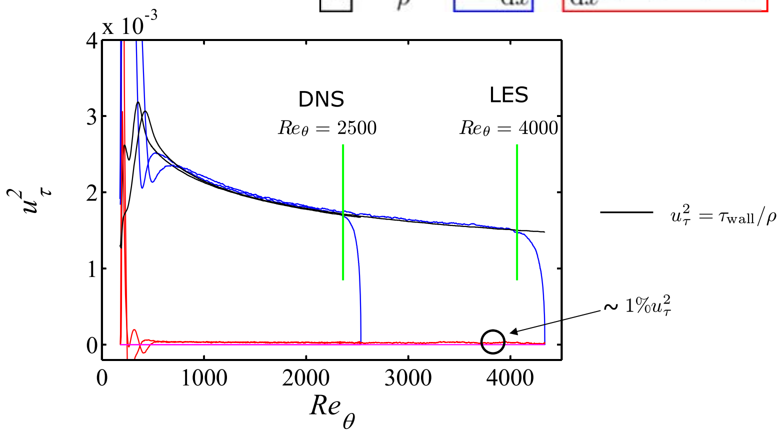
$$\theta = \int_0^\infty \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right)\mathrm{d}y$$

$$u_\tau^2 = \frac{\tau_w}{\rho} = U_\infty^2\frac{\mathrm{d}\theta}{\mathrm{d}x} - \frac{\mathrm{d}}{\mathrm{d}x}(\langle u'^2\rangle - \langle v'^2\rangle)$$

3. Dynamics and structure of wall-bounded turbulence

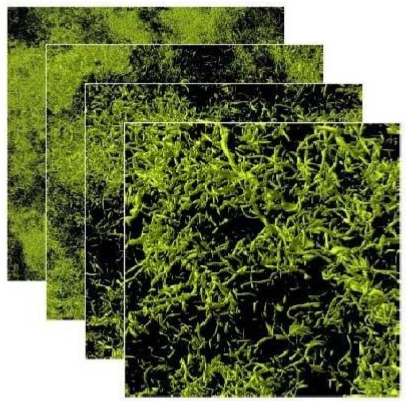
Integral Momentum Equation

- von Kármán equation $u_\tau^2 = \frac{\tau_w}{\rho} = U_\infty^2\frac{\mathrm{d}\theta}{\mathrm{d}x} - \frac{\mathrm{d}}{\mathrm{d}x}(\langle u'^2\rangle - \langle v'^2\rangle)$



Turbulent Structures

- Homogeneous isotropic turbulence



vortical structures

→ worms

Kaneda et al., Earth Simulator

Turbulent Structures

- Looking for (quasi-)coherent structures in wall turbulence, *i.e.* flow structures that are predictable at least in an ensemble sense.

S.K. Robinson (Annu. Rev. Fluid Mech. 1991) provides a list:

1. low-speed streaks below y^+=10
2. ejections of low-speed fluid from the wall
3. sweeps of high-speed fluid towards the wall
4. coherent vortical structures of various shapes (hairpins?)
5. internal shear layers up to y^+=80
6. near-wall pockets
7. backs which change streamwise velocity abruptly
8. outer-layer motion, valleys, bulges, intermittency.

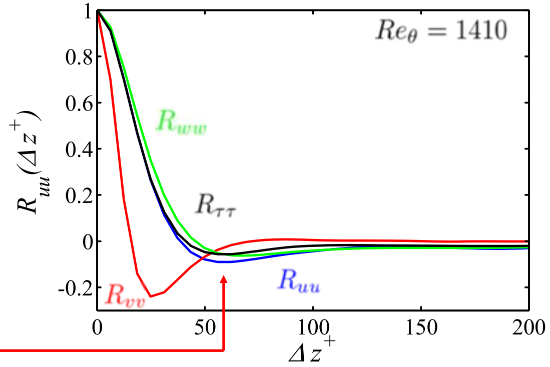
Turbulent Structures

- Quantify spatial/spanwise coherence of the fluctuations close to the wall: Spanwise two-point correlation

$$R_{uu}(\Delta z) = \frac{1}{u_{\rm rms}^2 L_z} \int u'(z)u'(z+\Delta z)\mathrm{d}z$$

first minimum: Spacing between low- and high-speed streak

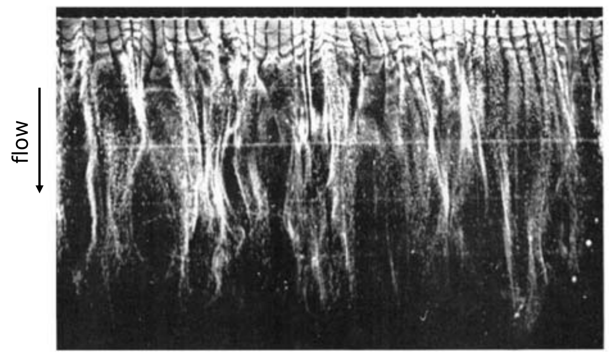
→ **Streak spacing: 100-120 plus units!**



Turbulent Structures

- Experiments for turbulent boundary layers:

1) Instantaneous top view close to the wall

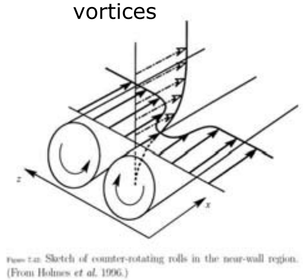


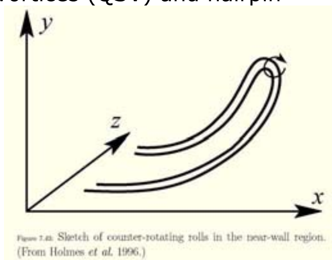
Kline et al. (1968)

Turbulent streaks with spacing Δz^+=100-120!

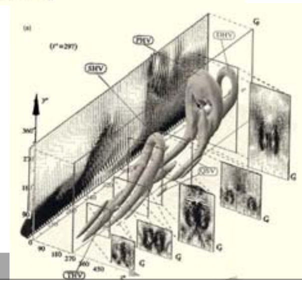
Turbulent Structures

- Streaks, quasi-streamwise vortices (QSV) and hairpin vortices





Hairpin vortex:



Adrian (*Phys. Fluids* 2007)

Turbulent Structures

- Near-wall dynamics:

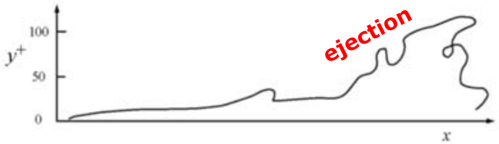
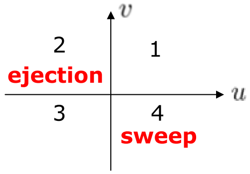


Figure 7.40: Dye streak in a turbulent boundary layer showing the ejection of low-speed near-wall fluid. (From the experiment of Kline *et al.* 1967.)

- Fluid stays close to the wall and suddenly bursts away → ejection!
- Quadrant analysis of (u-v) close to the wall ($y^+ \approx$ 15):



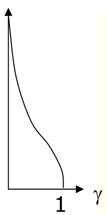
sweep: high-speed fluid towards the wall

ejection: low-speed fluid away from the wall

→ leading to turbulent production!

Turbulent Structures

- Large-scale organsiation: Irrotational and rotational fluid



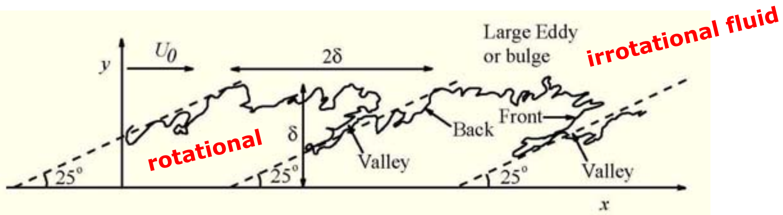


Figure 7.44: The large-scale features of a turbulent boundary layer at $Re_\theta \approx 4,000$. The irregular line—approximating the viscous superlayer—is the boundary between smoke-filled turbulent fluid and clear free-stream fluid. (From the experiment of Falco 1977.)

→ Intermittency γ (i.e. what fraction of time is a flow turbulent?)

Spectra...

- Start with two-point correlation (here spanwise):

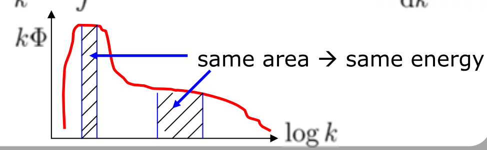
$$R_{uu}(\Delta z) = \frac{1}{u_{\rm rms}^2 L_z} \int u'(z)u'(z+\Delta z) \mathrm{d}z$$

- Perform Fourier transform → Power spectrum of u

$$\Phi = \mathcal{F}(R_{uu}) u_{\rm rms}^2 \frac{\Delta z}{\pi}$$
$$k = \frac{2\pi}{\Delta z}(0,1,2,\ldots)$$

- The wave length is related to the wave number: $\lambda = 2\pi/k$
- If x-axis plotted in log scale, use premultiplied form $k\Phi$

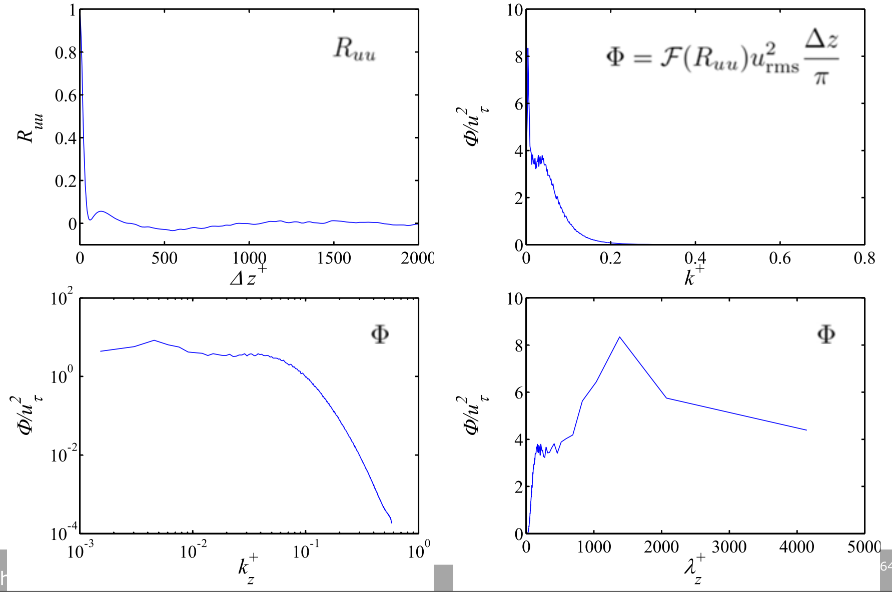
$$\int \Phi \mathrm{k} = \int k\Phi \frac{\mathrm{d}k}{k} = \int k\Phi \mathrm{d}(\log k) \quad \text{since} \quad \frac{\mathrm{d}\log k}{\mathrm{d}k} = \frac{1}{k}$$



Spectra...

DNS Schlatter (2010)

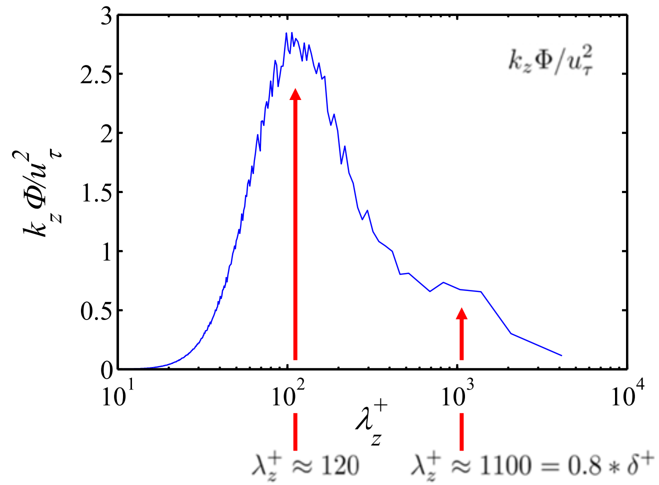
$Re_\theta = 4300 \quad Re_\tau = 1400 \quad y^+ = 7.5$



Spectra…

DNS Schlatter (2010)

