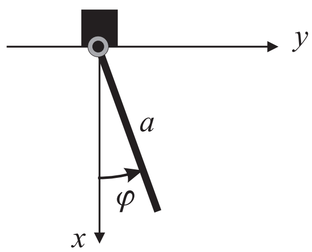
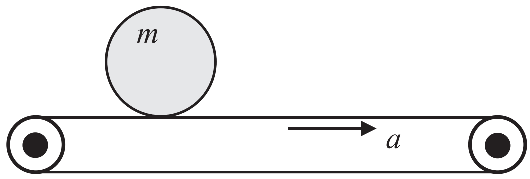
Rigid Body Dynamics, SG2150

Exam, 2011 10 22, kl 13.00-17.00

Calculational problems

Problem 1: A slender homogeneous rod of mass m and length a can rotate in a vertical plane about a fixed smooth horizontal axis through one endpoint. Find the equation of motion 1) using $\dot{\boldsymbol{L}} = \boldsymbol{M}$ and cylindrical coordinates and 2) using Lagrange method. 3) Find the angular frequency for small amplitude motion.

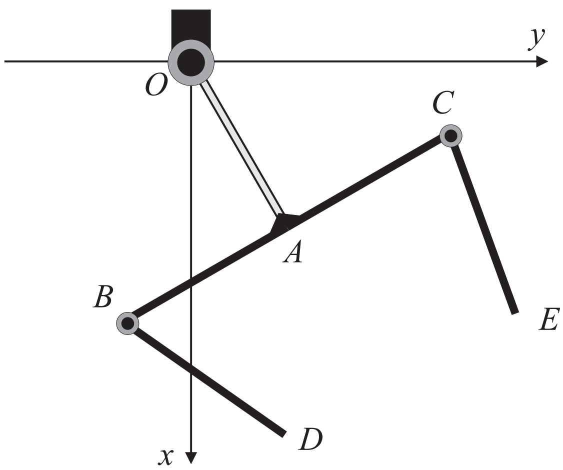




Problem 2: A straight circular cylinder of mass m and radius R is at rest on a rough horizontal conveyor belt. The axis of the cylinder is perpendicular to the direction of motion of the belt. The conveyor belt is then given a constant acceleration a. Find the Lagrangian that determines the motion of the center of mass of the cylinder. Find its translational acceleration from the Lagrange equation of motion.

Problem 3: Four slender homogeneous rods OA, BC, BD, and CE, constitute a planar mechanism. OA has length a and is light. It is fixed to BC at a right angle so that A is at the midpoint of BC. BD and CE each have mass m and length a while BC has mass $2m$ and length $2a$. There are smooth joints at B and C connecting the rods. Due to a smooth joint at O, OA can rotate about a fixed horizontal axis. The mechanism moves in a vertical plane.

Find the Lagrangian of the system. Find the approximation for small amplitude motion about the equilibrium and determine the M and the K-matrix. Find at least one angular eigen frequency of the system.



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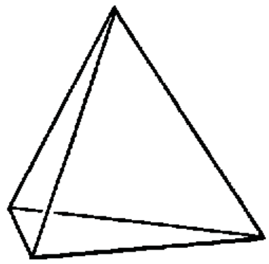
Idea problems:

Problem 4: Six identical slender homogeneous rods, each of mass m and length a are welded together at the endpoints so that they constitute the edges of a regular tetrahedron. Find the moment of inertia of this body with respect to an axis through the midpoint.



Problem 5: Use the equation $\dot{\boldsymbol{L}} = \boldsymbol{M}$ to find a simple approximation for the precession angular velocity ($\Omega = \dot{\psi}$) of the heavy fast symmetric top. Hint: use $\dot{\boldsymbol{e}} = \boldsymbol{\Omega} \times \boldsymbol{e}$ and assume $\boldsymbol{L}$ parallel to the axis of the top.

Problem 6: Find the motion of the free symmetric top in terms of suitable Euler angles. Discuss the difference between prolate and oblate bodies.

Each problem gives maximum 3 points, so that the total maximum is 18. Grading: 1-3 F; 4-5 FX; 6 E; 7-9 D; 10-12 C; 13-15 B; 16-18 A.
Allowed equipment: Handbooks of mathematics and physics. One A4 size page with your own compilation of formulas.

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