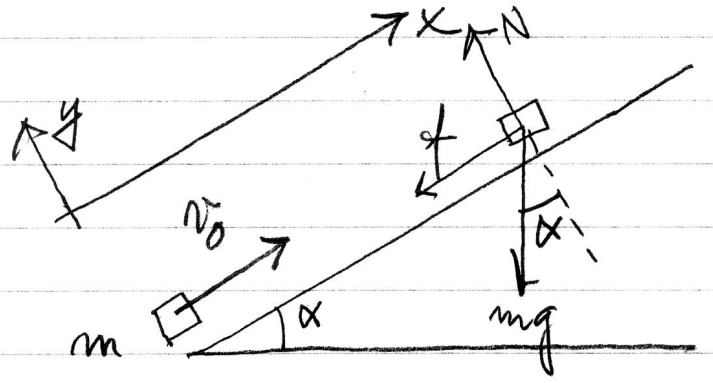
25/10 -08

Uppgift 1



$\uparrow$: $N - mg\cos\alpha = 0$, $N = mg\cos\alpha$

$$f = \mu N = \mu mg\cos\alpha$$

Hast. vall f. $x = l$

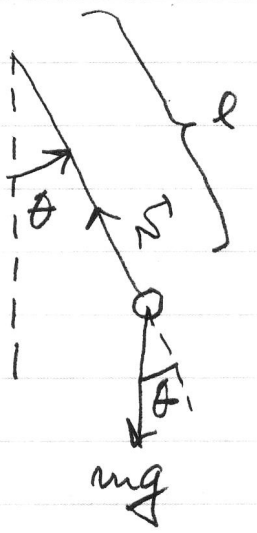
$$\underbrace{\frac{mv^2}{2}\Big|_{x=l}}_{=0} - \frac{mv_0^2}{2} = \int_0^l (-mg\sin\alpha - \mu mg\cos\alpha)\,dx$$

$$-\frac{mv_0^2}{2} = -mg(\sin\alpha + \mu\cos\alpha)\,l$$

$$l = \frac{v_0^2}{2g(\sin\alpha + \mu\cos\alpha)}$$

25/10-08

Uppgift 2



$\bar{e}_r$: $m(\ddot{r} - r\dot{\theta}^2) = mg\cos\theta - S$

$r = l = \text{konst.}$

$S = mg\cos\theta + ml\dot{\theta}^2$

Ber. $\dot{\theta}^2(\theta)$

$\bar{v} = l\dot{\theta}\bar{e}_\theta$

$$\frac{ml^2\dot{\theta}^2}{2} - \underbrace{\frac{ml^2\dot{\theta}^2}{2}\Big|_{\theta=\theta_0}}_{=0} = \int_{\theta_0}^{\theta} (-S\bar{e}_r + mg\cos\theta\bar{e}_r - mg\sin\theta\bar{e}_\theta)\cdot l\,d\theta\,\bar{e}_\theta$$

$$\frac{ml^2\dot{\theta}^2}{2} = -mgl\int_{\theta_0}^{\theta}\sin\theta\,d\theta = mgl(\cos\theta - \cos\theta_0)$$

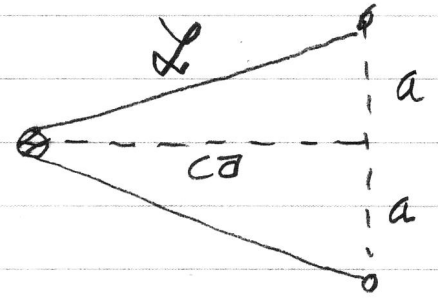
$$ml\dot{\theta}^2 = 2mg(\cos\theta - \cos\theta_0)$$

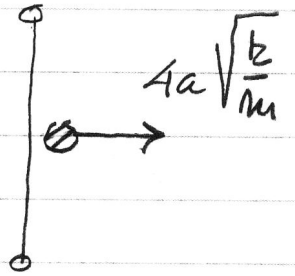
$\Rightarrow$ $S = 3mg\cos\theta - 2mg\cos\theta_0$

$S(0) = 3mg \Rightarrow \cos\theta_0 = 0,\ \theta_0 = \pm\frac{\pi}{2}$

25/10-08

Uppg 43





$T_0 = 0$, $V_0 = \frac{k}{2}(\Delta l)^2$ $\qquad$ $T = \frac{mv^2}{2} = 8ka^2$, $V = 0$

$\mathcal{L} = a\sqrt{1+c^2}$ enl. Pythagoras sats

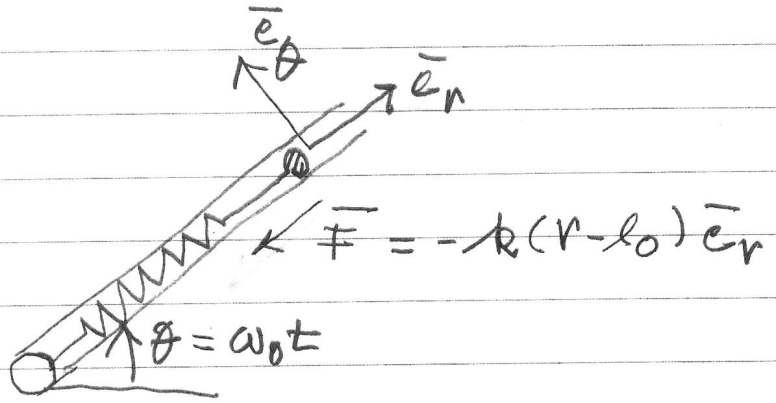
$\Delta l = 2\mathcal{L} - 2a = 2(\mathcal{L} - 1)$

$$0 + \frac{k}{2} \cdot 4a^2 (\sqrt{1+c^2} - 1)^2 = 8ka^2$$

$$(\sqrt{1+c^2} - 1)^2 = 4$$

$$c = \pm 2\sqrt{2}$$

25/10-08



$$\bar{e}_r: \quad m(\ddot{r} - r\dot{\theta}^2) = -k(r-l_0)$$

$$\ddot{r} + \frac{k}{m} r - r\dot{\theta}^2 = \frac{k}{m} l_0$$

$$\ddot{r} + \underbrace{\left(\frac{k}{m} - \omega_0^2\right)}_{\omega^2} r = \frac{k}{m} l_0$$

Enb. fekt $\omega^2 = 4\omega_0^2 \Rightarrow k = 5m\omega_0^2$

$$\ddot{r} + 4\omega_0^2 r = 5\omega_0^2 l_0$$

$$r = r_H + r_P, \quad r_P = \frac{5l_0}{4}$$

$$BV: \quad r(0) = r_H(0) + \frac{5l_0}{4} = \frac{5l_0}{4} \Rightarrow r_H(0) = 0$$

$$\dot{r}(0) = \dot{r}_H(0) + 0 = \frac{l_0\omega_0}{2}$$

$$r = \underbrace{A\cos 2\omega_0 t + B\sin 2\omega_0 t}_{r_H} + \frac{5l_0}{4}$$

5

25/10-08

BV $\Rightarrow$ $A = 0, B = \frac{l_0}{4}$

$$r = \frac{l_0}{4}(5 - 3\sin 2\omega_0 t)$$