$Re_\theta = 4300 \quad Re_\tau = 1400 \quad y^+ = 7.5$

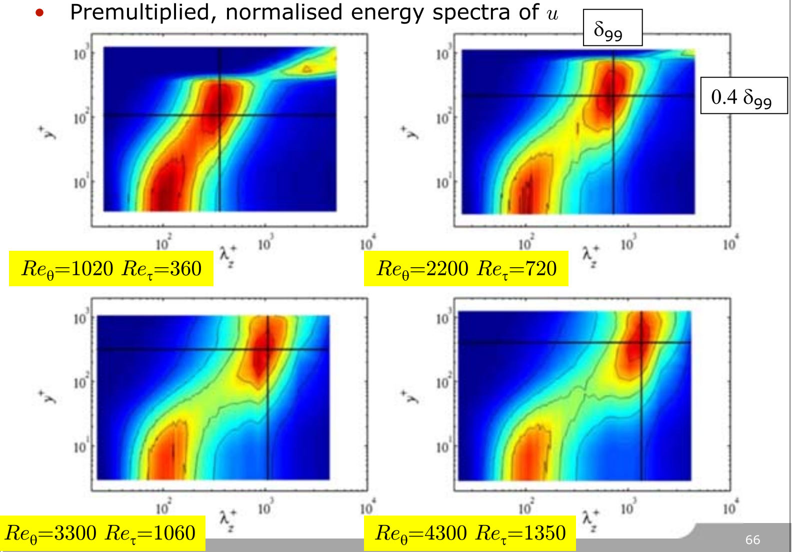


$\lambda_z^+ \approx 120 \qquad \lambda_z^+ \approx 1100 = 0.8 * \delta^+$

Spanwise Scales

DNS Schlatter (2010)

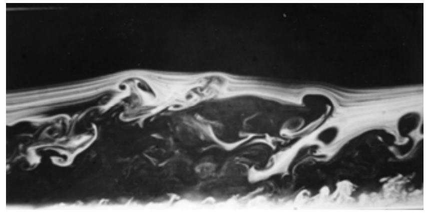
- Premultiplied, normalised energy spectra of u



Turbulent Structures

- Experiments for turbulent boundary layers:

2) Instantaneous side view



Jim Wallace

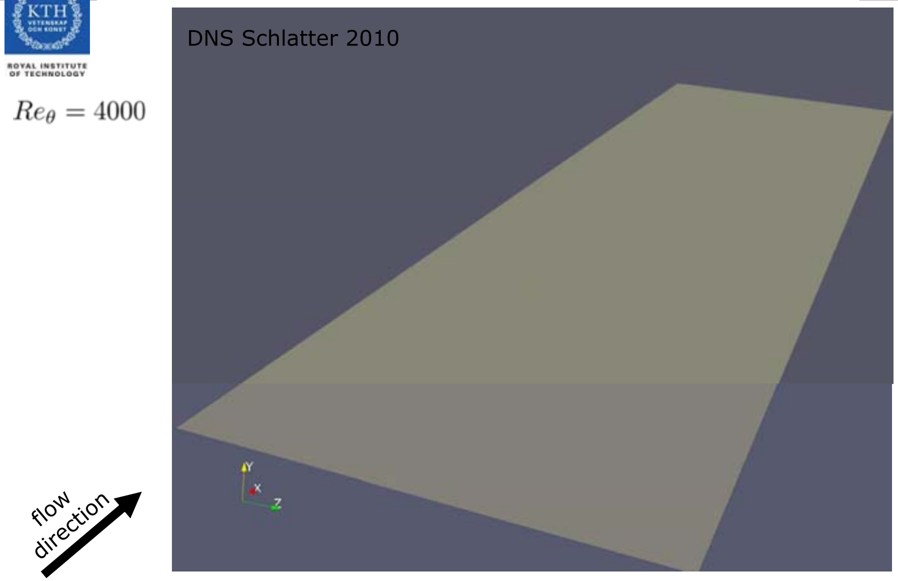
Coherent structures: Eddies and vortices of various sizes

Many open questions related to scaling, intensity, shape etc.

Hassan Nagib

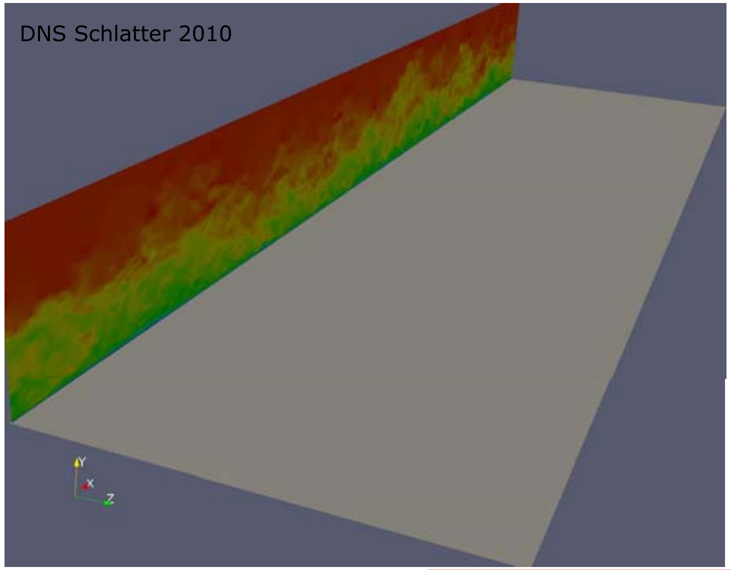
- corrugated boundary-layer edge
- large vortices further away
- small vortices close to the wall

$Re_\theta = 4000$



flat plate, $Re_\theta = 4000$

$Re_\theta = 4000$

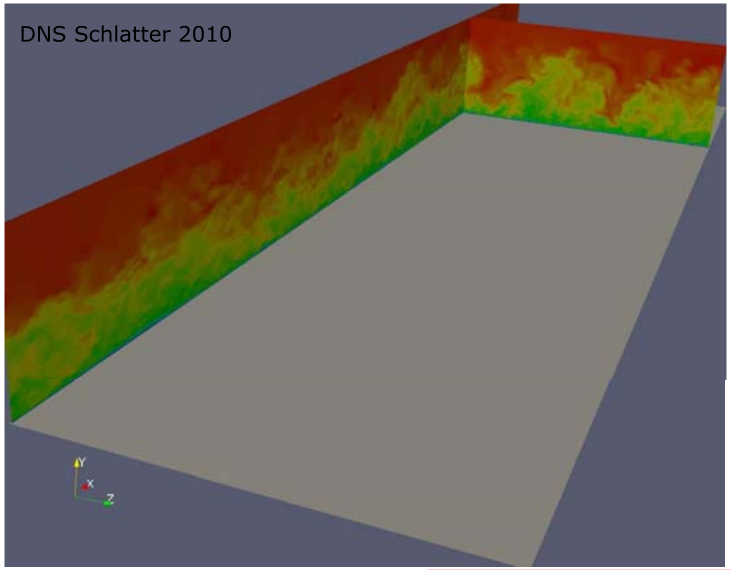


flow direction

streamwise velocity u

near-wall streaks and
larger structures further away

$Re_\theta = 4000$

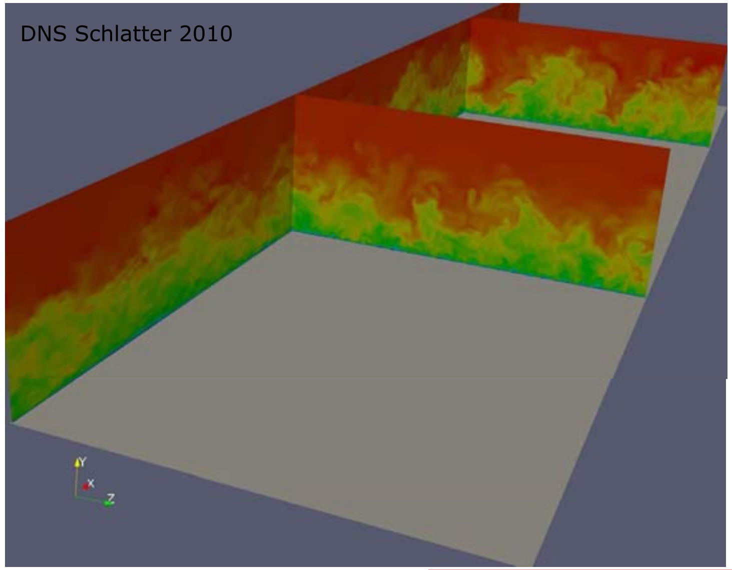


flow direction

streamwise velocity u

near-wall streaks and
larger structures further away

$Re_\theta = 4000$

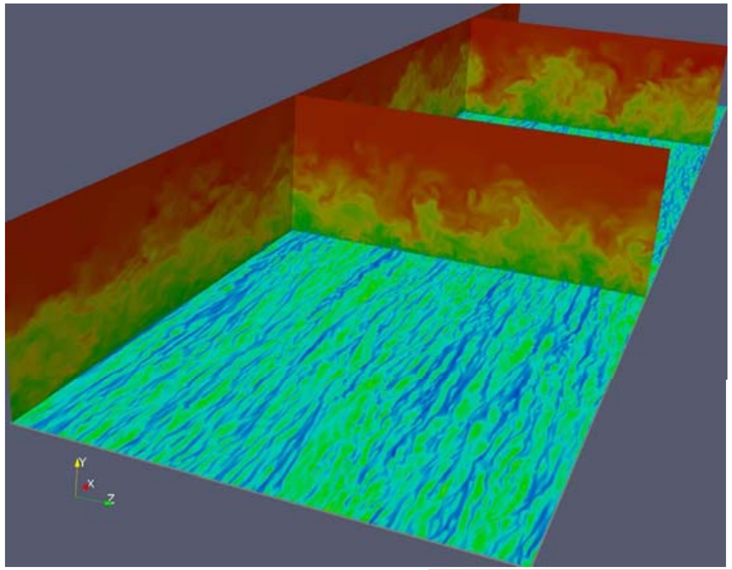


flow direction

streamwise velocity u

near-wall streaks and
larger structures further away

$Re_\theta = 4000$

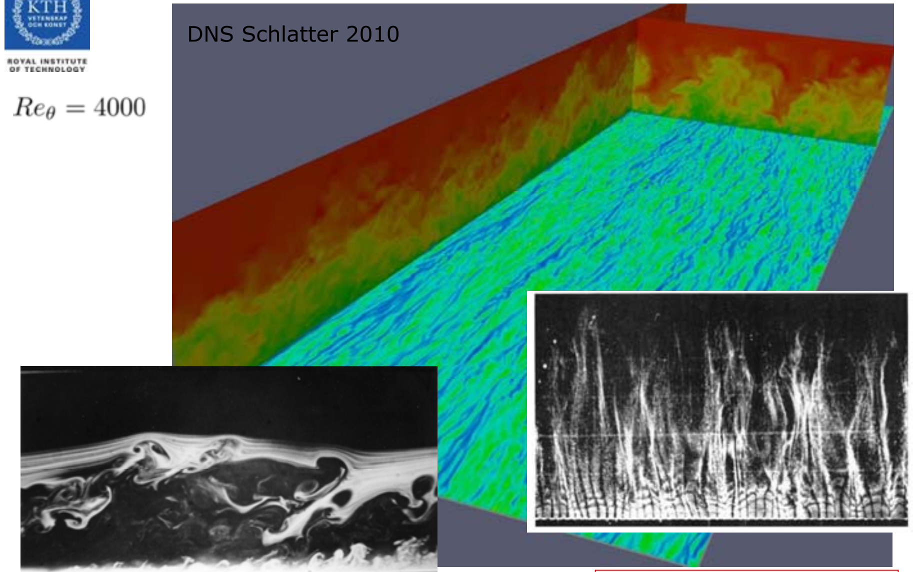


flow direction

streamwise velocity u

”modulation” of near-wall
structures by outer layer

$Re_\theta = 4000$

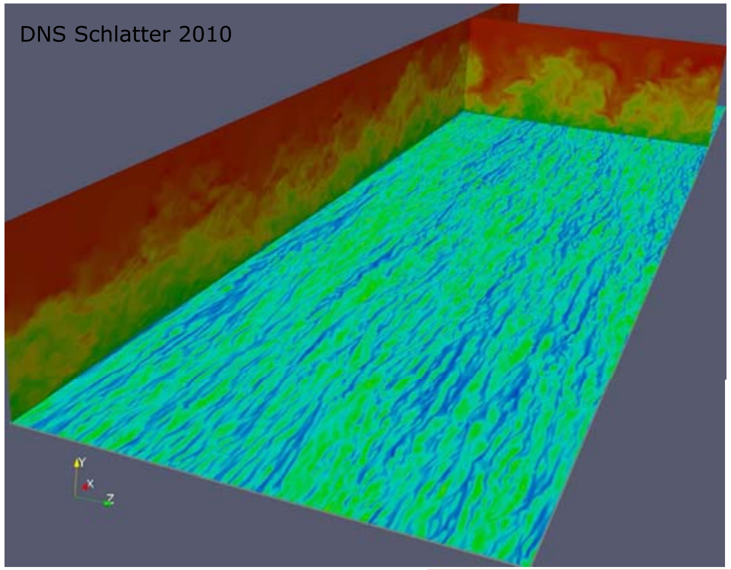


Hassan Nagib

streamwise velocity u

"modulation" of near-wall structures by outer layer

$Re_\theta = 4000$

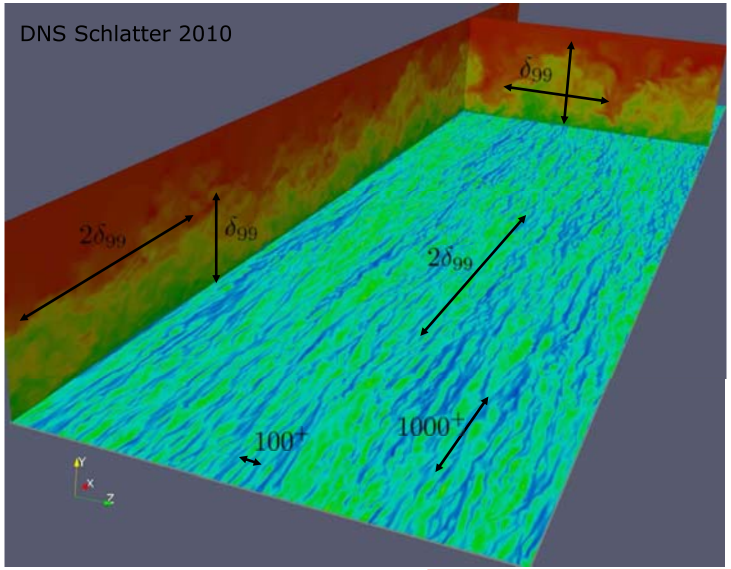


flow direction

streamwise velocity u

"modulation" of near-wall structures by outer layer

