Kungl Tekniska Högskolan **2008 10 03**

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Rigid Body Dynamics, SG2150

Hand in assigments 4, HT 2008

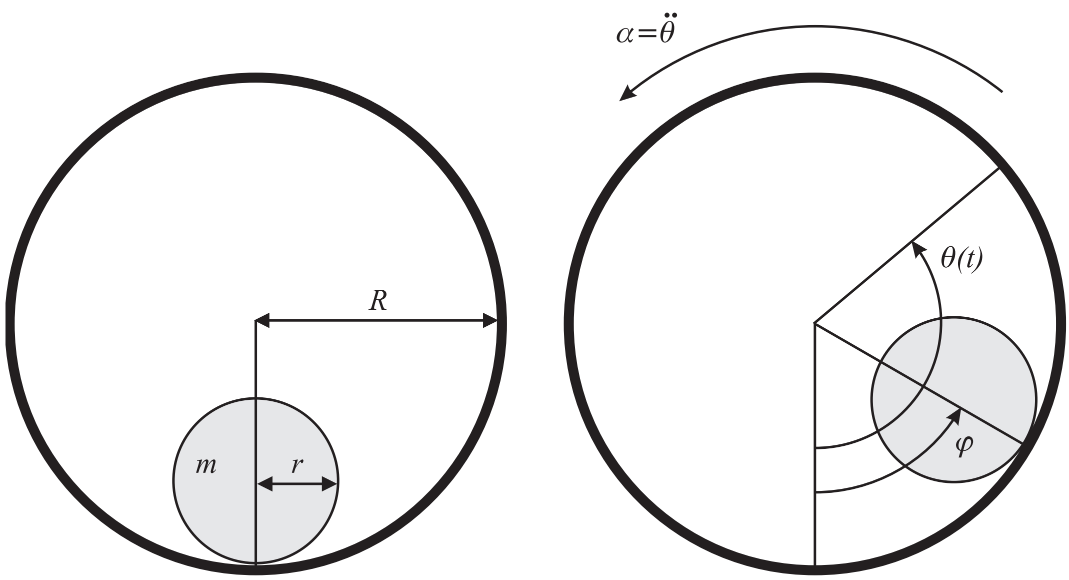
1) A homogeneous, straight, circular cylinder of mass m and radius $r(< R)$, rolls in a horizontal straight, circular cylindrical drum. The drum, which has radius R, and is rough on the inside can rotate about its fixed horizontal axis. Initially the drum and the cylinder are at rest with the cylinder in equilibrium at the bottom of the drum. By means of a controlled motor the drum is then given a constant angular acceleration α.

a) Find the equation of motion for the deflection angle φ of the cylinder using the Lagrange method with time dependent constraint. (Note that the drum has a prescribed motion and only supplies a time dependent constraint; it has no equation of motion.)

b) Calculate the deflection angular velocity $\dot{\varphi}$ as a function of the angle φ.

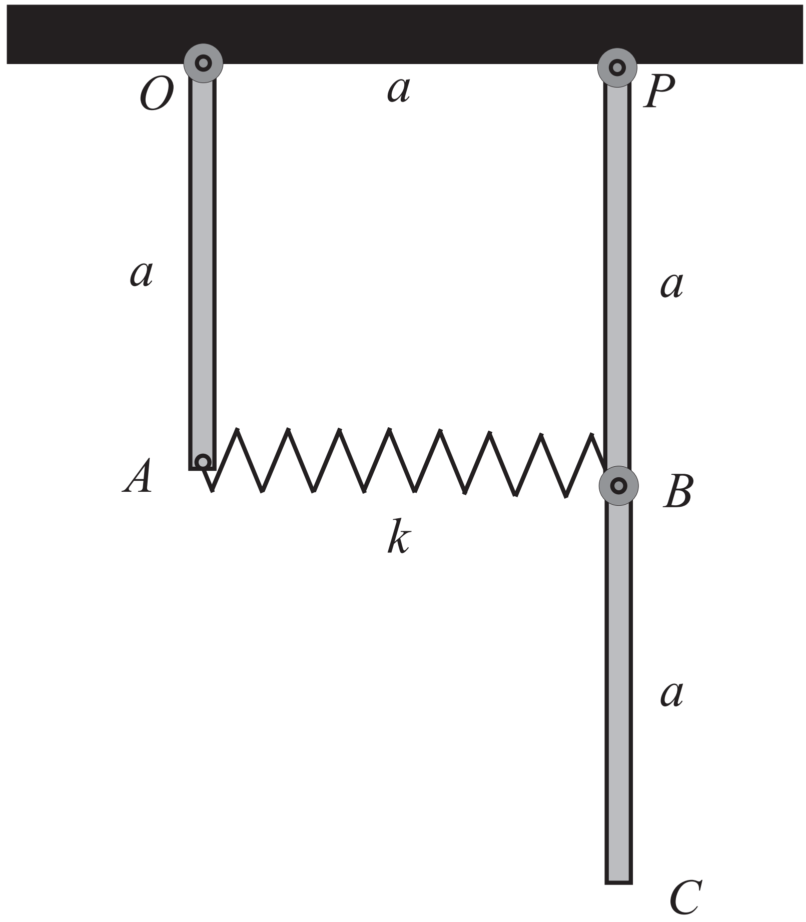
c) Because of the angular acceleration α of the drum the cylinder acquires a new equilibrium deflection angle. Calculate the value φ_0 of this angle.

