Hand in assignments, Set 3, SG2150, Rigid body dynamics, HT08

1) A current I flows through a long thin straight metal wire. A charged particle of mass m and charge q can move in the vacuum around the wire. Use the Lagrange method to find the equations of motion of the particle. Are there any cyclic coordinates and what are the corresponding conserved generalized momenta?

Hints: See Sec. 10.1 in The Theory of Lagranges Method. Use cylinder coordinates with the z-axis along the wire. The vector potential for the magnetic field from the current in the wire is then,

$$\boldsymbol{A} = -\frac{2I}{c} \ln \rho \, \boldsymbol{e}_z. \tag{1}$$

Other fields and forces are negligible.

2) Determine the Lagrange function $L(\psi, \theta, \varphi, \dot{\psi}, \dot{\theta}, \dot{\varphi}) = T - V$ for the heavy symmetric top. The top has mass m and the distance between the tip and the center of mass is ℓ. Which are the cyclic coordinates? Find the corresponding conserved generalized momenta? Express the energy in terms of the generalized momenta and find a first integral for the θ-motion.

Hints: this problem is often treated in texts on analytical mechanics. If you find it hard, consult one of them. It is advisable to use Maple as an aid to do the algebra.

HE