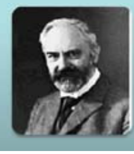
Scaling and Statistical Dynamics of Turbulent Structures (Part One)

Zhen-Su She

State Key Laboratory for Turbulence and Complex Systems
Peking University

Outline

1. Introduction: Scaling and self-organization



Why scaling?

- Multi-scale fluctuations (Kolmogorov)
- Boundary layer (Prandtl)

Definitions:

- Velocity Structure Functions: moments of velocity differences across a distance l
- Power law scaling: as l changes by a factor, the moment is rescaled by a power of that factor.

Three grand mathematics principles:

- Continuity (expansions, differentiation, etc)
- Invariance (equations, equalities, symmetry, etc)
- Similarity (geometry, topology, etc.)

1. Introduction: Scaling and self-organization

- Why is scaling important to us?
- Let us list the two research goals on turbulence:
  - More accurate predictions in science and engineering
    - Science: the study of mechanisms and structures
    - Engineering: CFD predictions in industry
  - Better philosophical understanding of the world around us
    - In 70s, chaos and nonlinear science
    - In 80s, coherent structures
    - In 90s, intermittency
- Understanding of how turbulent fluctuations arise (or sustained, or self-organized) may help to understand how the variety of other events are self-organized! – Universality!

1. Introduction: Scaling and self-organization

- A **power law** is a special kind of scaling law.
- Power laws appear in a wide variety of natural and man-made phenomena. For example:
  - frequencies of words in most languages
  - frequencies of family names
  - sizes of craters on the moon and of solar flares
  - earthquakes,
  - the popularity of books and music, etc…
- What does a power law state? This is not very clear! My guess:
  - It reflects the presence of a set of self-organized hierarchical structures, which is a **necessity** from a system perspective.
  - Turbulent fluctuations may be one of the most obvious system satisfying this principle of self-organization, compared to the systems mentioned above.

1. Introduction: Scaling and self-organization

- Scaling in homogeneous, isotropic turbulence:
  - Only multi-scale (space and time) property is concerned
  - It is related to the dynamics of the energy cascade
  - It ends with a multi-fractal description
  - Multi-fractality is general in natural (and social) science
- A chapter in the study of multi-fractality of turbulence: She-Leveque hierarchical scaling, its discovery and present status

VOLUME 72, NUMBER 3 PHYSICAL REVIEW LETTERS 17 JANUARY 1994

Universal Scaling Laws in Fully Developed Turbulence

Zhen-Su She and Emmanuel Leveque*
Department of Mathematics, University of Arizona, Tucson, Arizona 85721
(Received 12 July 1993)

北京大学 PEKING UNIVERSITY
LTCS 湍流与复杂系统国家重点实验室 State Key Laboratory of Turbulence and Complex Systems

1. Introduction: Scaling and self-organization

- Why is scaling important to us?
- Let us list the two research goals on turbulence:
  - More accurate predictions in science and engineering
  - Better philosophical understanding of the world around us
- Unfortunately, the above two goals drive two separate trucks of research (basic and applied). From an application perspective, scaling may be everything one needs!
- Can we try to design a research strategy which will help us to achieve the two goals simultaneously? This is what I would like to discuss today. – Connect scaling analysis to the description of the structural dynamics of complex, inhomogeneous flows!

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2. Scaling of intermittent fluctuation events

- Observations: nonlinear form for scaling exponents, issue for anomalous scaling
- Fluctuations models in energy dissipation (or energy cascade rate)
  - Log-normal model
  - Beta (mono-fractal) model
  - Random beta model
  - P (bi-splitting) model
  - etc…
- How can this be considered in connection to fluid structures?

$$\langle \delta v_\ell^p \rangle \sim \ell^{\zeta_p}, \quad \langle \epsilon_\ell^p \rangle \sim \ell^{\tau_p}.$$

$$\tau_p = -\tfrac{2}{3}p + 2[1 - (\tfrac{2}{3})^p].$$

$$\zeta_p = p/9 + 2[1 - (\tfrac{2}{3})^{p/3}].$$

$$\mu = 2/9.$$

$$D(h) = 1 + c_1(h - \tfrac{1}{9}) - c_2(h - \tfrac{1}{9})\ln(h - \tfrac{1}{9}),$$

$$c_1 = 3\left(\frac{1 + \ln\ln\frac{3}{2}}{\ln\frac{3}{2}} - 1\right), \quad c_2 = \frac{3}{\ln\frac{3}{2}}.$$

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2. Scaling of intermittent fluctuation events

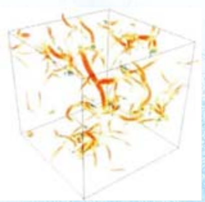
- Hierarchical structure formulation:
  - Finite amplitude of the most intermittent event
  - Discrete sequences
  - Invariance among the infinite sequence of events
- Estimate for the amplitude epsilon-infinity
- Geometrical interpretation for the amplitude epsilon-infinity
- Scaling is connected to the geometry of structures!

$$\epsilon_\ell^{(\infty)} \sim \delta E^\infty / t_\ell.$$

$$t_\ell \sim \bar{\epsilon}^{-1/3} \ell^{2/3}.$$

$$\epsilon_\ell^{(\infty)} \sim \bar{\epsilon} \left(\frac{\ell}{\ell_0} \right)^{-2/3} \sim \ell^{-2/3}.$$

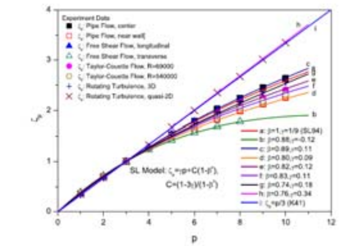
$$\tau_p = -\tfrac{2}{3}p + 2 + o(p) \quad (p \to \infty).$$

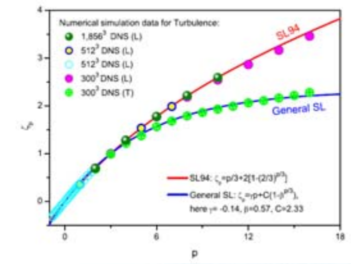


She et al., Nature, 1990.

2. Scaling of intermittent fluctuation events

- How does this formulation work for turbulence?
  - Since 1994, over 500 citations in more than 100 journals (in fact, #1 among PRL-published turbulence papers of the last 45 years)
  - Physicists are very enthusiastic (astrophysics in particular), but fluid mechanical community is not!
- Why? Homogeneous turbulence is an exception in nature!





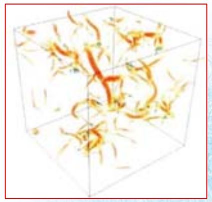
2. Scaling of intermittent fluctuation events

A fluid mechanical picture of those extremely intermittent events $\epsilon_\ell^{(\infty)}$ is as follows. Associated with a strong filamentary structure are quasi-two-dimensional spiral motions around the filament axis with only a small velocity component along the axis [19]. Such motions represent

This picture points out the nature of intermittency growth. It is the tendency towards the formation of local coherent structures that drives a strong deviation from the mean fluctuation level. In regions of these coherent

Intermittency = Coherent motion

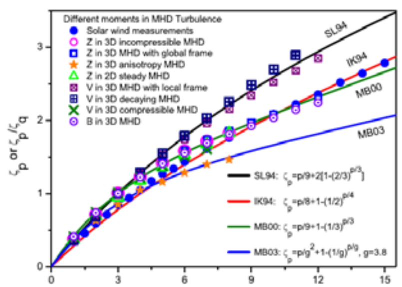
Hence, non-universality for anomalous scaling! More intermittency near the wall where coherence is stronger!

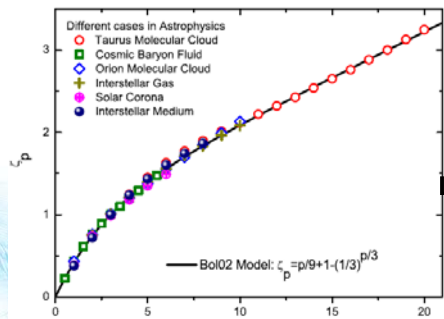


She et al., Nature, 1990.

2. Scaling of intermittent fluctuation events

- The theory has predictive power
  - MHD turbulence
  - Molecular clouds
  - Natural image
  - DNA anomalous composition





Larcy Law: exponent=1.74

2. Scaling of intermittent fluctuation events

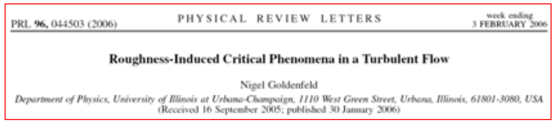
- Principle of self-organization:
  - Ensemble of fluctuations are organized around the mean or around the most intermittent events?
- We are led to suggestions to open minds to search for more relevant theoretical questions!

The importance of the turbulence ensemble manifests in ways to unambiguously evaluate statistical averages and to establish dynamical balances (and new equations) for a set of statistical variables.

She and Zhang, Acta Mecanica Sinica, 2009

3. Scaling for wall-bounded turbulence

- We will survey for a few recent theoretical work
  - Goldenfeld, PRL, 2006


PRL 96, 044503 (2006) PHYSICAL REVIEW LETTERS week ending 3 FEBRUARY 2006

Roughness-Induced Critical Phenomena in a Turbulent Flow

Nigel Goldenfeld

Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois, 61801-3080, USA
(Received 16 September 2005; published 30 January 2006)

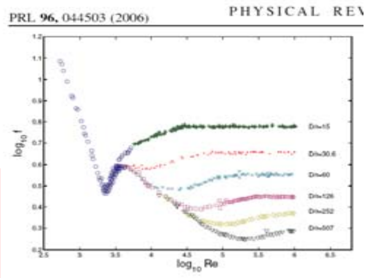


FIG. 1 (color online). Friction factor for turbulent flow in a rough pipe, as reported by Nikuradze [11]. The data were ...cted

Critical Phenomena		Turbulence	
Ferromagnet		Rough-pipe Flows	
Magnetization	$M(t,h)$	Friction factor	$f(Re,[r/R])$
Temperature	$M \sim H^{1/\delta}, t \to 0$	Reynolds number	$f \sim [r/R]^{1/3}, 1/Re \to 0$
External field	$M \sim t^{\beta}, H \to 0$	Roughness	$f \sim Re^{-1/4}, [r/R] \to 0$
Widom scaling		Goldenfeld scaling	
$M(t,h) = \|t\|^{\beta} f_M(h/t^{\Delta})$		$f = Re^{-1/4} g(Re^{3/4}[r/R])$	

2. Scaling of intermittent fluctuation events

- Summary:
  - Finite amplitude (rare) events are key to intermittency.
  - Hierarchical symmetry represents a form of the self-organization, which the turbulence ensemble and many other nonlinearly driven multi-scale systems possess, in one way or the other.
  - The concept of ensemble is equivalent to that of self-organization. Only then, RANS can make sense!
  - After confident description about multi-scale property, one needs to work on non-uniformity in space, which is everything when turbulence production and dissipation balance in non-trivial way.
- How should a study for scaling in inhomogeneous turbulence proceed?

3. Scaling for wall-bounded turbulence

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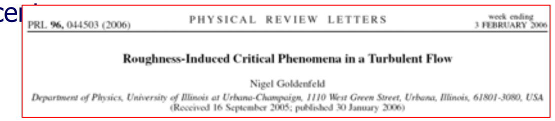
  - Goldenfeld, PRL, 2006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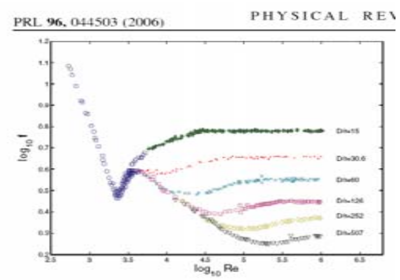
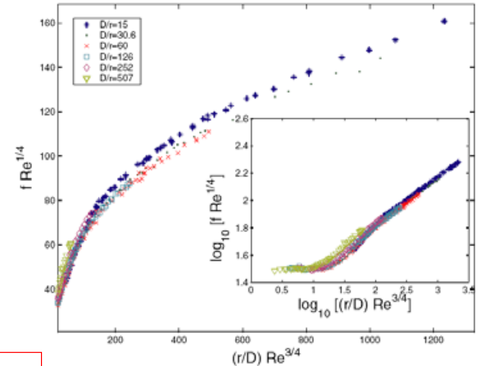


FIG. 1 (color online). Friction factor for turbulent flow in a rough pipe, as reported by Nikuradze [11]. The data were measured at different values of Re and r/D, and extracted



Friction factor, one integral quantity!

3. Scaling for wall-bounded turbulence

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  - Tao, PRL, 2009

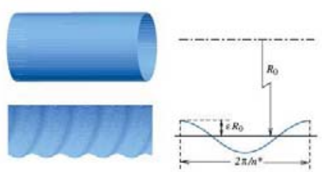
PRL 103, 264502 (2009) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2009

Critical Instability and Friction Scaling of Fluid Flows through Pipes with Rough Inner Surfaces

Jianjun Tao*

State Key Laboratory of Turbulence and Complex Systems, CAPT, Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, Beijing 100871, China

(Received 1 June 2009; published 28 December 2009)



$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial r}=-\frac{\partial P}{\partial x}+\frac{2}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}\right) \quad (1)$$

$$\frac{\partial U}{\partial x}+\frac{1}{r}\frac{\partial (Vr)}{\partial r}=0$$

with boundary conditions $U(x,R)=V(x,R)=0$.

$$U(x,r)=\frac{2}{R^2}\left(1-\frac{r^2}{R^2}\right)+\frac{\mathrm{Re}}{R^3}\frac{dR}{dx}\left[\frac{8}{225}-\frac{8}{225}\frac{r^2}{R^2}-\frac{4}{15}\frac{r^3}{R^3}+\frac{4}{15}\frac{r^4}{R^4}\right]. \quad (2)$$

$$-\frac{\partial \hat{P}}{\partial x}+\frac{2}{\mathrm{Re}}\left(\frac{\partial^2 \hat{U}}{\partial r^2}+\frac{1}{r}\frac{\partial \hat{U}}{\partial r}\right)=0 \quad \hat{U}(1)=0, \quad \left.\frac{\partial \hat{U}}{\partial r}\right|_{r=0}=0. \quad (3)$$

Three steps:
1)Determine approximate mean flow field with roughness;
2)Study its stability;
3)Determine critical Reynolds number and unstable modes.