d) Assume that the cylinder performs small oscillations about the new equilibrium. Find the angular frequency of these.



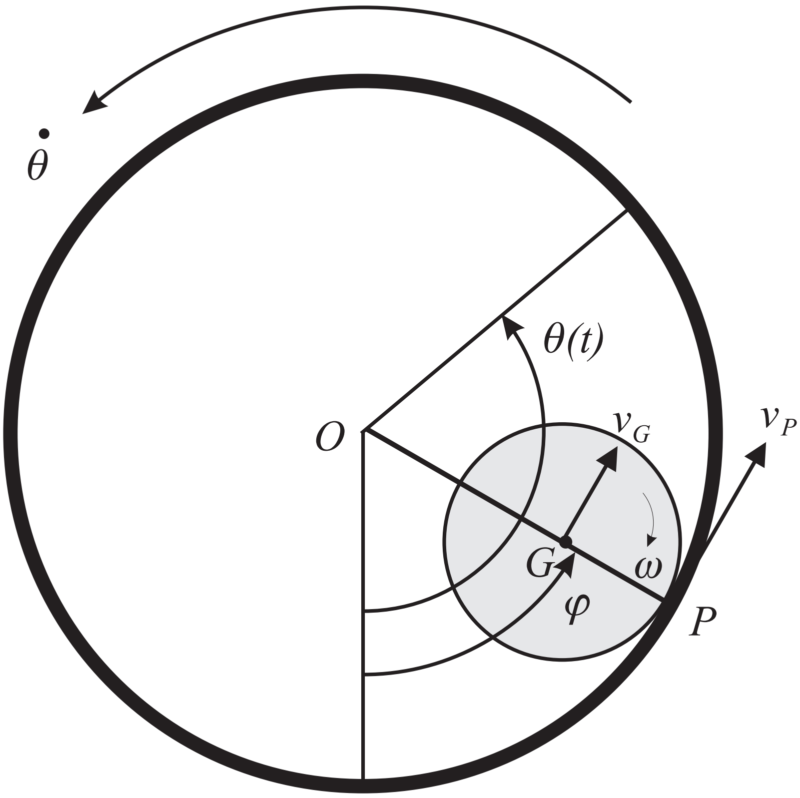
Don't miss problem 2 on next page!

2) A system consists of three equal homogenous rods OA, PB, and BC, of length a and mass m. Three rods are arranged according to the figure so that they all can swing in the same vertical plane about hinges of negligible friction at O, P, and B. O and P are fixed while B connects two of the rods. OA is thus a single physical pendulum, while PBC is a double pendulum. The end point A and the hinge at B are connected by a light straight spring of stiffness k and natural length a. the distance OP is also a, so that the equilibrium position of the system is as shown in the figure below. Find the angular frequencies for small oscillations about the equilibrium.



Hints and Answers:

The rolling constraint for the cylinder in the drum gives (a) $v_P = R\dot{\theta}$. By definition



of the angle φ we have (b) $v_G = (R-r)\dot{\varphi}$ for the cylinder C. of M. velocity. The connection formula for velocities in a rigid body finally gives (c) $v_P = v_G - r\omega$, where ω is the angular velocity of the cylinder. Equations (a)-(c) together give $\omega = (R/r)(\dot{\varphi} - \dot{\theta}) - \dot{\varphi}$, for the cylinder angular velocity.

The kinetic energy should be $T = \frac{1}{2}mv_G^2 + \frac{1}{2}J_G\omega^2$, the potential $V = mg(R-r)(1-\cos\varphi)$. The Lagrangian becomes

$$L = \frac{1}{2}m[\frac{3}{2}(R-r)^2\dot{\varphi}^2 - R(R-r)\dot{\varphi}\alpha t + \frac{1}{2}R^2\alpha^2 t^2] - mg(R-r)(1-\cos\varphi).$$

a) The equation of motion is $\frac{3}{2}(R-r)\ddot{\varphi} = \frac{1}{2}R\alpha - g\sin\varphi$.
b) The angular velocity as function of angle is obtained by multiplying the eq. of motion with $\dot{\varphi}$ followed by a time integration taking the initial conditions into account. One gets:

$$\dot{\varphi} = \pm\sqrt{\frac{2}{3}\frac{R\alpha}{R-r}\varphi + \frac{4}{3}\frac{g}{R-r}(\cos\varphi - 1)}.$$

c) The equilibrium angle is obtained by putting $\ddot{\varphi} = 0$ in the eq. of motion. This gives $\varphi_0 = \arcsin\left(\frac{R\alpha}{2g}\right)$.
d) Introduce $u = \varphi - \varphi_0$ in the equation of motion and expand around $u = 0$. One gets $\ddot{u} = -\omega_0^2 u$ where ω_0 is the required angular frequency. One finds that,

$$\omega_0^2 = \frac{2}{3}\frac{\sqrt{g^2 - (R\alpha/2)^2}}{R-r}.$$

Hints and answers for the three link pendulum:
The kinetic energy is, $T = T_{OA} + T_{PB} + T_{BC}$. The first two rods here have pure rotation about O and P respectively so their kinetic energies are $T = \frac{1}{2}\left(\frac{1}{3}ma^2\right)\dot{\varphi}_i^2$ for $i = 1, 2$. the link BC has $T_{BC} = \frac{1}{2}mv_G^2 + \frac{1}{2}\left(\frac{1}{12}ma^2\right)\dot{\varphi}_3^2$. Here v_G must be calculated. It depends on both φ_2 and φ_3 and their time derivatives. After assuming small deflection angles on finds the approximate kinetic energy,

$$T = \frac{1}{2}ma^2\left(\frac{1}{3}\dot{\varphi}_1^2 + \frac{4}{3}\dot{\varphi}_2^2 + \frac{1}{3}\dot{\varphi}_3^2 + \dot{\varphi}_2\dot{\varphi}_3\right).$$

The potential energy is

$$V = -\frac{a}{2}mg\left[\cos\varphi_1 + \cos\varphi_2 + (2\cos\varphi_2 + \cos\varphi_3)\right] + \frac{1}{2}k\left(|\boldsymbol{r}_A - \boldsymbol{r}_B| - a\right)^2.$$

The quadratic approximation near the equilibrium at $\varphi_i = 0,\ \ i = 1, 2, 3$ gives,

$$V = \frac{a}{2}\left[\left(\frac{mg}{2} + ka\right)\varphi_1^2 + \left(\frac{3mg}{2} + ka\right)\varphi_2^2 + \frac{mg}{2}\varphi_3^2 - 2ka\,\varphi_1\varphi_2\right].$$

The roots of the secular equation are complicated. For the limit $k = 0$ one should get: $\omega_1^2 = (3/2)(g/a)$, $\omega_{2,3}^2 = 3[1 \pm (2/7)\sqrt{7}](g/a)$. To study the roots for non-zero values of k one can introduce the dimensionless parameter $\epsilon = (ka)/(mg)$ in V and put $g = 10\,\mathrm{ms}^{-2}$ and $a = 1\,\mathrm{m}$, and then plot the roots $y = \omega^2$ as function of ϵ. The result is: