> *# Solution to Hand in assignments, batch 1, Problem 3, HT 2006:*

> $with(LinearAlgebra):$

> $R3p := Matrix(3, 3, [[\cos(\text{psi}), \sin(\text{psi}), 0], [-\sin(\text{psi}), \cos(\text{psi}), 0], [0, 0, 1]]):$

> $R1 := Matrix(3, 3, [[1, 0, 0], [0, \cos(\text{theta}), \sin(\text{theta})], [0, -\sin(\text{theta}), \cos(\text{theta})]]):$

> $R3f := subs(\text{psi} = \text{phi}, R3p):$

> $R := (R3f.R1).R3p;$

$$R := [[\cos(\phi)\cos(\psi) - \sin(\phi)\cos(\theta)\sin(\psi), \cos(\phi)\sin(\psi) + \sin(\phi)\cos(\theta)\cos(\psi), \sin(\phi)\sin(\theta)], [-\sin(\phi)\cos(\psi) - \cos(\phi)\cos(\theta)\sin(\psi), -\sin(\phi)\sin(\psi) + \cos(\phi)\cos(\theta)\cos(\psi), \cos(\phi)\sin(\theta)], [\sin(\theta)\sin(\psi), -\sin(\theta)\cos(\psi), \cos(\theta)]] \quad \textbf{(1)}$$

> $TrR := Trace(R);$

$$TrR := \cos(\phi)\cos(\psi) - \sin(\phi)\cos(\theta)\sin(\psi) - \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\theta)\cos(\psi) + \cos(\theta) \quad \textbf{(2)}$$

> $Ra := (1/2)*(R-Transpose(R));$

$$Ra := \left[\left[0, \frac{1}{2}\cos(\phi)\sin(\psi) + \frac{1}{2}\sin(\phi)\cos(\theta)\cos(\psi) + \frac{1}{2}\sin(\phi)\cos(\psi) + \frac{1}{2}\cos(\phi)\cos(\theta)\sin(\psi), \frac{1}{2}\sin(\phi)\sin(\theta) - \frac{1}{2}\sin(\theta)\sin(\psi)\right], \left[-\frac{1}{2}\sin(\phi)\cos(\psi) - \frac{1}{2}\cos(\phi)\cos(\theta)\sin(\psi) - \frac{1}{2}\cos(\phi)\sin(\psi) - \frac{1}{2}\sin(\phi)\cos(\theta)\cos(\psi), 0, \frac{1}{2}\cos(\phi)\sin(\theta) + \frac{1}{2}\sin(\theta)\cos(\psi)\right], \left[\frac{1}{2}\sin(\theta)\sin(\psi) - \frac{1}{2}\sin(\phi)\sin(\theta), -\frac{1}{2}\sin(\theta)\cos(\psi) - \frac{1}{2}\cos(\phi)\sin(\theta), 0\right]\right] \quad \textbf{(3)}$$

> $Vv := Vector([Ra[2, 3], -Ra[1, 3], Ra[1, 2]]);$

$$Vv := \left[\left[\frac{1}{2}\cos(\phi)\sin(\theta) + \frac{1}{2}\sin(\theta)\cos(\psi)\right], \left[\frac{1}{2}\sin(\theta)\sin(\psi) - \frac{1}{2}\sin(\phi)\sin(\theta)\right], \left[\frac{1}{2}\cos(\phi)\sin(\psi) + \frac{1}{2}\sin(\phi)\cos(\theta)\cos(\psi) + \frac{1}{2}\sin(\phi)\cos(\psi) + \frac{1}{2}\cos(\phi)\cos(\theta)\sin(\psi)\right]\right] \quad \textbf{(4)}$$

> $VaSq := Transpose(Vv).Vv;$

$$VaSq := \left(\frac{1}{2}\cos(\phi)\sin(\theta) + \frac{1}{2}\sin(\theta)\cos(\psi)\right)^2 + \left(\frac{1}{2}\sin(\theta)\sin(\psi) - \frac{1}{2}\sin(\phi)\sin(\theta)\right)^2 + \left(\frac{1}{2}\cos(\phi)\sin(\psi) + \frac{1}{2}\sin(\phi)\cos(\theta)\cos(\psi)\right. \quad \textbf{(5)}$$

$$+\frac{1}{2}\sin(\phi)\cos(\psi)+\frac{1}{2}\cos(\phi)\cos(\theta)\sin(\psi)\Big)^2$$

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> Valts := (1/4)*(2*sin(theta)^2*(1+cos(phi+psi)) + (1+cos(theta))^2*sin(phi
      +psi)^2);
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$$Valts := \frac{1}{2}\sin(\theta)^2(1+\cos(\phi+\psi))+\frac{1}{4}(1+\cos(\theta))^2\sin(\phi+\psi)^2 \tag{6}$$

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> diffe := simplify( VaSq-expand(Valts), trig);
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$$diffe := 0 \tag{7}$$

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> # That this difference is zero means that Valts is a simplified version of VaSq.
> Vabs := sqrt(Valts) :

> # Now comes the numerics. Euler angles from the birth date of a student.
> yy := 85.;
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$$yy := 85. \tag{8}$$

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> mm := 7.;
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$$mm := 7. \tag{9}$$

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> dd := 3.;
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$$dd := 3. \tag{10}$$

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> Vvv := subs(psi = yy*evalf(Pi)/180., subs(theta = mm*evalf(Pi)/180., subs(phi = dd
      *evalf(Pi)/180., Vv)));
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$$Vvv := \begin{bmatrix} 0.06616196942 \\ 0.05751372264 \\ 0.9956661731 \end{bmatrix} \tag{11}$$

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> Vabsv := evalf(subs(psi = yy*evalf(Pi)/180., subs(theta = mm*evalf(Pi)/180., subs(phi
      = dd*evalf(Pi)/180., Vabs))));
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$$Vabsv := 0.9995180655 \tag{12}$$

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> Trv := evalf(subs(psi = yy*evalf(Pi)/180., subs(theta = mm*evalf(Pi)/180., subs(phi = dd
      *evalf(Pi)/180., TrR))));
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$$Trv := 1.062085008 \tag{13}$$

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> RotVect := evalf(ScalarMultiply(Vvv, 1/Vabsv));
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$$RotVect := \begin{bmatrix} 0.06619387054 \\ 0.05754145386 \\ 0.9961462505 \end{bmatrix} \tag{14}$$

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> angRotVect := ScalarMultiply( Vector( [evalf(arccos(RotVect[1])),
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$evalf(\arccos(RotVect[2])), evalf(\arccos(RotVect[3]))]), 180./evalf(\mathrm{Pi}));$

$$angRotVect := \begin{bmatrix} 86.2045954330543509 \\ 86.7012954676246182 \\ 5.03174868006127252 \end{bmatrix} \quad \mathbf{(15)}$$

> $rotang1 := \arcsin(Vabsv) * 180./evalf(\mathrm{Pi});$

$$rotang1 := 88.22111002 \quad \mathbf{(16)}$$

> $angsum := yy + dd;$

$$angsum := 88. \quad \mathbf{(17)}$$

> $rotang2 := \arccos((Trv-1)/2) * 180./evalf(\mathrm{Pi});$

$$rotang2 := 88.22110975 \quad \mathbf{(18)}$$

> *# The very small difference between Rotang1 and Rotang2 is due to the fact that the theta angle is only 7 degrees.*