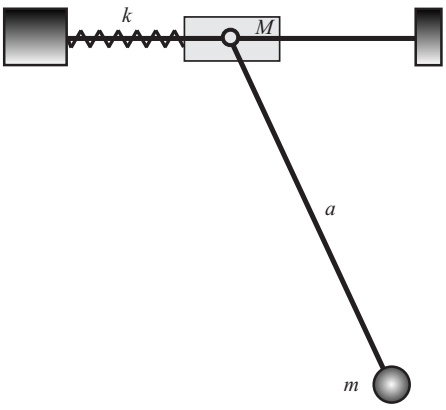
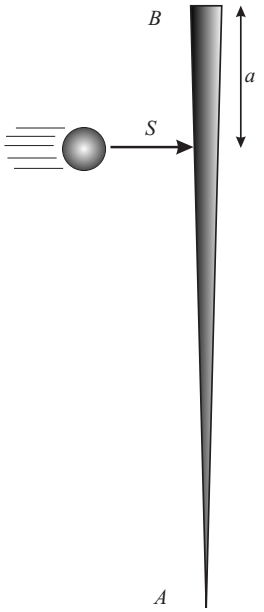
KTH Mechanics **2010 01 15**

Rigid Body Dynamics, SG2150
Exam, 2010 01 15, kl 09.00-13.00

Calculational problems

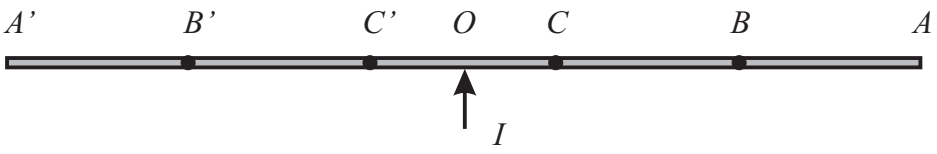
Problem 1: A mathematical pendulum *i.e.* a particle of mass m in a light rod of length a, is attached to a block that slides along a horizontal track. The light rod can rotate about a horizontal axis perpendicular to the track in a vertical plane. The block has mass $M = 8m$ and is attached to one end of a horizontal spring of stiffness $k = 9mg/a$ which is fixed at the other end. Find the exact Lagrangian for the system. Also find its quadratic approximation and the periods for small oscillations about equilibrium.





Problem 2: A bat has the shape of a long narrow (straight circular homogeneous) cone. A person holding the the bat at the pointed end A of the cone hits a ball that transfers an impact S to the bat in a direction perpendicular to the axis of the cone. Assume that the bat has length one meter. Find the distance a from the thick end B where the ball should be hit if there is to be no reaction impulse in the hand of the batter.

Problem 3: A linkage consists of five identical thin rods each of mass m and length a. They are hinged to each other at the endpoints so that the linkage as a whole can deform in a plane. Initially the linkage is placed on a horizontal plane, at rest in a straight configuration, when it is stuck at the midpoint O in a direction perpendicular to the linkage. The impulse delivered is I. Assume symmetry about the axis through the midpoint perpendicular to the linkage. Find the angular velocities of the rods immediately after impact.



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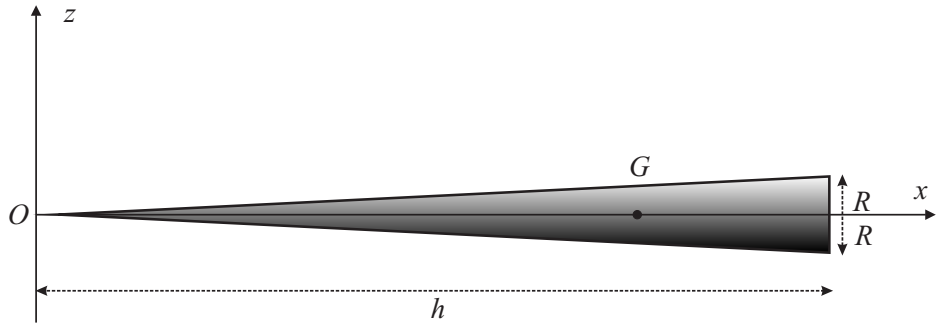
Idea problems:

Problem 4: Find the position of the center of mass G, and the moment of inertia J_z with respect to a perpendicular axis, for a long narrow (slim) cone with respect to an origin O at the vertex of the cone. Assume that the cone is straight, circular and homogeneous of height h and base radius R ($R \ll h$).



Problem 5: A rigid body rotates about a fixed point O. Derive a formula for the components of the angular momentum vector $\boldsymbol{L}_O$ given the components of the angular velocity vector $\boldsymbol{\omega}$ and the masses m_i and position vectors $\boldsymbol{r}_i$ of the particles of the body.

Problem 6: Derive Euler's dynamic equations for a rigid body that rotates about a fixed point O. Consider the components of the vector equation $\dot{\boldsymbol{L}} = \boldsymbol{M}$ in a body fixed principal axes system.

Each problem gives maximum 3 points, so that the total maximum is 18. Grading: 1-3 F; 4-5 FX; 6 E; 7-9 D; 10-12 C; 13-15 B; 16-18 A.
Allowed equipment: Handbooks of mathematics and physics. One A4 size page with your own compilation of formulas.

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