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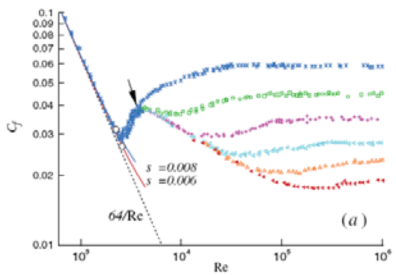
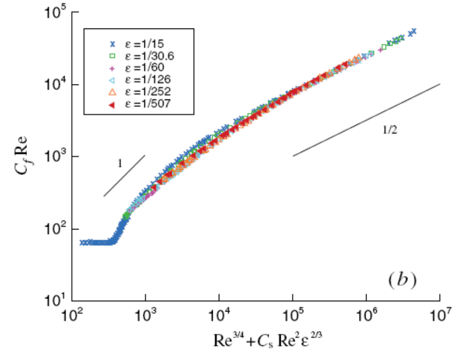
- Roughness shape factor is important!
- A new way to collapse!





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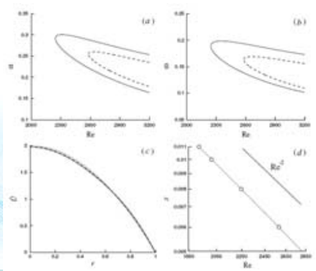
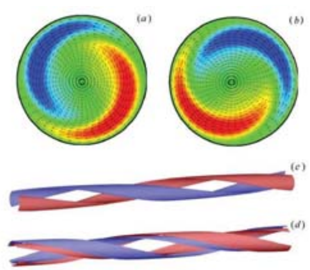
$$\hat{U}=2(1-r^2)-\left(\frac{\mathrm{Re}}{2}S\right)^2 g(r), \quad (4)$$

$$\tilde{u},\tilde{v},\tilde{w},\tilde{p}=\mathbb{R}[F(r),iG(r),H(r),J(r)]e^{i(m\phi+\alpha x-\omega\tau)}, \quad (5)$$

Neutral curves:





Unstable modes

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  - L'vov, Procaccia, Rodenko, PRL, 2008

PRL 100, 054504 (2008) PHYSICAL REVIEW LETTERS week ending 8 FEBRUARY 2008

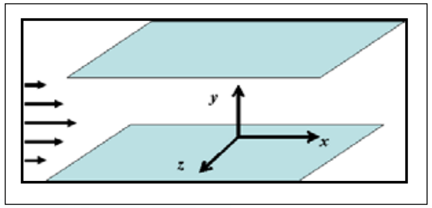
Universal Model of Finite Reynolds Number Turbulent Flow in Channels and Pipes

Victor S. L'vov, Itamar Procaccia, and Oleksii Rudenko

Department of Chemical Physics, The Weizmann Institute of Science, Rehovot 76100, Israel

(Received 30 May 2007; published 8 February 2008)

We will call it LPR theory!



Kim, Moin & Moser (1987) :Ret=183
Hoyas & Jimenez (2006): Ret=2003

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\partial}{\partial y}\left[\overline{u'v'}+\nu\frac{\partial U}{\partial y}\right],$$

PHYSICS OF FLUIDS 18, 011702 (2006)

Scaling of the velocity fluctuations in turbulent channels up to Re_τ=2003

Sergio Hoyas and Javier Jiménez[a)]

School of Aeronautics, Universidad Politécnica de Madrid, 28040 Madrid, Spain

(Received 25 October 2005; accepted 1 December 2005; published online 11 January 2006)

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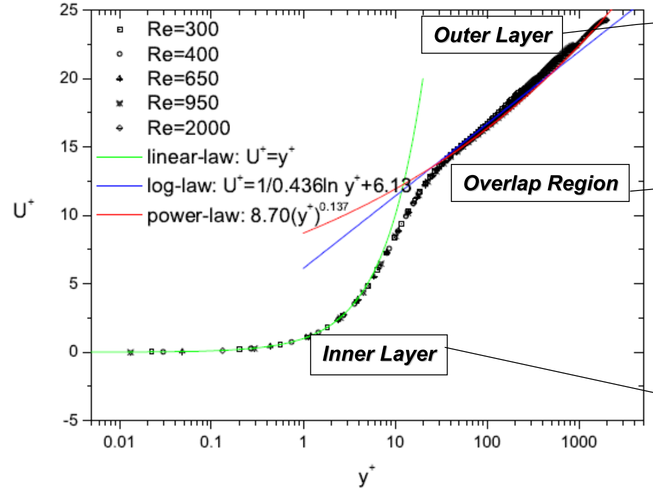
3. Scaling for wall-bounded turbulence



Defect Law

$$\frac{U_C - U}{u_0} = g[\frac{y}{D}] = g[\bar{y}]$$

Log Law or Power Law ?

$$U^+ = \frac{1}{\kappa} ln y^+ + B$$

$$U^+ = C[Re](y^+)^{\gamma[Re]}$$

Law-of-Wall

$$U^+ = f[\frac{y u_\tau}{\nu}] = f[y^+]$$

(Prandtl,1932; Barenblatt *et al*,1993; Zagarola *et al*,1997)

3. Scaling for wall-bounded turbulence

- We will survey for a few recent theoretical work
  - L'vov, Procaccia, Rodenko, PRL, 2008

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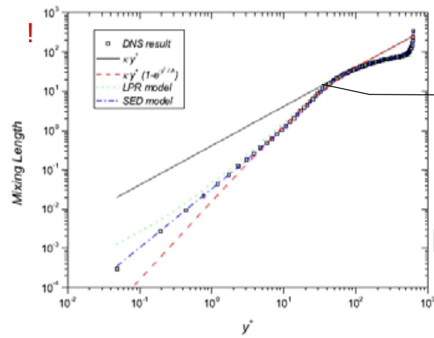
We will call it LPR theory!



L'vov et al (2008): "we argued that the controversy between log law and power law is moot, stemming from a rough estimate of the scaling function…"

LPR theory has proposed an estimate for actual spatial variation.

3. Scaling for wall-bounded turbulence

Log-Law V.S. Power-Law

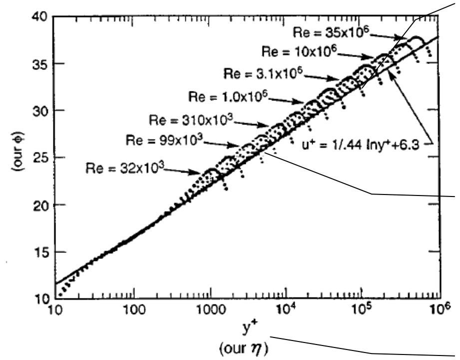


FIG. 1. The Princeton data exhibit the splitting of the data according to the Reynolds number and the rise of the curves above their envelopes in the ln η, φ plane, as mandated by the power law.

Zagarola et al (1997): "At sufficient high Reynolds numbers, the mean velocity profile in the overlap region is found to be better represented by a log law than a power law…"

Barenblatt & Chorin (1998): "If this collection looks to you, dear reader, like a single straight line, then we have lost the argument…"

George (2007): "the idea of a universal log law for wall-bounded flows is not supported by either the theory or the data."

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$$V^+(z^+) = \kappa^{-1}\ln(z^+) + B, \quad (1)$$

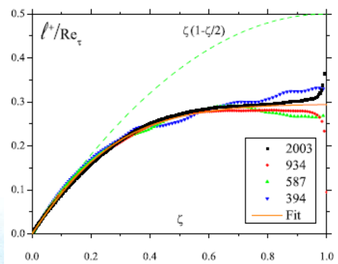
$$V^+(z^+) = C(Re_\tau)(z^+)^{\gamma(Re_\tau)}. \quad (2)$$

$$S^+ + W^+ = 1 - \zeta, \quad \zeta/L. \quad (3)$$

$$S^+W^+ \approx \varepsilon_K^+; \quad \varepsilon_K^+ = K^{+3/2}/[\kappa_K \ell_K^+]. \quad (5)$$

$$W^+ = (\kappa S^+ \ell^+)^2 r_W^{-3/2}, \quad (6)$$

$$\ell^+ \equiv [\ell_W^{+3}(\zeta)\ell_K^+(\zeta)]^{1/4} = \sqrt[4]{W^{+3} r_W^3 / S^{+3}\varepsilon_K^+}. \quad (7)$$



Using DNS data to determine length function!

3. Scaling for wall-bounded turbulence

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  - L'vov, Procaccia, Rodenko, PRL, 2008

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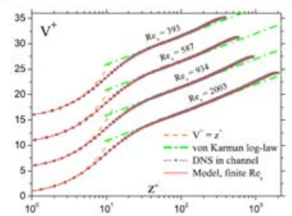

$$S^+ + W^+ = 1 - \zeta, \quad \zeta/L. \quad (3)$$

$$W^+ = (\kappa S^+ \ell^+)^2 r_W^{-3/2}, \quad (6)$$

$$r_W W^+ \approx \kappa_W \ell_W^+ \sqrt{K^+} S^+, \quad r_W(z^+) \equiv \left(1 + \frac{\ell_{\rm buf}^{+6}}{z^{+6}}\right)^{1/6}. \quad (4)$$

$$\frac{\ell^+(\zeta)}{\mathrm{Re}_\tau} = \ell_s \left\{1 - \exp\left[-\frac{\tilde{\zeta}}{\ell_s}\left(1 + \frac{\tilde{\zeta}}{2\ell_s}\right)\right]\right\} \quad (9)$$

$$S^+ = \frac{\sqrt{1 + (1-\zeta)[2\kappa\ell^+(\zeta)]^2 / r_W(z^+)^{3/2}} - 1}{2[\kappa\ell^+(\zeta)]^2 / r_W(z^+)^{3/2}}. \quad (8)$$

3. Scaling for wall-bounded turbulence

- More complex physics:
  - Boundary layers
  - Rough wall effects
  - Buoyancy effects
  - Compressibility effects
  - Rotation effects
  - MHD turbulence
  - etc…

- How will a systematic scaling analysis look like?
- How does the dynamics of structure play in the analysis?

3. Scaling for wall-bounded turbulence

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$$S^+ W^+ \approx \varepsilon_K^+; \quad \varepsilon_K^+ = K^{+3/2}/[\kappa_K \ell_K^+]. \quad (5)$$

- Comments:
  - Progress: address properties of fluctuations!
  - Yet, artificial wall-function and length function
  - Good agreements with DNS data!

$$r_W W^+ \approx \kappa_W \ell_W^+ \sqrt{K^+} S^+, \quad r_W(z^+) \equiv \left(1 + \frac{\ell_{\rm buf}^{+6}}{z^{+6}}\right)^{1/6}. \quad (4)$$

- features:
  - Not scaling exponents, but scaling functions (including constants) are targets for modeling!
  - Analysis is not systematic; no rule to follow! Extension to more complex situations is not obvious!

Scaling and Statistical Dynamics of Turbulent Structures (Part Two)

Zhen-Su She

State Key Laboratory for Turbulence and Complex Systems
Peking University

Outline

1. Introduction: Scaling and self-organization
2. Scaling for intermittent fluctuations
3. Scaling for wall-bounded turbulent flows
4. Structural Ensemble Dynamics (SED) approach
5. Application to channel and boundary layers
6. Open questions

5. Application of SED for channel and boundary layer flow

- Mean momentum equation : → $S^+ + W^+ = 1 - y.$

  $S^+ \equiv dU^+/dy^+$

  $W^+ \equiv -\overline{u'v'}^+$
- Eddy viscosity hypothesis: (Boussinesq, 1877) → $\nu_T = W^+/S^+.$
- Mixing length model: (Prandtl, 1932) → $l_M = W^{+1/2}/S^+.$
- Von Karman log-law → $l_M^0 = \kappa y^+$

  $U^+ = \kappa^{-1} ln(y^+) + B,$

Classical modeling is a top-down approach!
SED propose to determine them closely from empirical data!

4. Structure Ensemble Dynamics (SED) approach:

1. Existence of an ensemble → Sub-ensemble decomposition, if necessary!
2. Balance equations
   - mass, momentum, energy, etc. → Essence of the physics principles!
3. State functions
   - main variables → Solution of the closure problem!
4. Order functions
   - a combination of correlation functions → dynamics of fluctuation structures!

We will illustrate how LPR theory can be improved!

4. Structure Ensemble Dynamics (SED) approach:

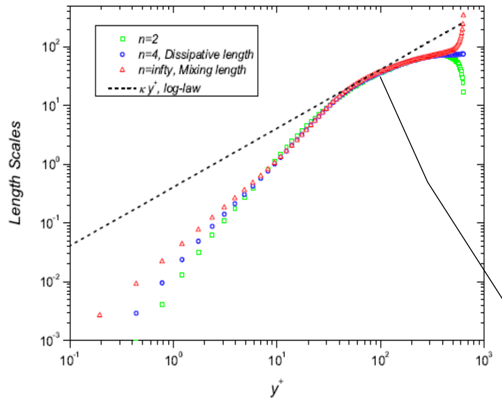
$$\varepsilon^+ = f(S^+, W^+, \ell^+) = W^{+(1+\frac{n}{2})} S^{+(1-n)} \ell^{+(-n)}$$

- Length is important!
- Scaling analysis: Express the energy dissipation in terms of mean shear, Reynolds stress and a length scale.
- We find an expression for the length function, depending on n.

$$\ell^+ = W^{+(\frac{1}{n}+\frac{1}{2})} S^{+(\frac{1}{n}-1)} \varepsilon^{+(-\frac{1}{n})},$$

4. Structure Ensemble Dynamics (SED) approach:

- Looking for the law governing the variation of the length function.



Dissipative Length (n=4)

$$\ell_\nu^+ = [(\frac{W^{+3}}{S^{+3}})/\varepsilon^+]^{1/4}.$$

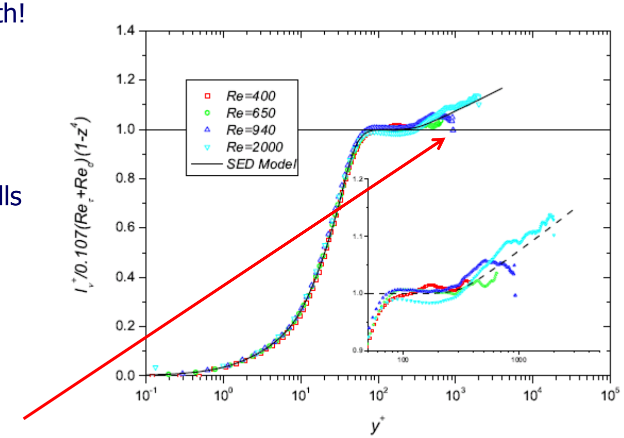
Mixing Length (n=∞)

$$l_M = W^{+1/2}/S^+.$$

Collapse in logarithm region, an evidence for decoupling of mean-field property from fluctuation structures.

5. Application of SED for channel and boundary layer flow

- Now, we try to model the dissipation length!
- Near-wall behavior:
  - a transition function
  - Its modeling calls for an order function!
- Discover non-trivial Reynolds number effects!



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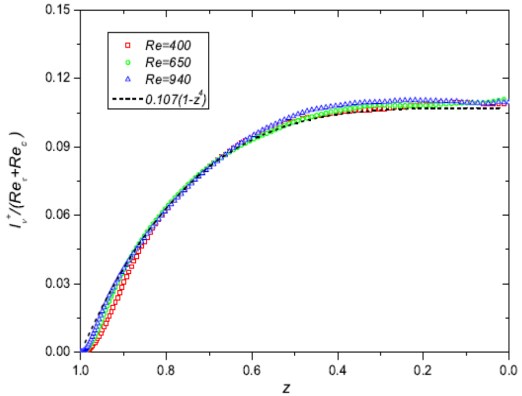
5. Application of SED for channel and boundary layer flow

- Now, we try to model the dissipation length!
- Central behavior:
  - 1-z^4
  - A Re-dependence
- Geometrical interpretation underway!



z=1-y is the distance to the center of the channel

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4. Structure Ensemble Dynamics (SED) approach:

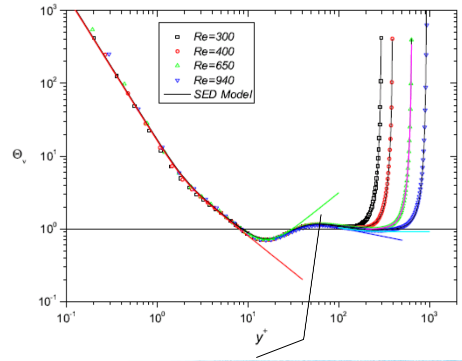
- An order-function is defined

$$\frac{\ell_{n_1}^+}{\ell_{n_2}^+} = (\frac{\varepsilon^+}{S^+W^+})^{\frac{1}{n_2}-\frac{1}{n_1}}.$$

- It displays a few transition from the wall to the center – rich physics captured!

Dissipation-shear ratio

$$\Theta_\nu = \frac{\varepsilon^+}{S^+W^+}.$$



Almost equal to 1 in log-layer.

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5. Application of SED for channel and boundary layer flow

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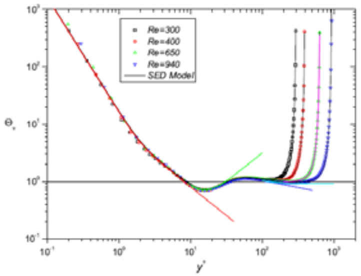


Dissipation-shear ratio

$$\Theta_\nu = \frac{\varepsilon^+}{S^+W^+}.$$

$$\Theta_\nu = 16(y^+)^{-2}\left(1+(\frac{y^+}{2})^4\right)^{\frac{1}{4}}\left(1+(\frac{y^+}{16})^4\right)^{\frac{2}{4}}\left(1+(\frac{y^+}{41})^4\right)^{-\frac{1.25}{4}}\left(1+(\frac{y^+}{170})^4\right)^{\frac{0.25}{4}}\left(1+(\frac{z}{0.26})^{-2.6}\right)^{\frac{2}{2.6}},$$

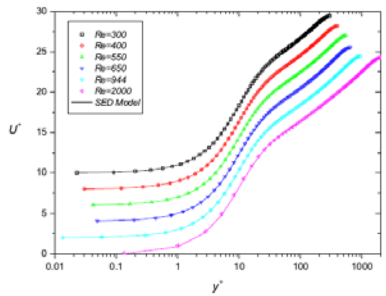
a refined description for all Reynolds number! **Why?**

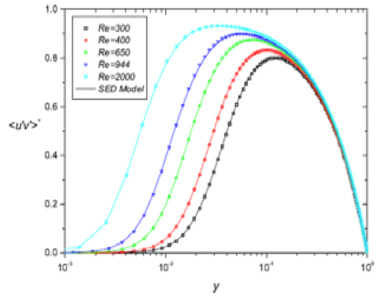
5. Application of SED for channel and boundary layer flow

- A complete closure from SED, with a specification of a length function and an order function
- Maximum relative error: 0.5%, better than LPR.

$$S^+ + W^+ = 1 - y$$
$$\Theta_\nu \mathcal{P}^+ = \varepsilon^+$$
$$\mathcal{P}^+ = S^+W^+$$
$$\varepsilon^+ = (W^{+3}/S^{+3})/\ell_\nu^{+4}.$$

$$l_M^{SED} = \ell^+_\nu \Theta_\nu^{1/4}.$$



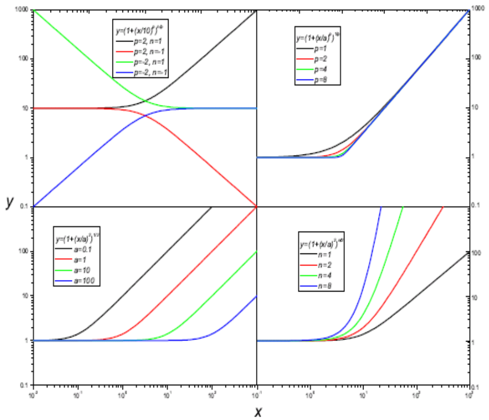


4. Structure Ensemble Dynamics (SED) approach:

- Modeling transition behavior for order functions in terms of a set of rational functions:

$$B(x) = c\left(1+(\frac{x}{a})^p\right)^{n/p},$$

a: transition point
p: sharpness of transition
n: transition scaling



Multiple transition points:

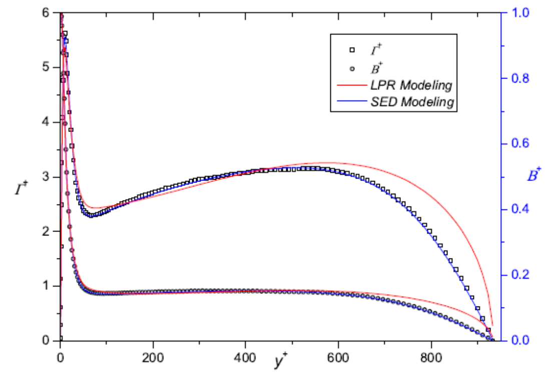
$$f(x) = M(x)\prod_i B_i(x) = M(x)\prod_i\left(1+(\frac{x_i}{d_i})^{p_i}\right)^{n_i/p_i}.$$

5. Application of SED for channel and boundary layer flow

- A sensitive comparison:

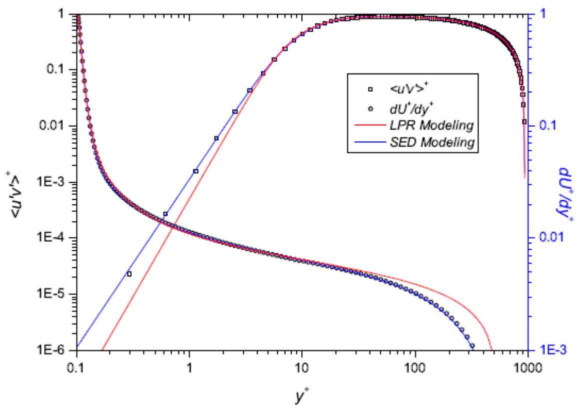
$$\Gamma^+ = y^+dU^+/dy^+$$
$$\mathcal{B}^+ = y^+/U^+dU^+/dy^+$$

- Discrepancy near the center in LPR model is corrected!
- This is because at the center, it is the turbulent transport that balances the energy dissipation. The equality between the mean-shear production and the dissipation is flawed there!



5. Application of SED for channel and boundary layer flow

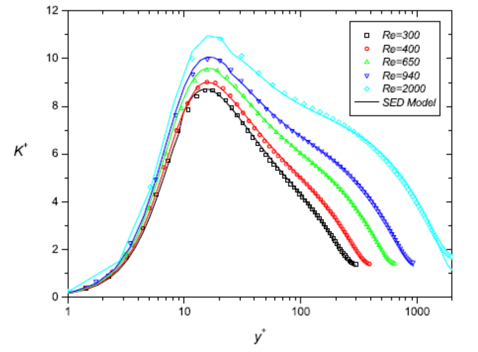
- A sensitive comparison:
- Discrepancy of W near wall and S at the cente LPR model near the wa are also corrected!



5. Application of SED for channel and boundary layer flow

- Prediction of the kinetic ene (or rms velocity).

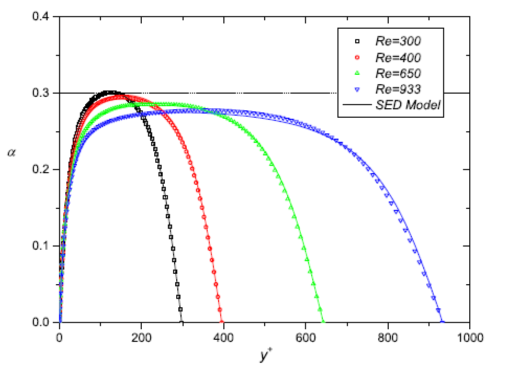
$$k^+ = W^+/\alpha_k$$



5. Application of SED for channel and boundary layer flow

- Bradshaw function is anothe example of the order functi

$$\alpha_k = W^+/K^+$$

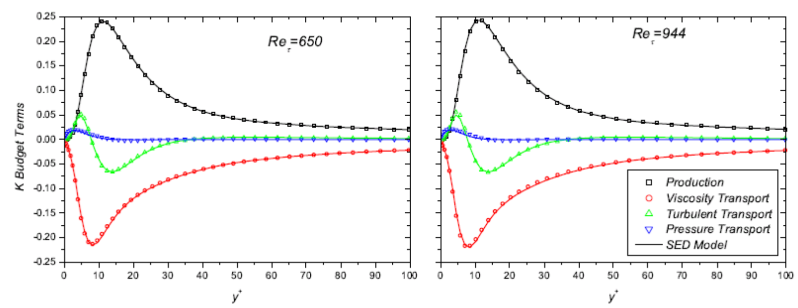


$$\alpha_k = B_\alpha \left(1 + \frac{64}{y^+}^{1.26} Re^{-0.2}\right)^{-1/1.2} \left(1 + \frac{0.78}{z}^{2} Re^{-0.4}\right)^{-1/2}$$

5. Application of SED for channel and boundary layer flow

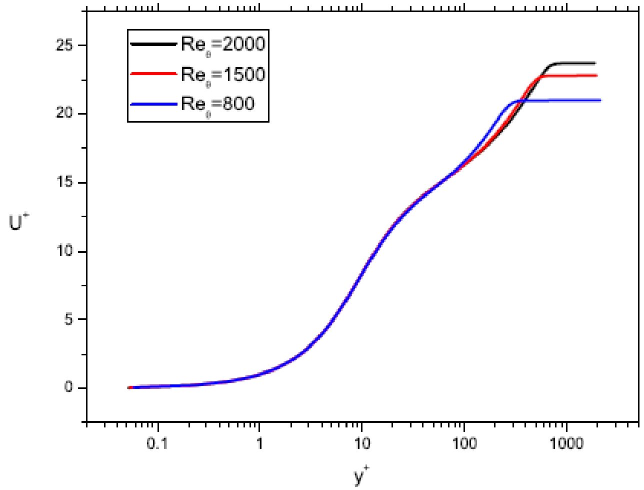
- Towards a complete second-order closure

$$k^+ = W^+/\alpha_k$$
$$\mathcal{C}^+ = \Theta_C S^+ W^+$$
$$\Pi^+ = (\Theta_\nu - \Theta_C - 1) S^+ W^+.$$



5. Application of SED for channel and boundary layer flow

- **P Schlatter and Q Li, 2010**



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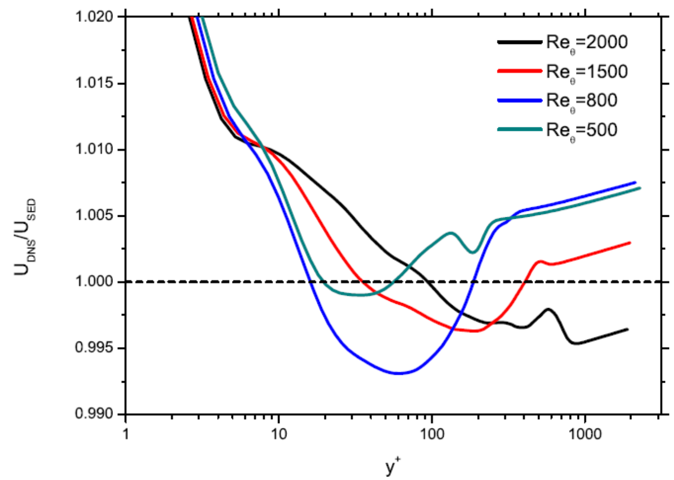

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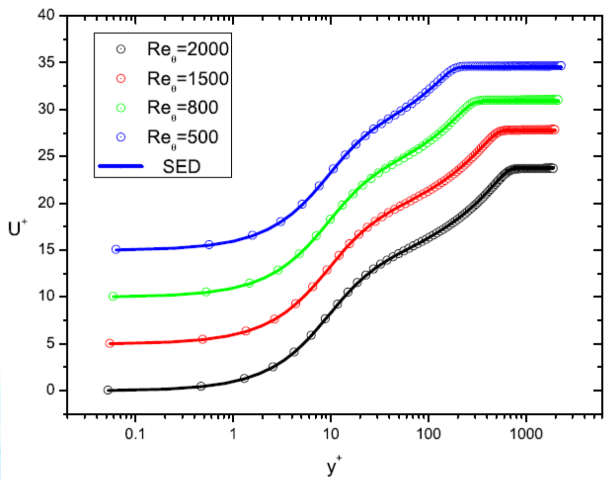
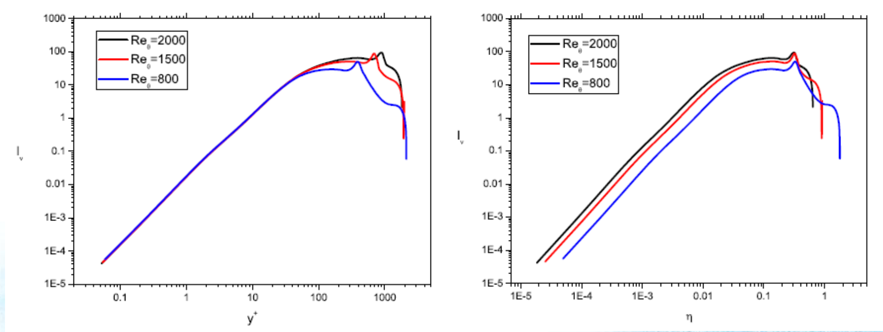


5. Application of SED for channel and boundary layer flow

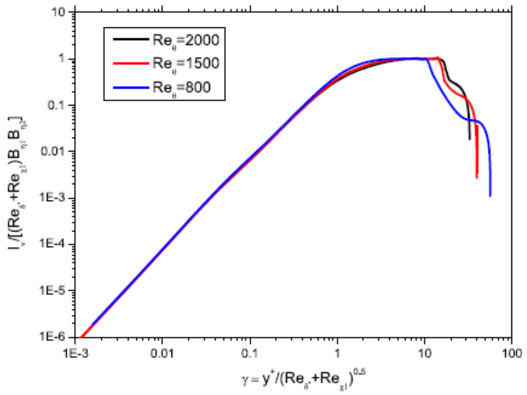
- **X Chen and Q Li, 2010**

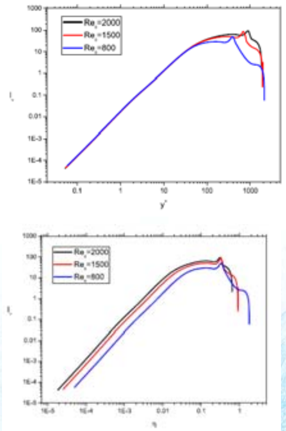
The dissipation Length function:



5. Application of SED for channel and boundary layer flow

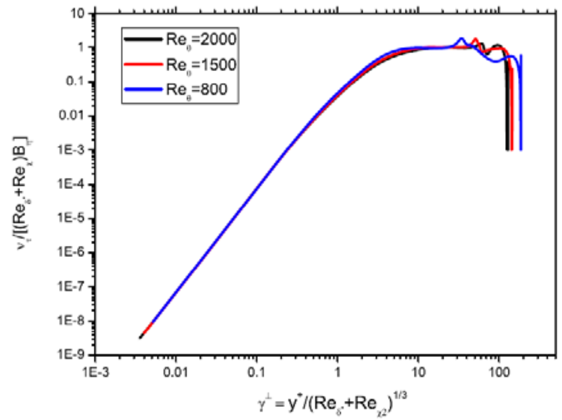
- **X Chen and Q Li, 2010**

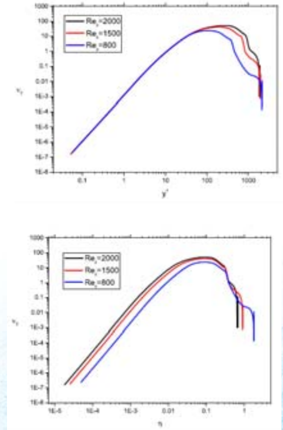




5. Application of SED for channel and boundary layer flow

- **X Chen and Q Li, 2010**

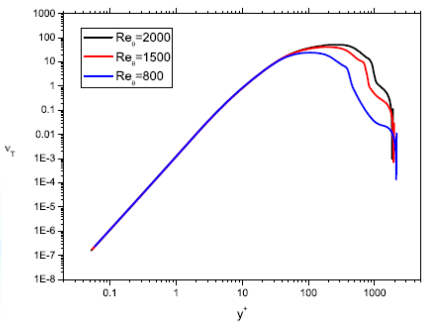


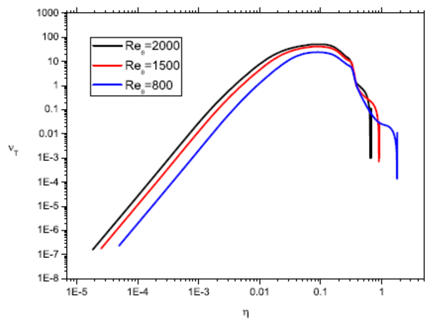


5. Application of SED for channel and boundary layer flow

- **X Chen and Q Li, 2010**

Eddy viscosity function





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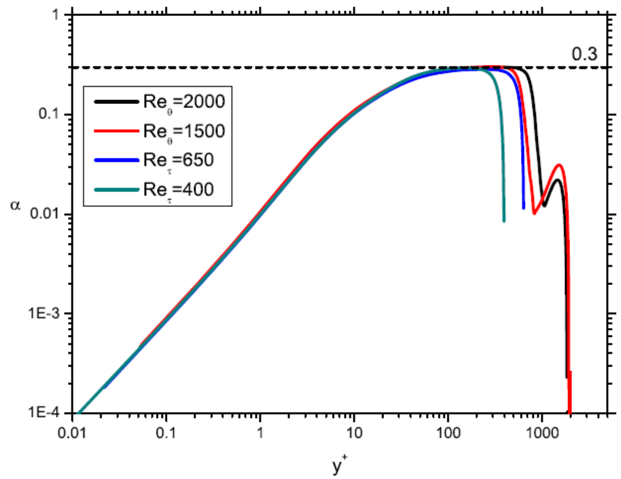
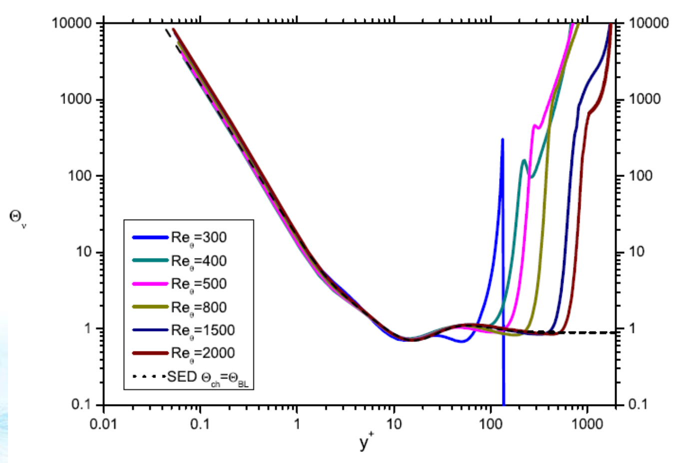
Bradshaw function



5. Application of SED for channel and boundary layer flow

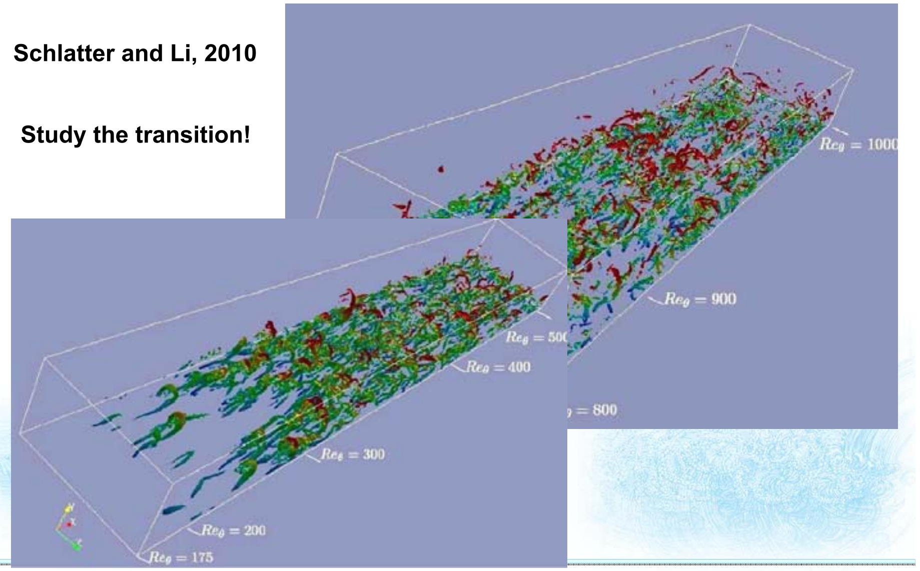
Shear-Dissipation ratio function



5. Application of SED for channel and boundary layer flow

Schlatter and Li, 2010

Study the transition!



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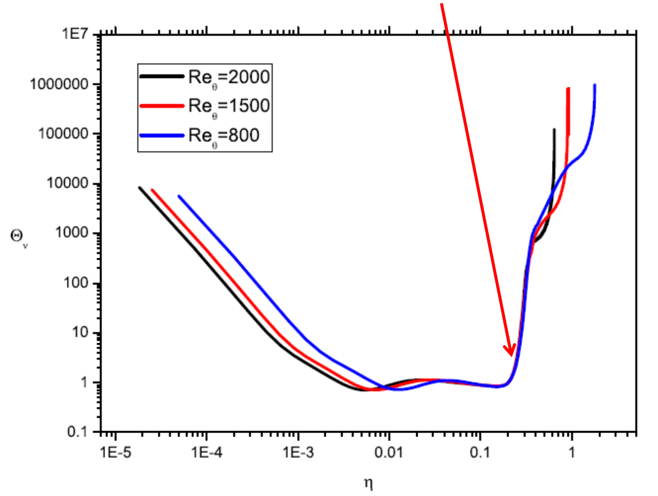
5. Application of SED for channel and boundary layer flow

Good collapse at a fixed eta!



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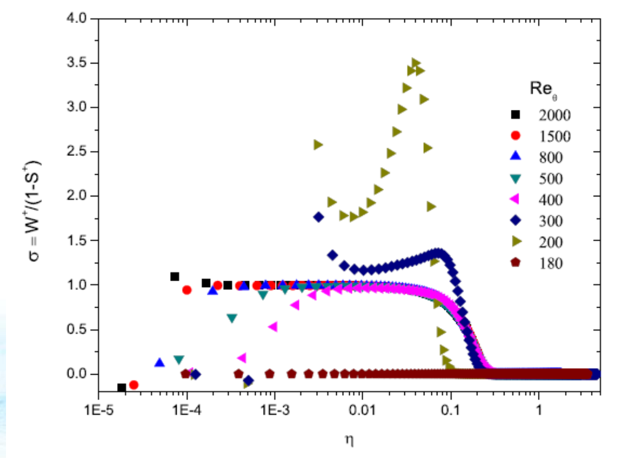
- **X Chen and Q Li, 2010**

This is an ensemble description!



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6. Summary and open questions

- SED is a new theoretical framework, designed to readdress the old closure problem of turbulence
- It is a platform to analyze DNS (and experimental) data.
- What I have shown is only the first part of SED, namely, a systematic revelation of spatial and parametric variations. The second part will come to interpret the variations, which will emphasize physical mechanisms and role of structures.
- Then, a connected study of structure-profile would become possible!
- **More exciting things would come, when many complex flows are analyzed, and laws behind the variations are revealed!**

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6. Summary and open questions

- A word about controversy between log-law and power-law
- It is a wrong question. Why?
- There are more than one possible expansions at the limit of high Reynolds numbers, and hence more than one possible leading terms. Log-law or power-law can be both correct!
- The real question is not the determination of the leading term, but a consistent expansion for work out higher order terms. This is what a beautiful applied mathematics theory should do.
- SED is moving towards the direction, when underlying physical constraint behind the algebraic structures of the order functions get derived.

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