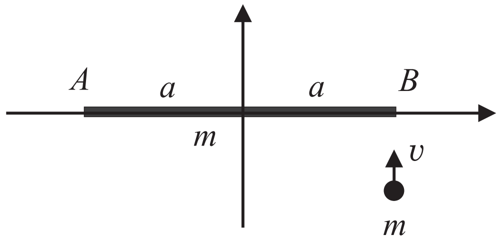
KTH Mechanics **2010 10 21**

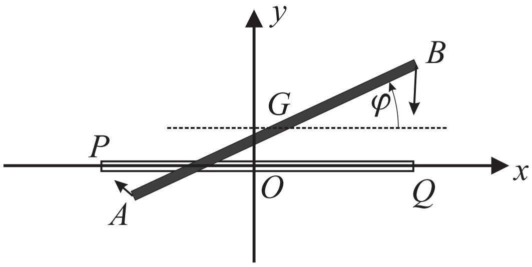
Rigid Body Dynamics, SG2150

Exam, 2010 10 21, kl 09.00-13.00

Calculational problems

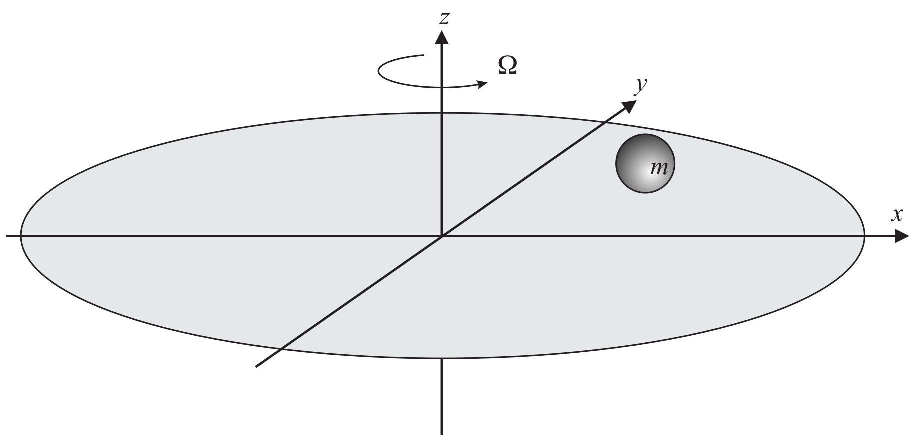
Problem 1: A slender rod AB of mass m and length $2a$, rests on a smooth horizontal plane. A small ball of the same mass m impacts with the rod at its endpoint B and sticks there. Just before impact the ball has a velocity v in the horizontal plane perpendicular to the rod. Find the angular velocity of the body (rod plus ball) and the velocity of the point A, just after impact.





Problem 2: A slender rod AB of mass m and length $2a$, can move in a smooth horizontal plane. The endpoints of the rod are attached with elastic strings in such a way that at equilibrium the rod goes between the points P and Q a distance $2a$ apart and the forces on the rod are then zero. When the rod is displaced in the plane the force on the endpoints A and B pull them back towards P and Q respectively. The strength of these forces are proportional to the distances ($|\boldsymbol{F}_A| = k|AP|$, $|\boldsymbol{F}_B| = k|BQ|$). Find the kinetic energy, the potential energy and the equations of motion of the rod. Also find the angular frequencies for small amplitude motions. Use center of mass coordinates x, y and angle φ as generalized coordinates. Hint: the potential energy of the force $\boldsymbol{F}_A$ is $V_A = (k/2)(\boldsymbol{r}_A - \boldsymbol{r}_P)^2$.

Problem 3: A rough horizontal table rotates with constant angular velocity Ω about a fixed vertical axis. Find the equations of motion for the center of mass of a ball, of mass m and radius R, that rolls on the table. Solve them. Hints: Use a fixed Cartesian coordinate system. Find the conditions for rolling without slipping by expressing the velocity of a point of the table and equating it to the velocity of the lowest point on the ball. Use the vector equations of motion, $m\ddot{\boldsymbol{r}}_G = \boldsymbol{F}$, $\dot{\boldsymbol{L}}_G = -R\boldsymbol{e}_z \times \boldsymbol{F}$. Use $J_G = 2mR^2/5$.



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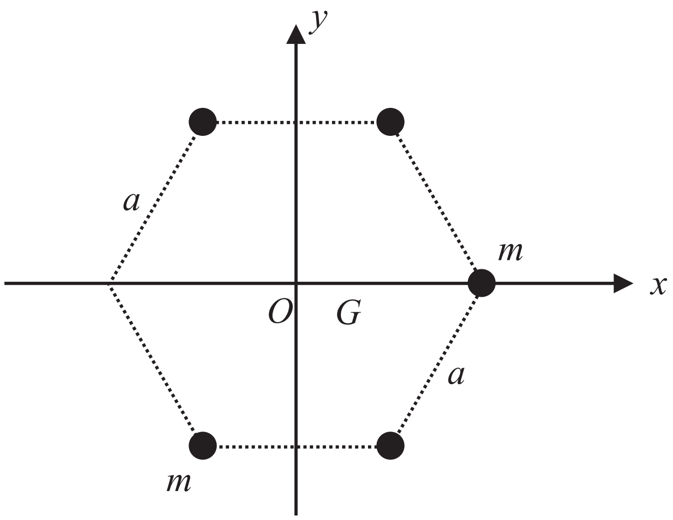
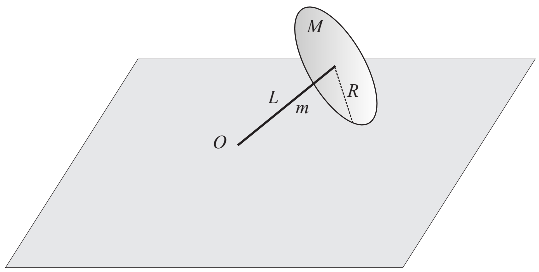
Idea problems:

Problem 4: Five particles, each of mass m, are placed at the corners of a regular hexagon of side length a. Find the position of the center of mass G in the coordinate system of the figure. Find the inertia matrix $\mathbf{J}_G$, with respect to the same axes, but with origin at the center of mass.



Problem 5: A rigid body is made from a thin circular disc of radius R and mass M with a slender rod of length L and mass m attached at the center on one side of the disc. The rod is perpendicular to the plane of the disc. This body is placed on a rough horizontal plane so that the free end of the rod and a point on the periphery of the disc is in contact with the table. It is then set in rolling motion with absolute value of angular velocity given by ω. Find its kinetic energy.



Problem 6: Derive Lagrange's equations of motion for one particle, either from a variational principle, or by projecting Newton's second law on the tangent vectors of the allowed motions.

Each problem gives maximum 3 points, so that the total maximum is 18. Grading: 1-3 F; 4-5 FX; 6 E; 7-9 D; 10-12 C; 13-15 B; 16-18 A.
Allowed equipment: Handbooks of mathematics and physics. One A4 size page with your own compilation of formulas.

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