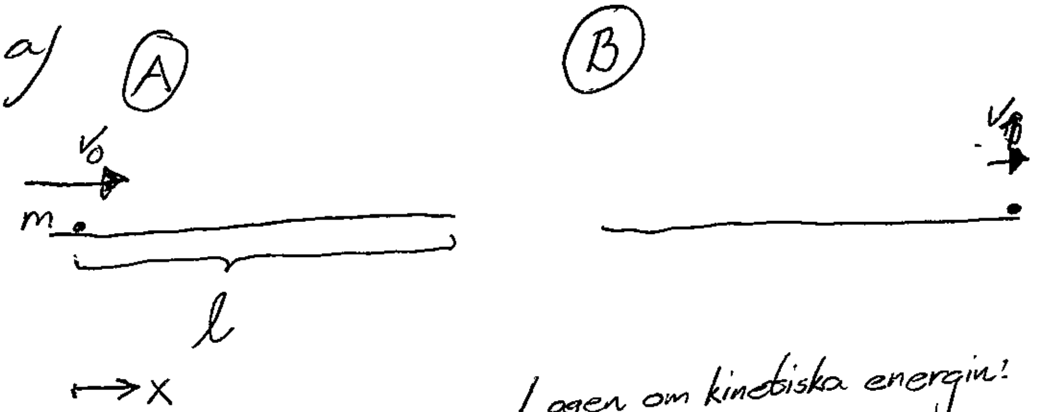
SG1102 Tentamen 120529, lösningsförslag

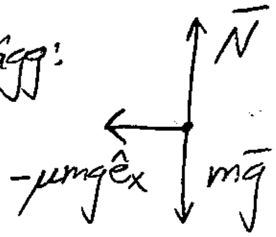
1. a)



Lagen om kinetiska energin:

$$U_{A-B} = T_B - T_A$$

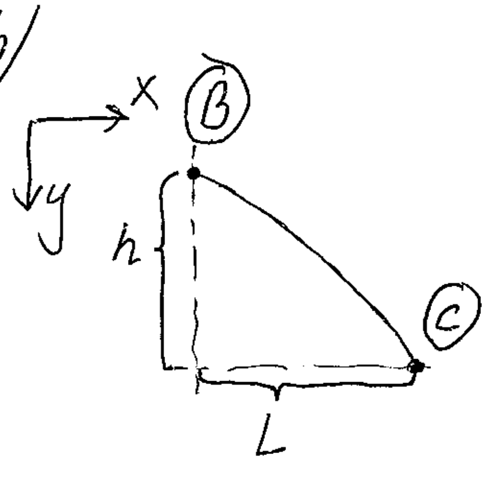
frilägg:



$$U_{A-B} = -\mu m g l \qquad T_A = \frac{mv_0^2}{2} \qquad T_B = \frac{mv_1^2}{2}$$

$$-\mu m g l = \frac{mv_1^2}{2} - \frac{mv_0^2}{2} \Rightarrow \underline{\underline{v_1 = \sqrt{2\left(\frac{v_0^2}{2} - \mu g l\right)}}} \qquad (1)$$

b)



frilägg: $\downarrow m g \hat{e}_y$

$$NII: \; m\ddot{x} = 0 \Rightarrow \dot{x} = C_1 \qquad x = C_1 t + C_2$$

$$m\ddot{y} = mg \Rightarrow \dot{y} = gt + D_1 \qquad y = \frac{gt^2}{2} + D_1 t + D_2$$

B.V: $B(t=0)$: $\dot{x} = v_1 \quad \dot{y} = x = y = 0 \Rightarrow C_1 = v_1 \quad C_2 = D_1 = D_2 = 0$

i C gäller: $x(t_c) = L \quad y(t_c) = h$

$$\Rightarrow v_1 t_c = L \Rightarrow t_c = \frac{L}{v_1} \qquad y(t_c) = h \Rightarrow \frac{g t_c^2}{2} = h$$

