Study questions

(SG2221 Wave motions and hydrodynamic stability, 7.5 credits)

Oral/written examination

The examination is composed of an alternating oral and written test. The test is performed individually and will last for about 30 to 45 minutes. A list of study questions, which can be seen as a study guide, is found below from where questions will be taken and asked at the individual test. Note that some questions can be answered very shortly, which is fine, the emphasis at the test will lie on the physical understanding

Surface waves

1. Which are the "two" balancing forces giving rise to *surface* waves? Discuss different types of surface waves.
2. Discuss the assumptions made when deriving the governing equations of surface gravity waves. What do the individual assumptions imply? What is the final set of equations?
3. What are the boundary conditions for surface gravity waves and what are their physical meaning?
4. Derive the dispersion relation for surface gravity waves. What does the relation tell you? Apply the deep and shallow water approximations.
5. How is the group speed defined, in 2D and in 3D?
6. Discuss the particle orbits in surface gravity waves. Apply the deep and shallow water approximations. Is the particle velocity positive or negative below the wave crest?
7. Show that the total wave energy is

   $$E_{total} = \frac{1}{2}\rho g a^2$$

   and discuss the travelling speed of energy.
8. Discuss on the following:
   a. surface tension
   b. capillary waves
   c. taking into account surface tension, how are the gov. eqs. of surface gravity waves affected?
9. Derive the dispersion relation for surface capillary-gravity waves?
10. When is surface tension important? Apply the capillary length for reasoning.
11. Calculate the group speed for capillary-gravity waves and discuss the limiting cases.
12. Discuss phase speed versus group speed. How can the superposition of two sinusoids with the same amplitude but with slightly different wave number illustrate

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this? What is the physical difference and how do the wave patterns differ between dropping a stone in a pond and observing a raindrop falling into the pond?

13. What do we observe if we travel with the group speed assuming the depth H uniform everywhere? How is this shown mathematically?
14. Using the shallow water equations, describe what happens when a wave
    a. passes over a step in the seabed
    b. approaches the shoreline (how is this linked to Tsunamis?)

Continuously stratified fluids

15. What is the meaning of the Brunt-Väisälä/buoyancy frequency (*N*) in continuously stratified fluids?
16. What does it mean when a fluid is horizontally isotropic?
17. Show that the internal wave frequency in cont. strat. fluids can be expressed as: $\omega = N \cos \theta$.
    a. what conclusions can be drawn?
    b. what are the limiting cases, and what do they imply?
18. Show that the group speed vector is orthogonal to the phase speed vector.
19. Show that the particle motion is orthogonal to the wave number vector.

Bénard

20. Discuss the Bénard problem, what is the important parameter and what is its physical interpretation?
21. What is meant by a so *called exchange of stability*? How is this expressed for increasing Rayleigh numbers?
22. Discuss Bénard's eigenvalue problem and what information can one extract by solving this problem?

Taylor-Couette

23. Describe the Taylor-Couette problem and identify the important physical parameter?
24. What assumptions have to be made in order to arrive at the same mathematical eigenvalue problem as in the Bénard problem?
25. How many steady state instability patterns may be observed in a carefully performed experiment?

Shear flow instability

26. Derive the equations governing the evolution of small amplitude perturbations in a parallel shear flow. Assume wave-like perturbations.
27. Show that a linear mechanism is required for perturbation energy growth.
28. Derive Rayleigh equation (inviscid stability of a parallel flow) and discuss its properties.
29. Demonstrate Rayleigh inflection point criterion and give a physical interpretation.

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30. What is the inviscid algebraic instability? Can you relate it to the transient energy growth observed at finite Reynolds numbers?
31. Kelvin-Helmhotz instability. Discuss the dispersion relation for the case of two streams of the same fluid.
32. Give a physical interpretation of the vortex sheet instability and compare it with the case of a piecewise mixing layer.
33. Viscous stability. Demonstrate Squire's theorem and enunciate the main results from classic eigenvalue analysis for simple wall-bounded shear flows.
34. Why velocity profiles with no inflection point, stable as the Reynolds number tends to infinity, become unstable when the effect of viscosity is considered?
35. Introduce the non-modal analysis of the stability problem. Consider bounds to the operator exponential.
36. Show with an example that the superposition of non-orthogonal eigenfunctions can lead to transient energy amplification.
37. How can the solution to the OrrSommerfeld-Squire system be projected onto the space spanned by the system eigenfunctions?
38. Explain how to compute the largest possible amplification of the perturbation kinetic energy over a time period t for parallel shear flows.
39. Describe how to compute the initial disturbance giving largest possible amplification. Lift-up effect.
40. Describe how to compute the largest possible response of the flow to a time harmonic forcing.
41. Discuss the sensitivity of the eigenvalues in non-normal system.
42. Introduce the concept of numerical range and use it to compute the time derivative of the perturbation kinetic energy.
43. Use Burger's equation to introduce the concept of spatial stability.
44. Discuss the concept of absolute/convective instability.
45. Non-linear effects. Introduce the definition of global/conditional/linear critical Reynolds number.
46. Explain the idea of a secondary instability analysis.
47. Discuss transition scenarios in wall-bounded shear flows.

Good luck!

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