Rigid body dynamics, SG2150

Hand in assignments, batch 3, HT 2009

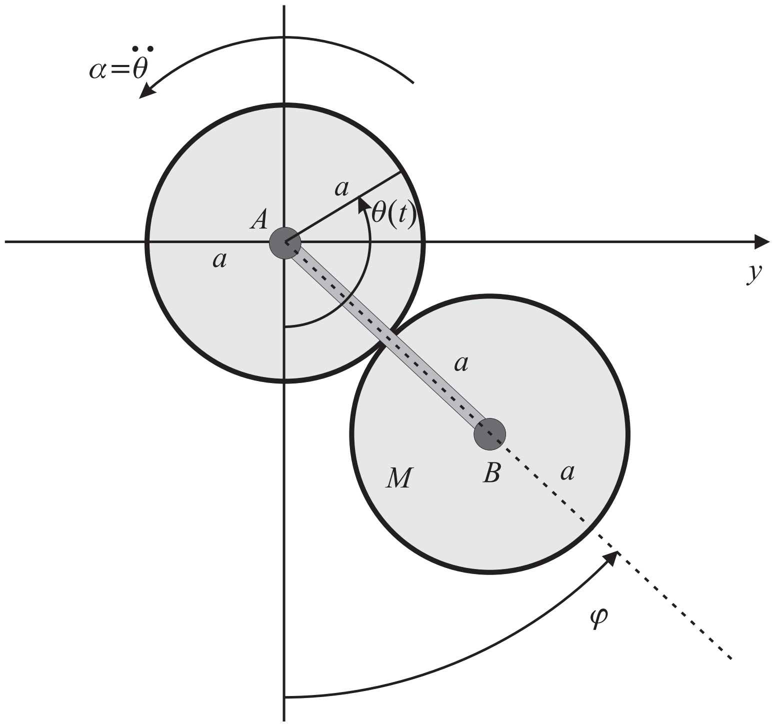
Due Tuesday 13/10

1) Two homogeneous circular discs in a vertical plane, both of radius a and mass M, with rough circumferences, are kept in contact by a linkage AB between the midpoints. The linkage is a thin homogeneous rod of mass m and length $2a$. The midpoint of one of the discs is fixed and a rotational motion about this midpoint is enforced in such a way that its orientation angle is $\theta(t)$. Assume that $\alpha = \ddot{\theta}$ is constant. Find the equations of motion and calculate the angular velocity $\dot{\varphi}$ of the linkage as a function of the angle φ.

Assume that before $t = 0$ the system is at rest with the linkage AB hanging vertically down. How large must the angular acceleration α be in order for the linkage AB to become horizontal in the ensuing motion?

Answer:

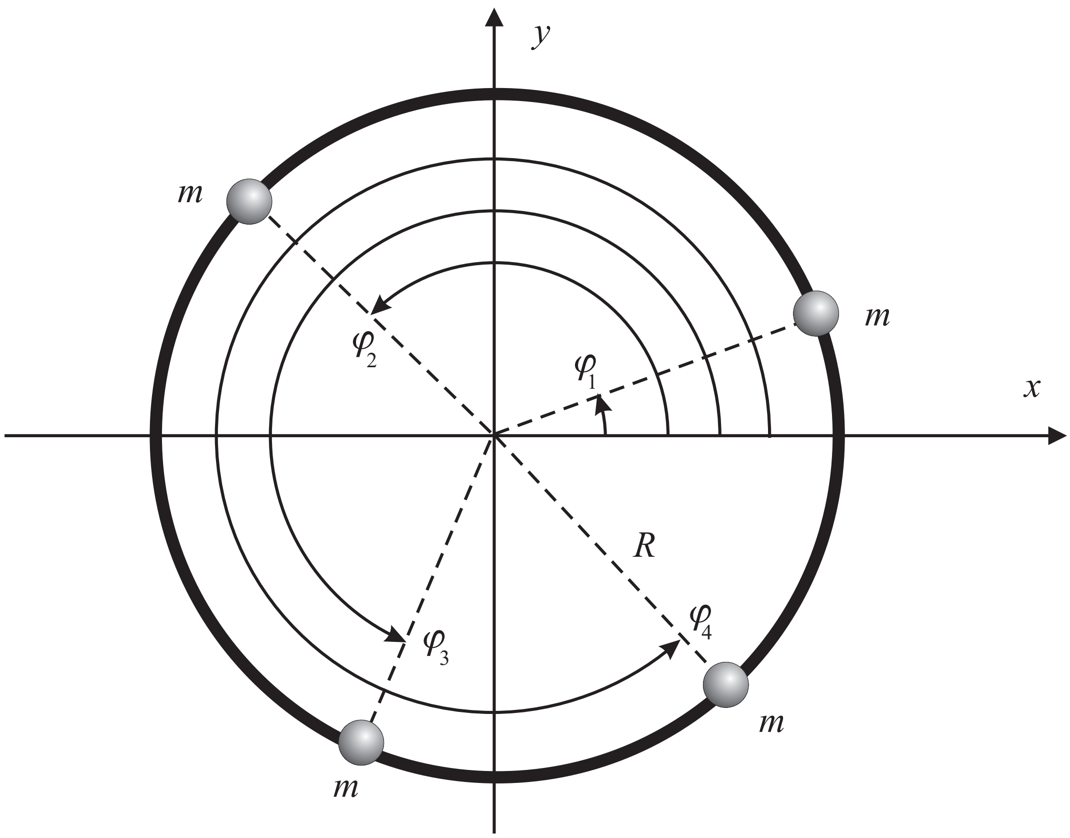
$$\alpha > \frac{4g}{\pi a}\left(1 + \frac{m}{2M}\right)$$



2) Four particles, each of mass m, can slide along a fixed horizontal smooth ring of radius R. Four light springs run along the ring connecting adjacent particles, and each of these springs have stiffness k. The springs are such that they have their natural length when the angle between the vectors from the center of the ring to adjacent particles are $\pi/2$. At equilibrium the particles therefore form a square.

Find the angular (eigen) frequencies of the system for oscillations near equilibrium.

Hints: Introduce cylinder coordinates $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ for the particles according to the figure. Find the kinetic and potential energies in terms of these. Note that these are not zero at the equilibrium positions. Therefore change to new coordinates u_1, u_2, u_3, u_4 according to $\varphi_1 = u_1$, $\varphi_2 = u_1 + \frac{\pi}{2} + u_2$, $\varphi_3 = u_1 + 2\frac{\pi}{2} + u_3$, $\varphi_4 = u_1 + 3\frac{\pi}{2} + u_4$, and write the Lagrangian function in terms of these. Then use the theory for coupled oscillations and Maple to find the characteristic frequencies.



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