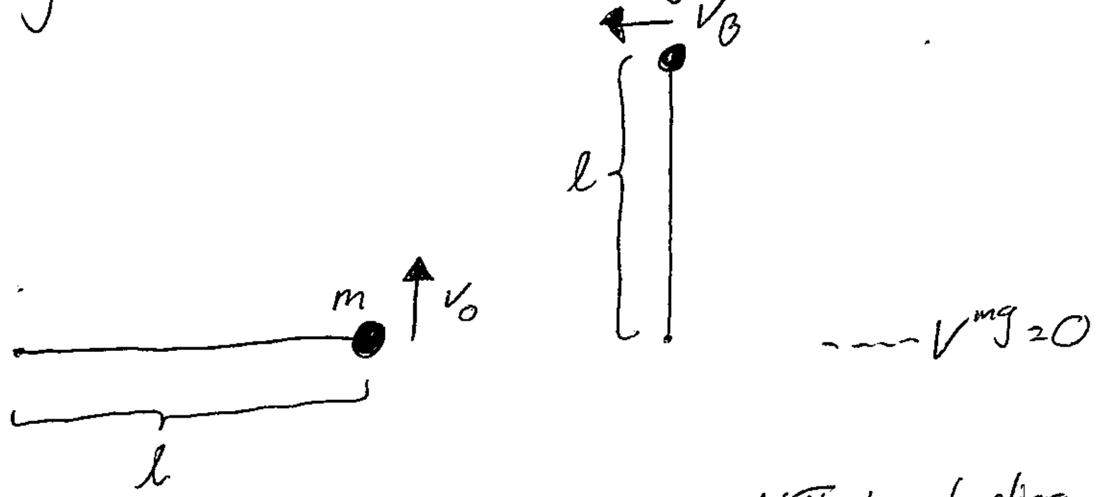
1 b/ folyt.

$$\Rightarrow \frac{gL^2}{2v_1^2} = h \Rightarrow v_1 = \sqrt{\frac{gL^2}{2h}} \quad (2)$$

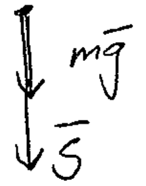
$$(1,2) \Rightarrow \sqrt{\frac{gL^2}{2h}} = \sqrt{v_0^2 - 2\mu g l}$$

$$\underline{\underline{v_0 = \sqrt{2\mu g l + \frac{gL^2}{2h}}}}$$

2. läge A: läge B:



friläg i B: $\hat{e}_t$ $\hat{e}_n$



NII i naturliga komponenter:

$\hat{e}_n$: $\dfrac{mv_B^2}{l} = S + mg$

$S \geq 0 \Rightarrow$ linan sträckt

$S = 0 \Rightarrow \dfrac{mv_B^2}{l} = mg \Rightarrow v_B = \sqrt{gl}$ (1)

försumma luftmotståndet $\Rightarrow$ konservativt!

Lagen om mekaniska energin:

$$T_A + V_A = T_B + V_B$$

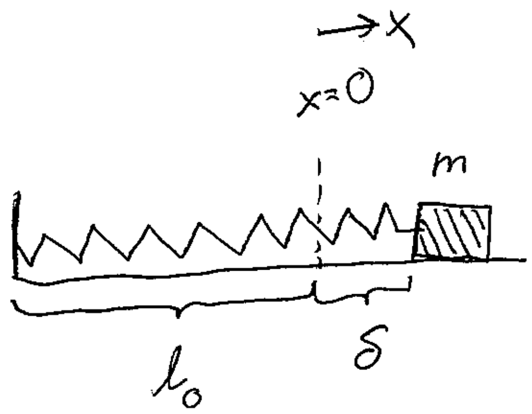
$$\Rightarrow \frac{mv_0^2}{2} = \frac{mv_B^2}{2} + mgl \Rightarrow v_B^2 = v_0^2 - 2gl \quad (2)$$

$$(1,2) \Rightarrow \sqrt{gl} = \sqrt{v_0^2 - 2gl}$$

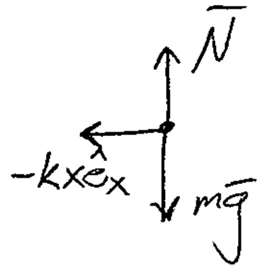
$$gl = v_0^2 - 2gl$$

$$\underline{\underline{v_0 = \sqrt{3gl}}}$$

3.



frilägg:



$$N\mathrm{II}:\ m\ddot{x} = -kx$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{k}{m}},\quad x = A\sin\omega_n t + B\cos\omega_n t$$

$$B.V:\ t=0 \Rightarrow x=\delta,\ \ \dot{x}=0 \Rightarrow A=0,\ B=\delta$$

$$\dot{x} = -\delta\omega_n \sin\omega_n t \qquad \ddot{x} = -\delta\omega_n^2\cos\omega_n t$$

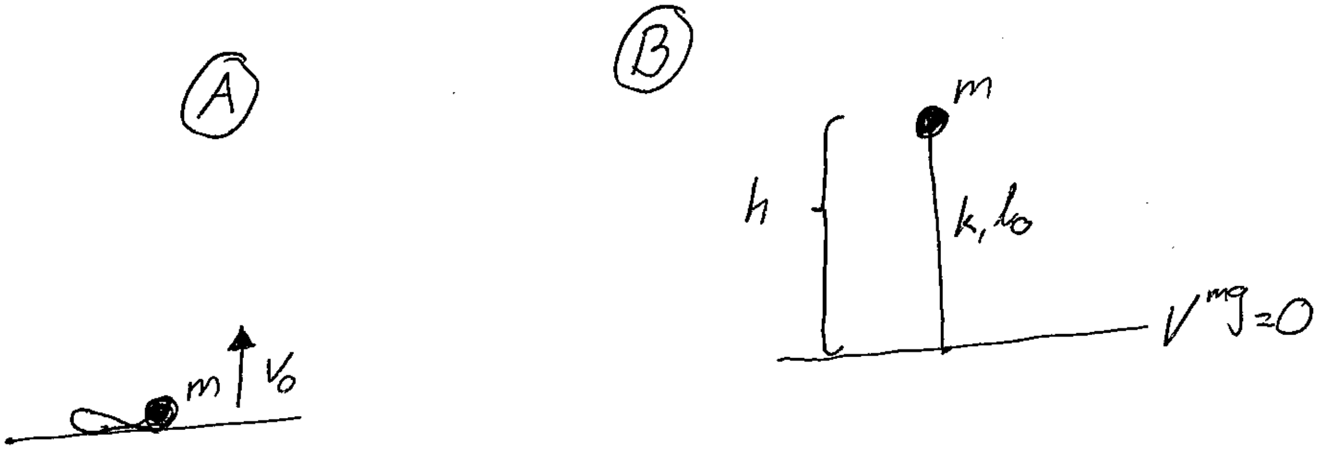
$$|\ddot{x}|_{max} = \delta\omega_n^2$$

givet: $|\ddot{x}|_{max} = g \Rightarrow \delta\omega_n^2 = g$

$$\delta = \frac{g}{\omega_n^2} = \frac{mg}{k}$$

svar: $\underline{\delta = \frac{mg}{k}}$

4.



fall 1: $h < l_0$

Lagen om mekaniska energins bevarande:

$$T_A + V_A = T_B + V_B$$

$$\frac{m v_0^2}{2} = mgh \Rightarrow h = \frac{v_0^2}{2g}$$

linan sträcks precis om $\frac{v_0^2}{2g} = l_0 \Rightarrow v_0 = \sqrt{2 g l_0}$

fall 2: $h \geq l_0$:

$$T_A + V_A = T_B + V_B$$

$$\frac{m v_0^2}{2} = mgh + \frac{k(h - l_0)^2}{2}$$

$$h^2 \frac{k}{2} + h(mg - k l_0) + \frac{k l_0^2}{2} - \frac{m v_0^2}{2}$$

$$h^2 + h\left(\frac{2mg}{k} - 2l_0\right) + l_0^2 - \frac{mv_0^2}{k} = 0$$

$$h = l_0 - \frac{mg}{k} \pm \sqrt{\left(l_0 - \frac{mg}{k}\right)^2 + \frac{mv_0^2}{k} - l_0^2}$$

$$h = l_0 - \frac{mg}{k} \pm \sqrt{\cancel{l_0^2} + \frac{m^2g^2}{k^2} - \frac{2mgl_0}{k} + \frac{mv_0^2}{k} - \cancel{l_0^2}}$$

positiva roten är sökt h!

$$h = l_0 - \frac{mg}{k} + \sqrt{\frac{m}{k}\left[\frac{mg^2}{k} - 2gl_0 + v_0^2\right]}$$

<u>svar:</u> fall 1, $v_0 \leq \sqrt{2gl_0}$: $h = \dfrac{v_0^2}{2g}$

fall 2: $v_0 \geq \sqrt{2gl_0}$: $h = l_0 - \dfrac{mg}{k} + \sqrt{\dfrac{m}{k}\left[\dfrac{mg^2}{k} - 2gl_0 + v_0^2\right]}$