$Re_\theta = 4000$

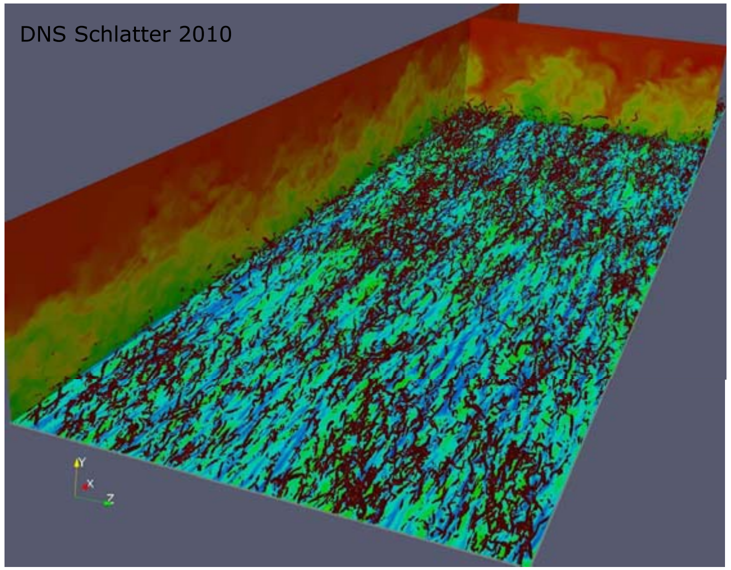


flow direction

streamwise velocity u

"modulation" of near-wall structures by outer layer

$Re_\theta = 4000$

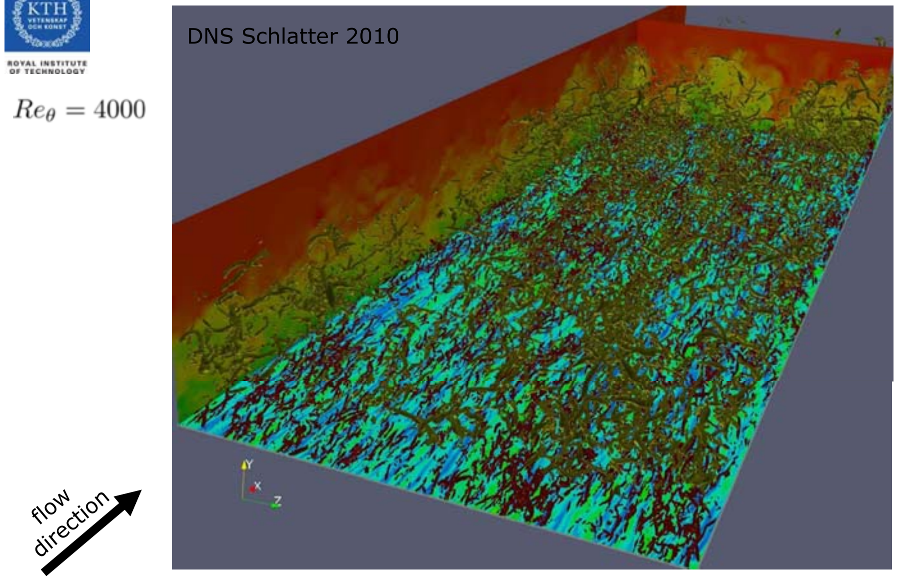


flow direction

λ_2 = -0.008

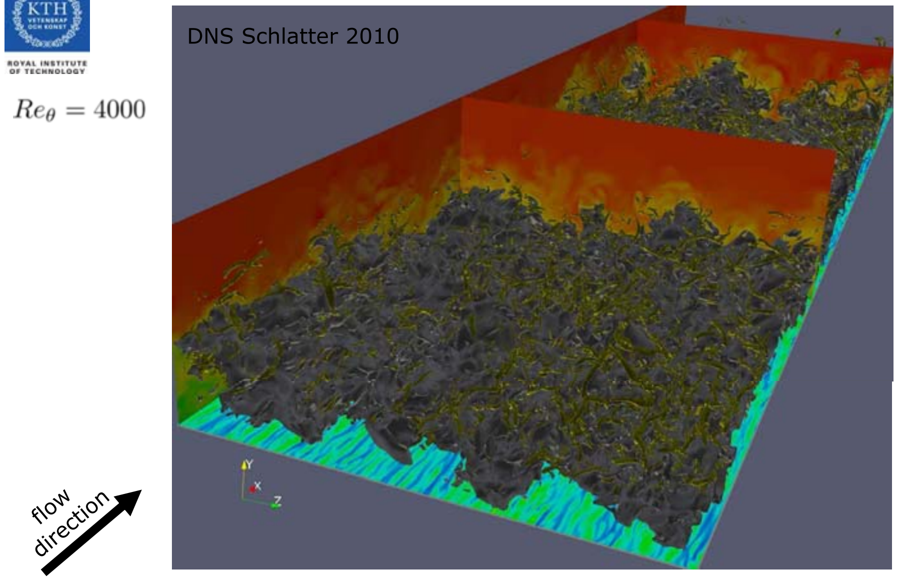
near the wall: high-speed region correlated with high vorticity region

$Re_\theta = 4000$



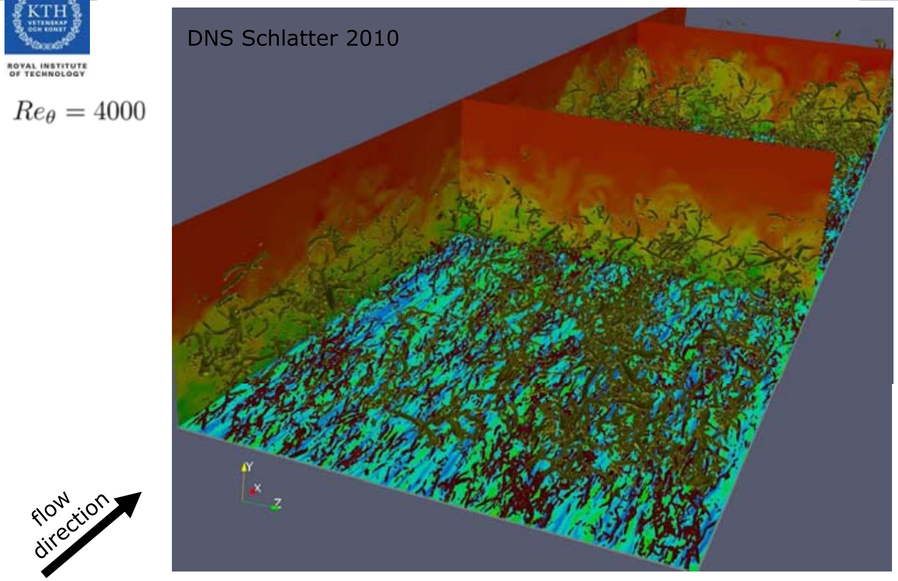
away from the wall: low-speed region correlated with high vorticity region

$Re_\theta = 4000$



strongly corrugated "edge" of the boundary layer

$Re_\theta = 4000$

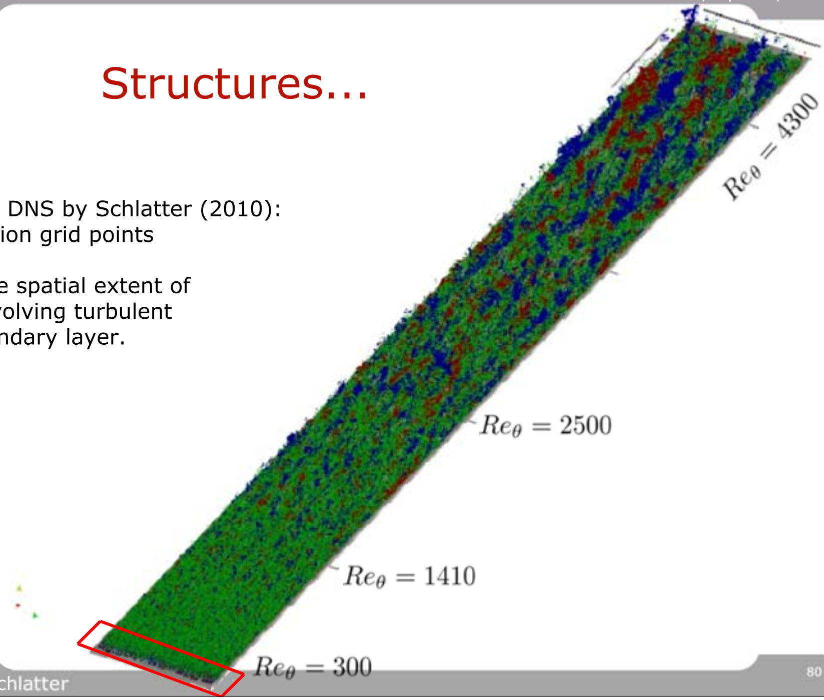


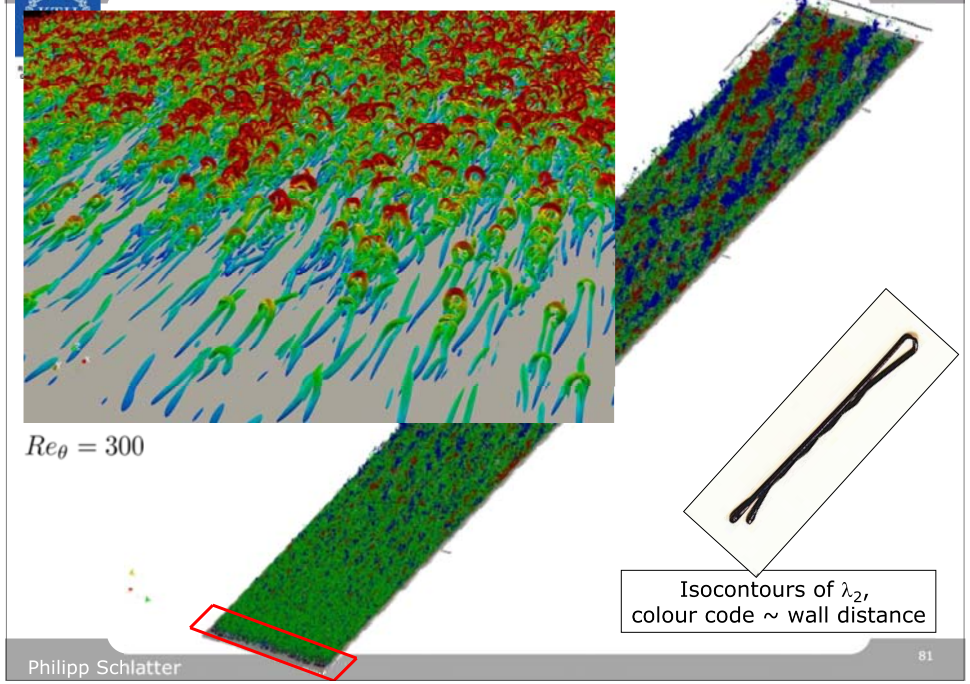
away from the wall: low-speed region correlated with high vorticity region

Structures...

Recent DNS by Schlatter (2010):
7.5 billion grid points

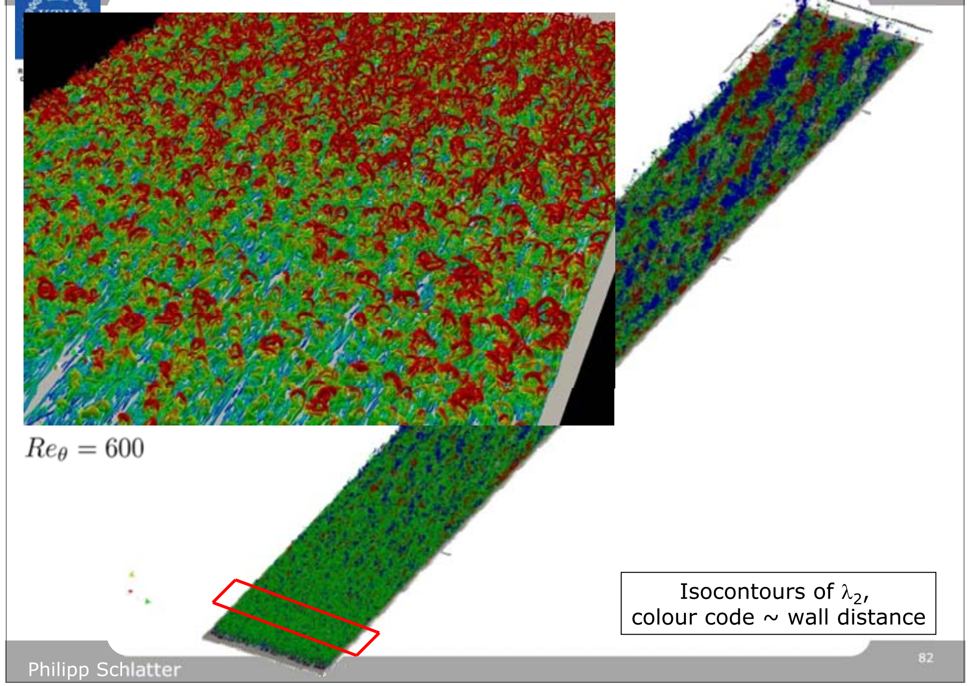
→ large spatial extent of a evolving turbulent boundary layer.





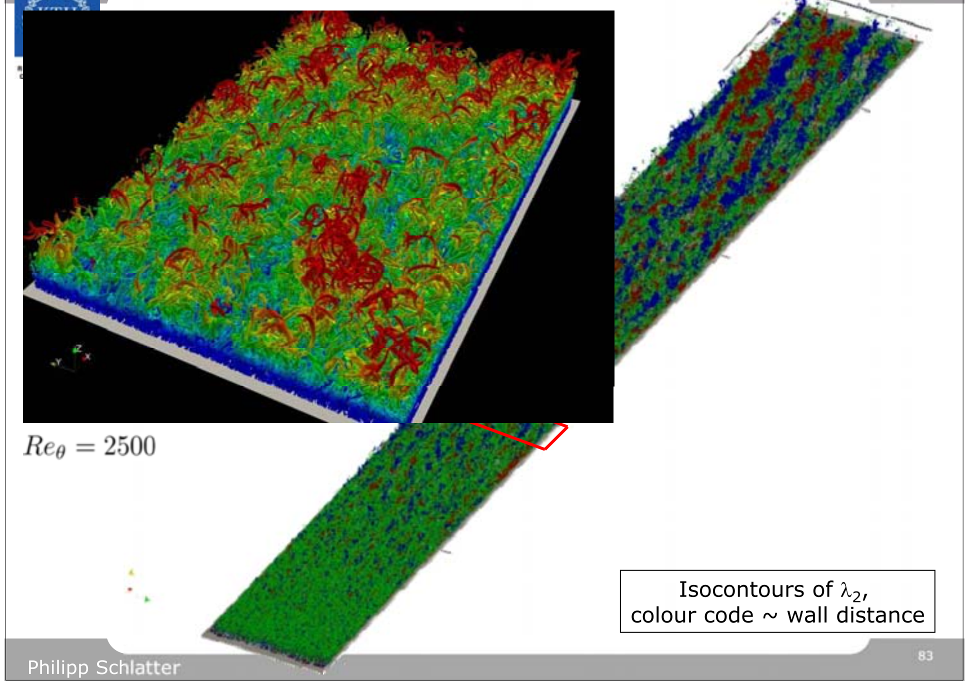
$Re_\theta = 300$

Isocontours of λ_2,
colour code ~ wall distance



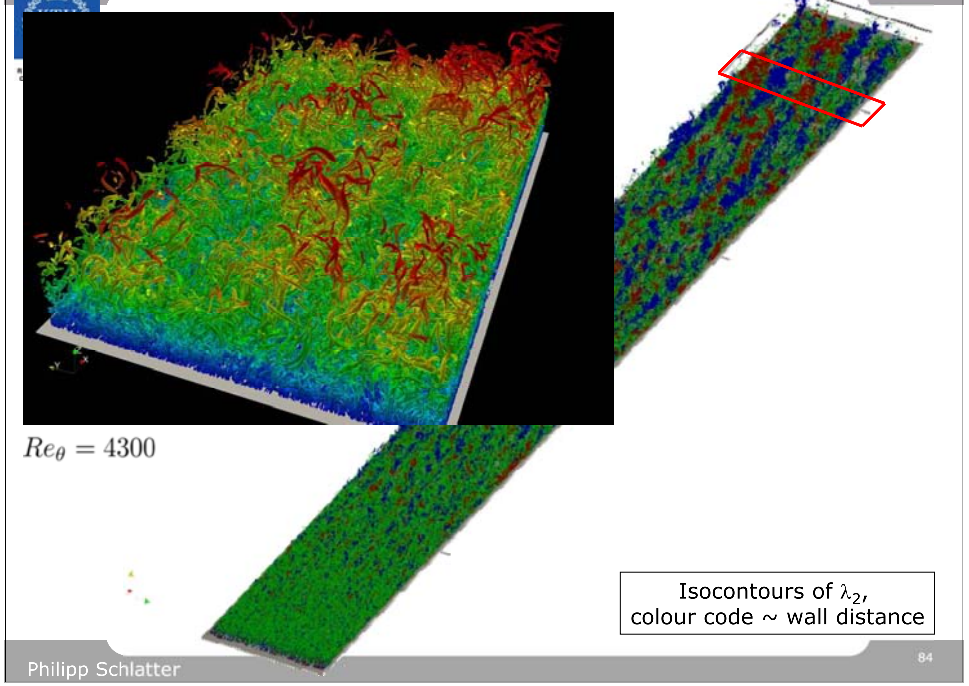
$Re_\theta = 600$

Isocontours of λ_2,
colour code ~ wall distance



$Re_\theta = 2500$

Isocontours of λ_2,
colour code ~ wall distance



$Re_\theta = 4300$

Isocontours of λ_2,
colour code ~ wall distance

Turbulent Scales

- Controversies and uncertainties over fundamental issues:
  - **mean velocity**: power law or logarithmic law
  - **scaling of Reynolds stresses**
- Key physical element: **Interaction between inner and outer regions**
  - How do turbulent structures (spectra, structure function etc.) vary with Reynolds numbers ?
  - Physical understanding of turbulence needs scale separation. DNS still only possible at quite low *Re*
- Scaling important for turbulence models
- Turbulence theories: high-Re asymptotics

"Some" References (NOT COMPLETE!)

- **Simulation data**:
  - Kim, Moin & Moser, JFM 1987: First channel DNS at Re_τ=180
  - Spalart, JFM 1988: "Reference" boundary-layer data up to Re_θ=1410
  - Moser, Kim & Mansour, PoF 1999: Channel data up to Re_τ=590
  - del Álamo & Jiménez, PoF 2003: Large-scale structures and scaling up to Re_τ=950
  - Hoyas & Jiménez, PoF 2006: Channel data and scalings up to Re_τ=2000
  - Schlatter et al., PoF 2009: Boundary-layer data up to Re_θ=2500
  - Schlatter et al., IJHFF 2010: Boundary-layer data up to Re_θ=4300 (Re_τ=1400)
- **Experimental data**:
  - Erm & Joubert, JFM 1991: Measurements with various trips for low-Re boundary layers
  - de Graaf & Eaton, JFM 2000: low-Re boundary-layer measurements
  - Tsuji et al., JFM 2007: Pressure measurements
  - Hutchins & Marusic, JFM 2007: Large-scale structures in boundary layers
  - Zagarola & Smits, JFM 1998: Superpipe data at high Reynolds numbers
  - Metzger & Klewicki, PoF 2001: Velocity scalings including high Re boundary layers (atmospheric boundary layers)
  - Monkewitz et al., PoF 2007: Composite velocity profile for boundary layers

4. Homework problem

Estimate ε in channel flow (1)

Consider a channel with half-height h, width L_z and bulk velocity U_b

- Work done by pressure:

$$W = (p + \Delta p)(2hL_z)U_b - p(2hL_z)U_b$$
$$= \Delta p(2hL_z)U_b$$

(Why can this be written like that?)

- Thermodynamics: first law for adiabatic processes

$$\Delta E = W$$

- Increase of inner energy is dissipation into heat

$$\Delta E = \varepsilon_{\text{tot}} \rho L_x (2h) L_z$$

- **What is the total dissipation composed of?**

$$\varepsilon_{\text{tot}} = ?$$

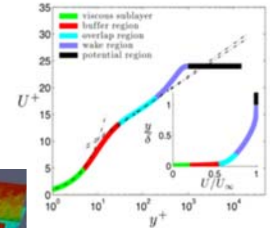
Estimate ε in channel flow (2)

- Total dissipation: $\varepsilon_{\text{tot}} = \dfrac{\Delta p U_b}{\rho L_x}$
- Integral force balance: Pressure gradient = wall shear
  $\dfrac{\Delta p}{L_x} = \dfrac{\tau_w}{h} \qquad \tau_w := \mu \left.\dfrac{\mathrm{d}U}{\mathrm{d}y}\right|_w$
- Thus we get $\varepsilon_{\text{tot}} = \tau_w \dfrac{U_b}{\rho h}$
- and with the definition of the skin friction coefficient
  $c_f = \tau_w / (\frac{1}{2}\rho U_b^2) \quad \Rightarrow \quad \varepsilon_{\text{tot}} = c_f \dfrac{U_b^3}{2h}$
- **Questions:**
  1. List all assumptions during the derivation
  2. What is the total dissipation composed of?
  3. Can you give a rough estimate for $\varepsilon = 2\nu\langle s'_{ij} s'_{ij}\rangle$?
     Use DNS data as justification or slide "Turbulence budget"!

5. Summary

Summary

- Reynolds average: $u = U + u'$
  $U = \langle u \rangle \, , \quad \langle U \rangle = U \, , \quad \langle u' \rangle = 0$
- Viscous scaling: $u_\tau = \sqrt{\tau_w/\rho} \qquad \ell_* = \nu/u_\tau$
  $U^+ = U/u_\tau \qquad y^+ = y/\ell_*$
- Law of the wall: $U^+ = \dfrac{1}{\kappa}\log y^+ + B$
  $U^+ = y^+$



- Structure of near-wall turbulence: