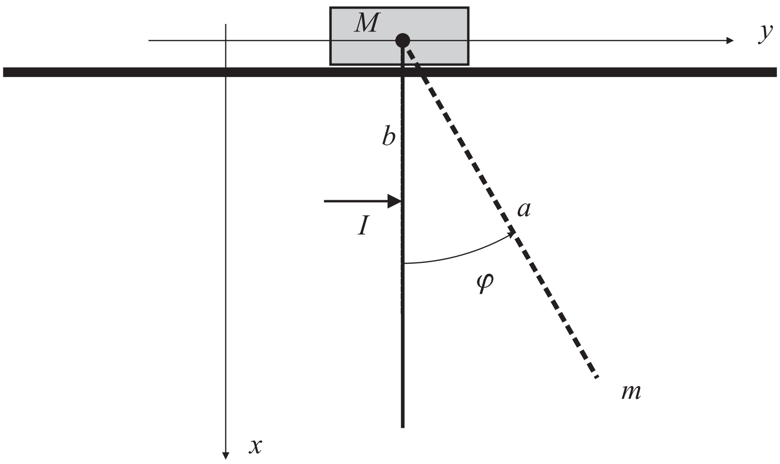
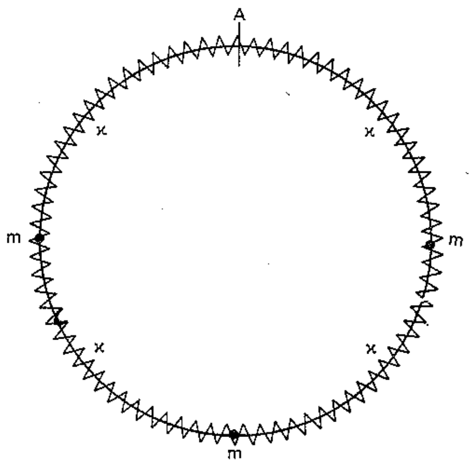
# Rigid Body Dynamics, SG2150
Exam, 2013 10 25, kl 14.00-18.00

## Calculational problems

**Problem 1:** A block of mass $M$ can slide freely along a straight horizontal track. A rod of mass $m$ and length $a$ is hinged with one end at the center of mass of the block so that it can rotate freely about a horizontal axis perpendicular to the track. At the equilibrium position, with the rod straight down and the system at rest, the rod is impacted with an impulse $I$ perpendicular to the rod and parallel to the track at a distance $b$ below the hinge. Find the values of $b$ that a) makes the angular velocity of the rod $\dot{\varphi}$ zero after the impact and b) makes the translational velocity $\dot{y}$ zero after impact.




**Problem 2:** Find the angular frequencies for the system shown: three particles, all of mass $m$, move freely along a smooth horizontal fixed ring. Four equal springs of stiffness $\kappa$ along the smooth ring connect the particles and a fixed point $A$ on the ring. The figure shows the equilibrium positions. (Hint: the secular equation factors into a linear and a quadratic; no need to solve a cubic.)

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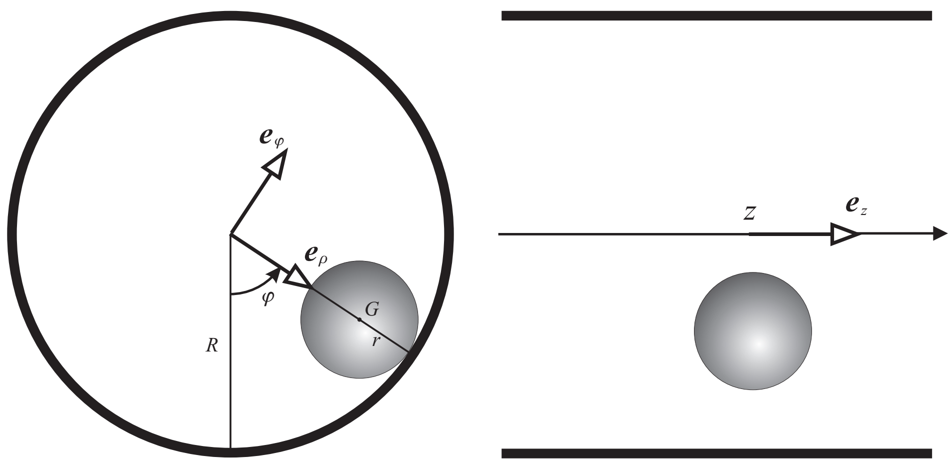
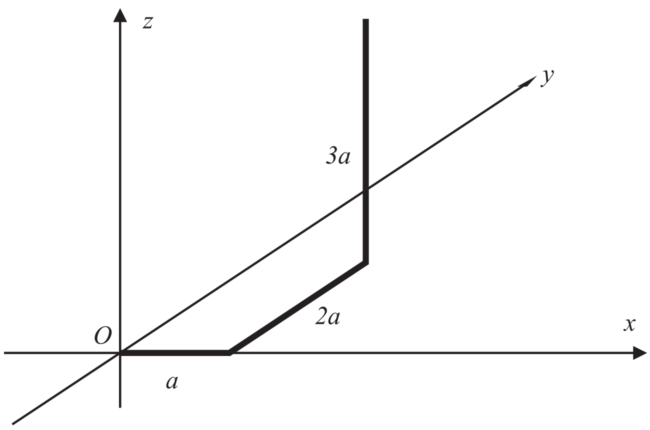
**Problem 3:** A ball of mass $m$ and radius $r$ rolls inside a fixed horizontal cylindrical pipe of radius $R$. Use cylindrical coordinates $\rho, \varphi, z$ for the center of mass of the ball, with $z$-axis along the axis of the cylinder, as indicated in the figure below. Also use cylindrical components of the angular velocity vector, $\boldsymbol{\omega} = \omega_\rho\, \boldsymbol{e}_\rho + \omega_\varphi\, \boldsymbol{e}_\varphi + \omega_z\, \boldsymbol{e}_z$. Find the equations of motion, in particular, the equation for the $\varphi$ motion, which decouples from the others, and the (two) relations between $\dot{\varphi}$, $\omega_\rho$ and $\omega_\varphi$, and their time derivatives, $\dot{\omega}_\rho$ and $\dot{\omega}_\varphi$.




**Idea problems:**

**Problem 4:** Three rods of lengths $a, 2a$, and $3a$ and mass $m, 2m$, and $3m$ respectively, are welded together at the end points at right angles, see figure below. Find the inertia matrix $\mathbf{J}_O$ of the body in the indicated coordinate system, with origin at one end of the rod of length $a$.




**Problem 5:** Derive Euler's dynamic equations for the components of the angular velocity vector of a rigid body in the body fixed principal axes system. Solve them for the special case of a free symmetric top.

**Problem 6:** Derive the Euler-Lagrange equation from the variational principle of least action, i.e. assuming that the action $S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q})\, \mathrm{d}t$ has a minimum for the real path $q(t)$ between given fixed end points $q_1 = q(t_1),\ q_2 = q(t_2)$.

*Each problem gives maximum 3 points, so that the total maximum is 18. Grading: 1-3 F; 4-5 FX; 6 E; 7-9 D; 10-12 C; 13-15 B; 16-18 A.*

Allowed equipment: Handbooks of mathematics and physics. One A4 size page with your own compilation of formulas.

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