KTH
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Course Project

Jets and Wakes

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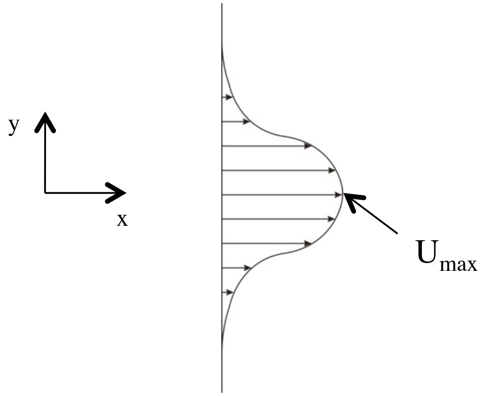
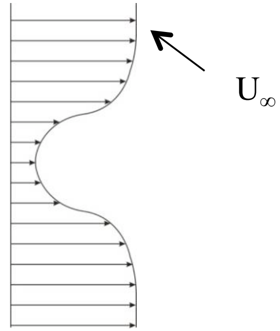
Shervin Bagheri

Jet



Wake



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- Governing equations are just as for Blasius boundary layer
- Boundary Conditions:

  Jet: $u \longrightarrow 0 \quad \text{as} \quad y \longrightarrow \pm\infty$

  Wake: $u \longrightarrow U_\infty \quad \text{as} \quad y \longrightarrow \pm\infty$

- For high Re, parallel flow can be assumed

  Jet: $U = U(y) = sech^2(y)$

  Wake: $U = U(y) = 1 - \lambda sech^2(y)$

- Disturbance equations are derived from Navier-Stokes equations by:
  - Decomposing velocities
  - Subtracting meanflow
  - Linearizing

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$$\frac{\partial u_i}{\partial t} + \bar{U}_j \frac{\partial u_i}{\partial x_j} + u_j \frac{\partial \bar{U}_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\nabla^2 u_i$$

- Governing stability equations are the
  - Orr-Sommerfeld equation
  - Squire equation

- They can be derived from the disturbance equations by assuming parallel flow

$$\left[\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)\nabla^2-\bar{U}''\frac{\partial}{\partial x}-\frac{1}{Re}\nabla^4\right]v=0$$

$$\left[\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}-\frac{1}{Re}\nabla^2\right]\eta=-\bar{U}'\frac{\partial v}{\partial y}$$

- Boundary Conditions:

$$v=\frac{\partial v}{\partial y}=\eta=0 \quad \text{as} \quad y\rightarrow\pm\infty$$

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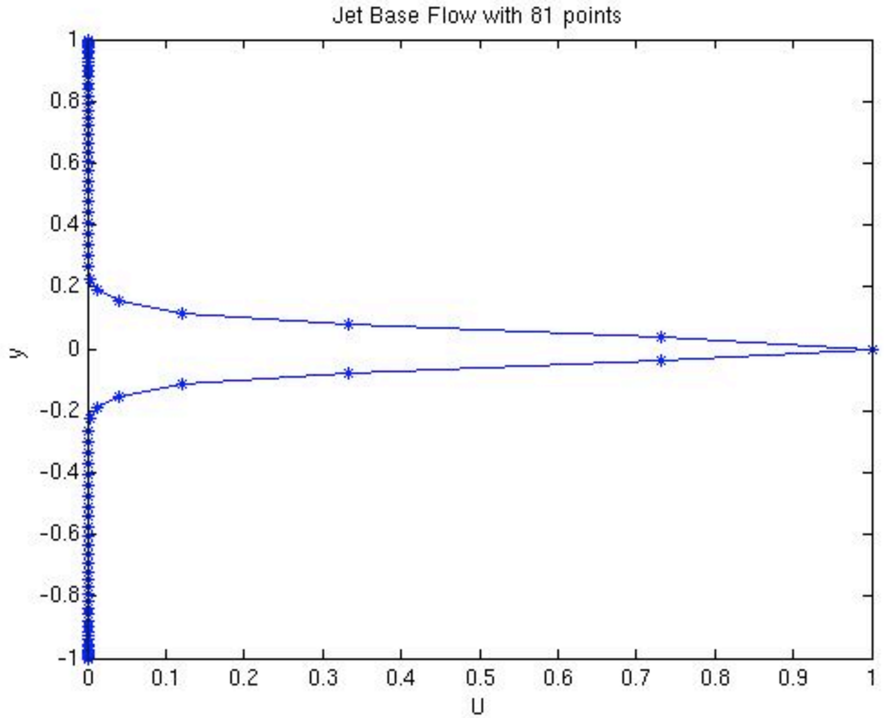
- Base flow of the laminar Jet



- Chebycheff collocation points not very suitable for this problem because resolution is worst in the middle

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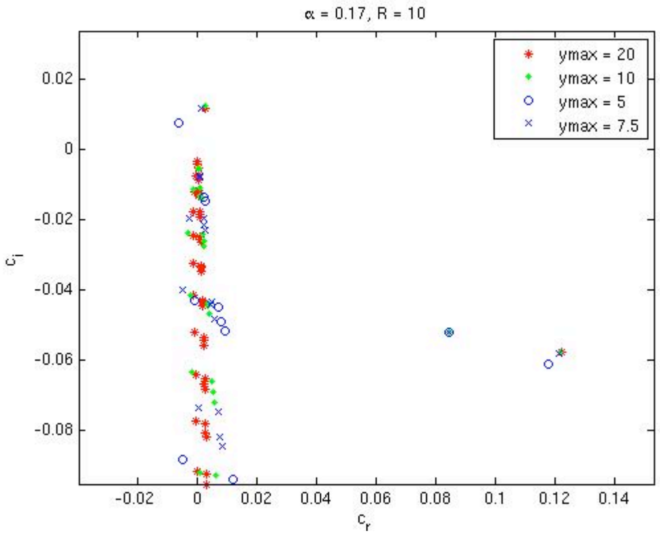
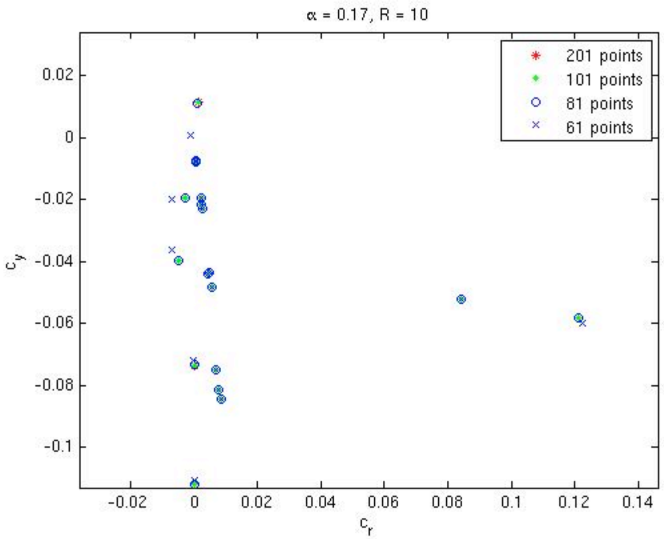
- Convergence tests





- High resolution of 201 points
- Height of box (= 2*Ymax) was varied
- Height of box was chosen as 15 which corresponds to Ymax=7.5
- Resolution was varied

⇨ Ymax = 7.5 and 81 points

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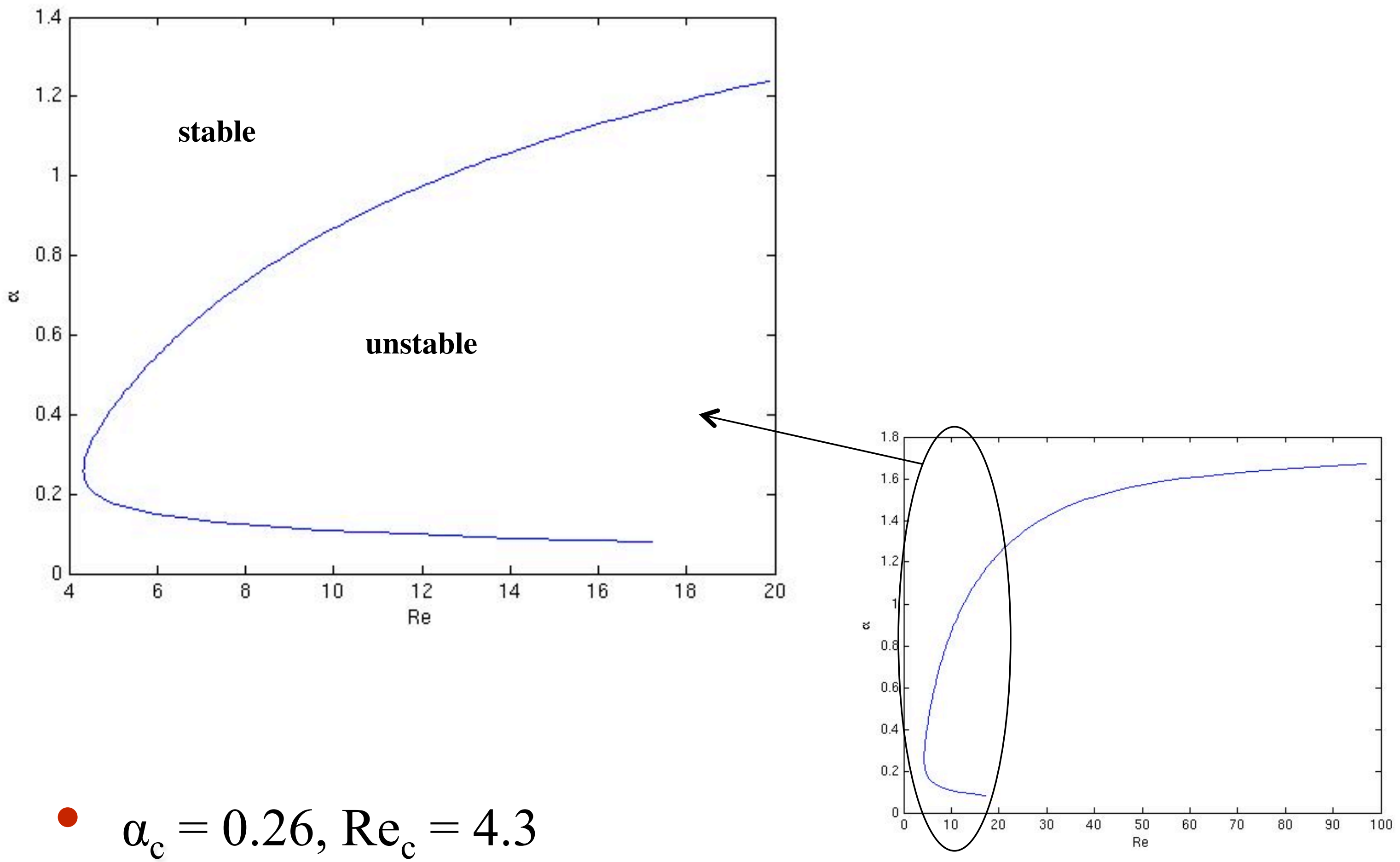
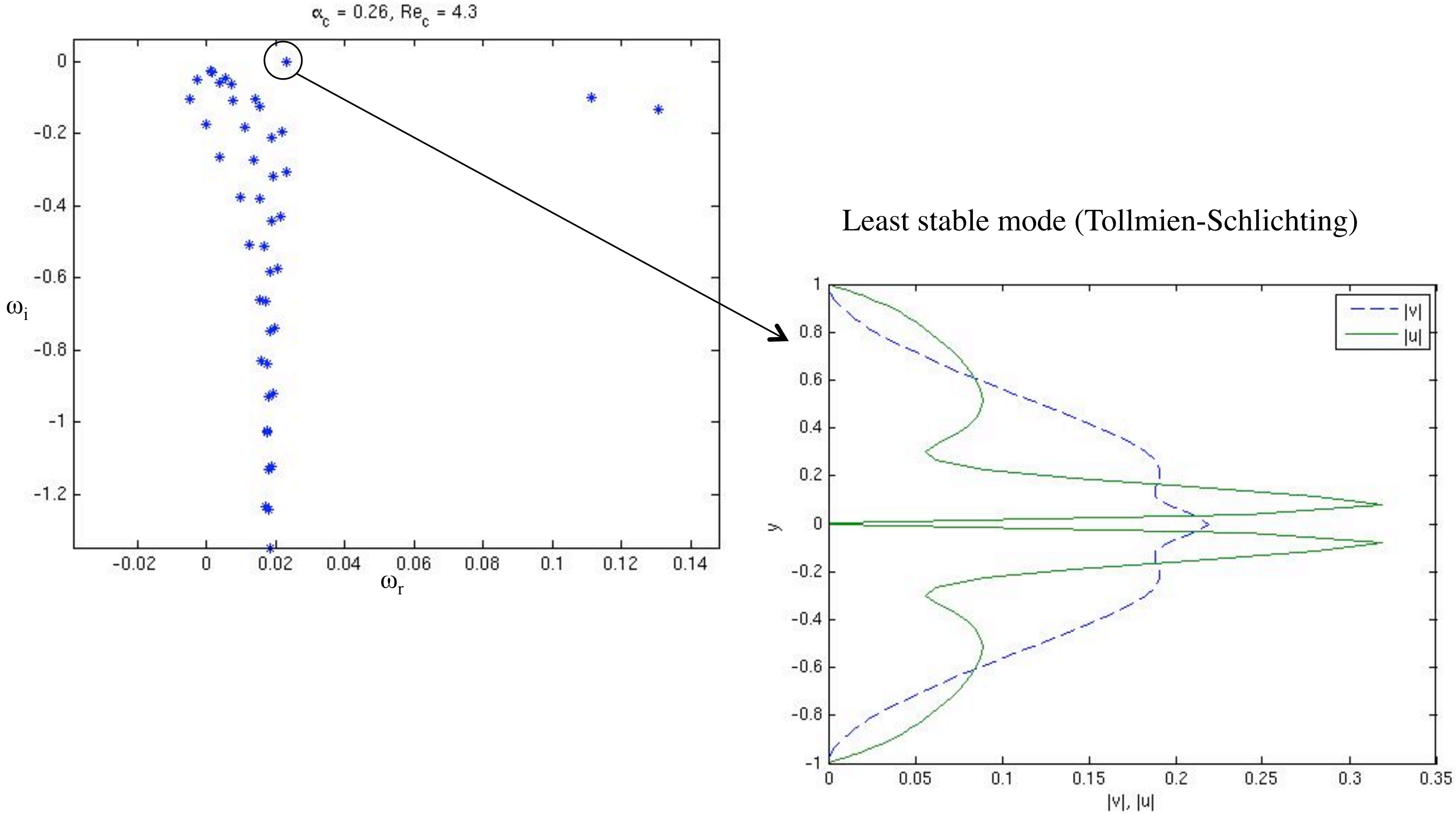
- Neutral stability of laminar jet

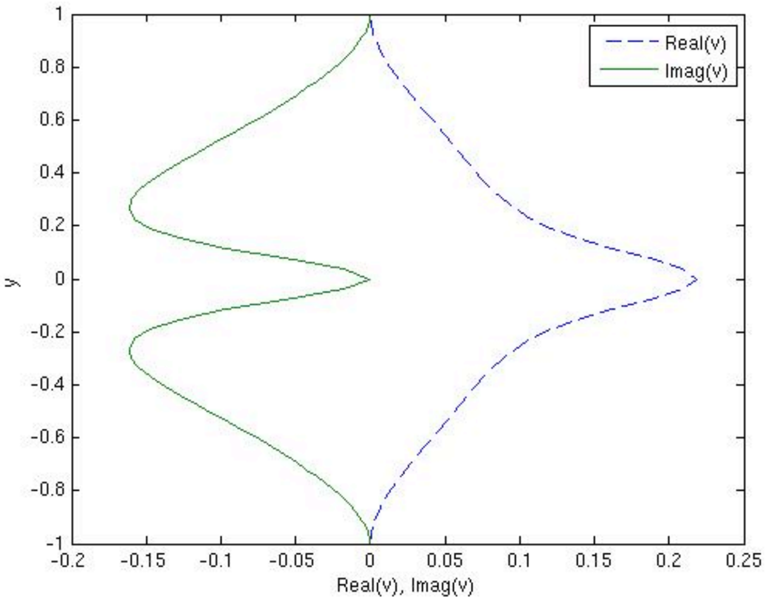


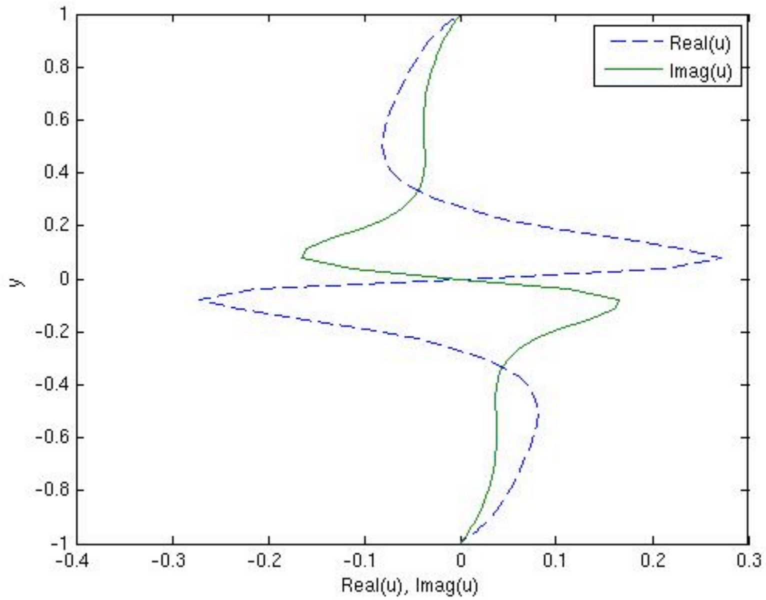
- $\alpha_c = 0.26$, $Re_c = 4.3$

- Spectra and Eigenfunctions of the jet for the critical conditions $\alpha_c = 0.26$, $Re_c = 4.3$

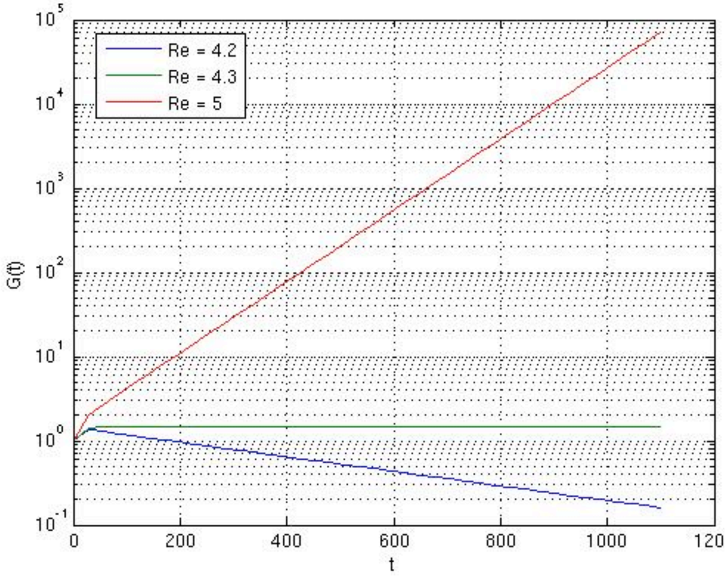


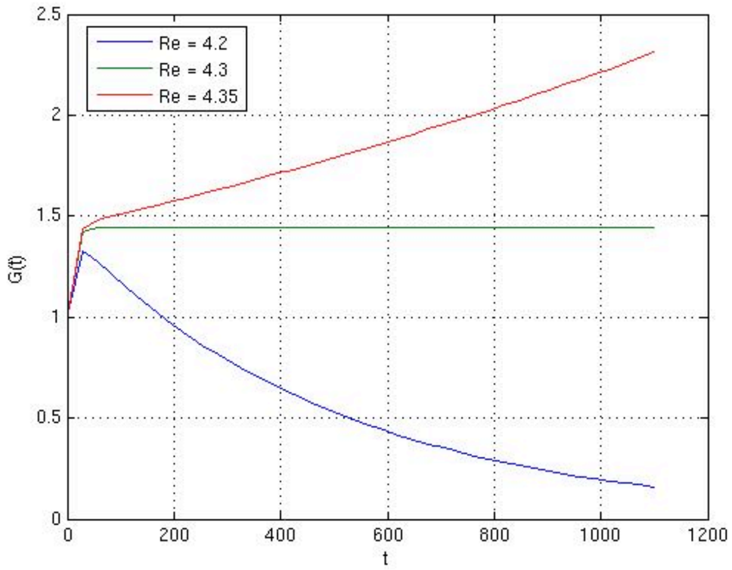
- Real and imaginary parts of eigenfunctions for the critical values $\alpha_c = 0.26$, $Re_c = 4.3$

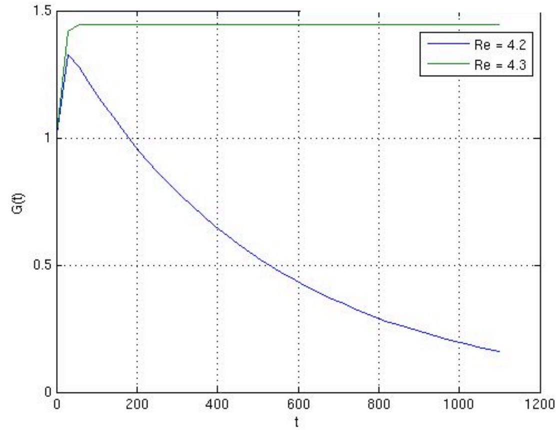




- Transient Growth (α_c = 0.26) stable, neutral and unstable case







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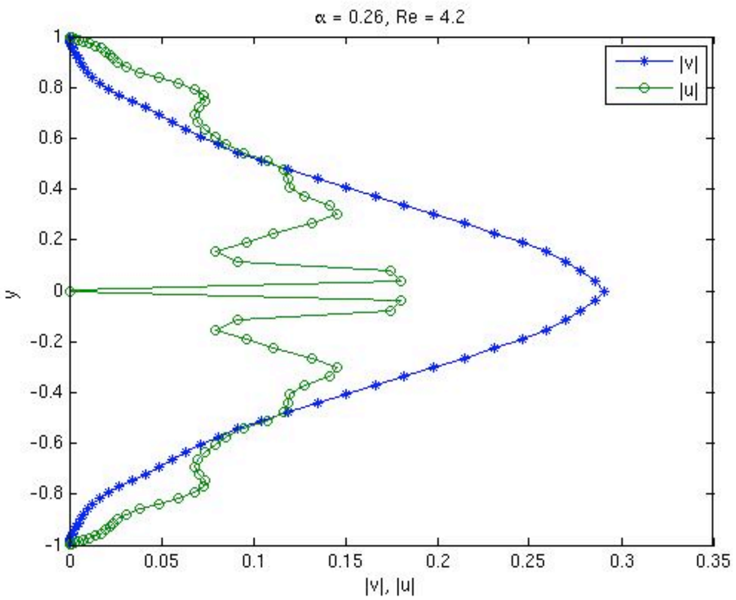
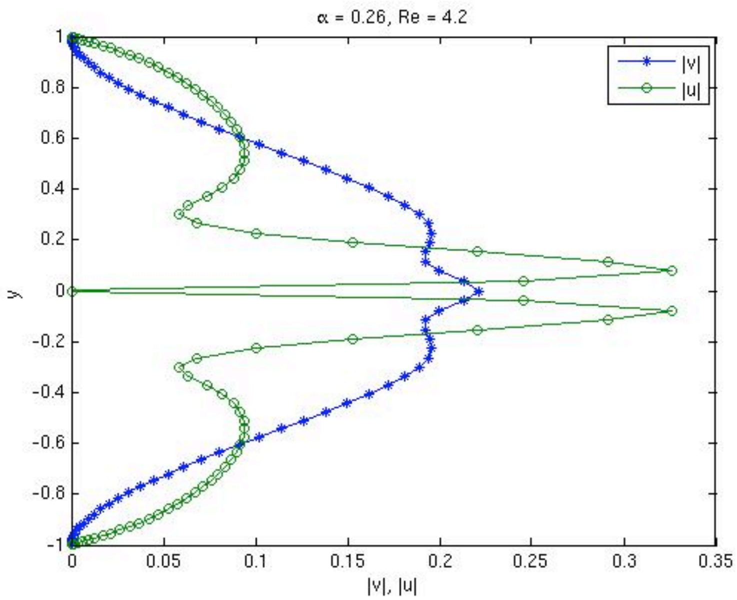
- Optimal disturbance and optimal response for the stable case

Optimal disturbance at t_0



Optimal response at T



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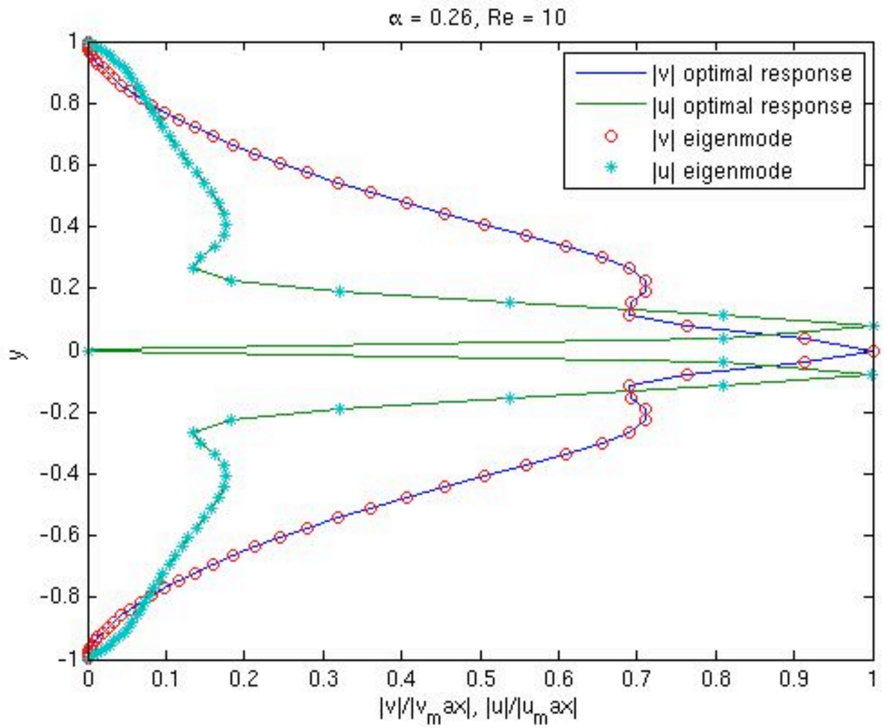
- Optimal response and eigenmode for an unstable case (Re = 10)



- Eigenmode and developed optimal disturbance are identical

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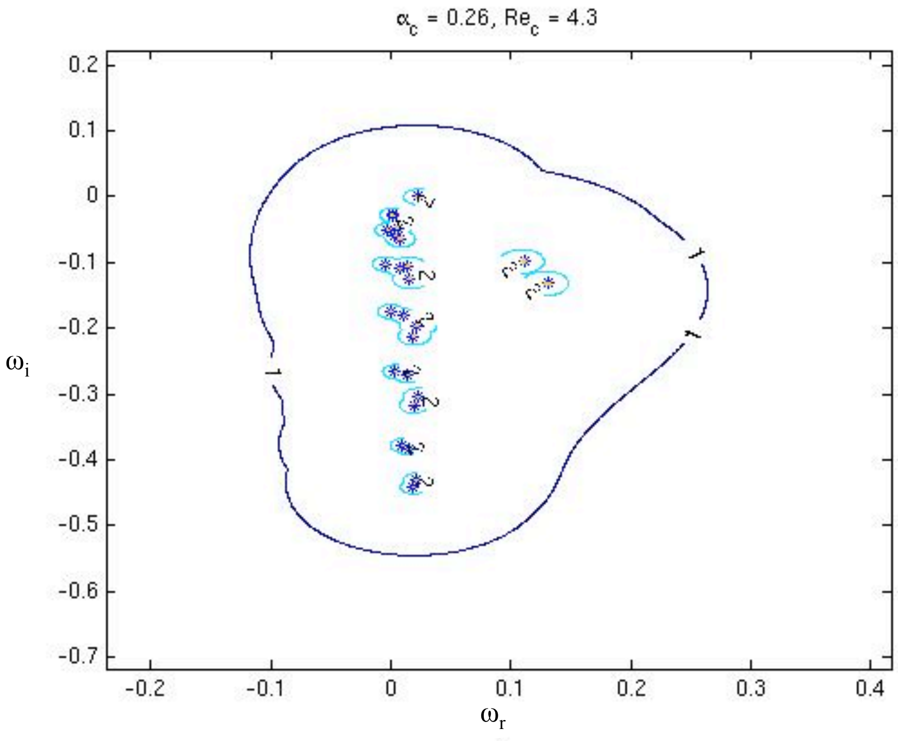
- Pseudospectra of the stable case
  $\alpha = 0.26$, $Re = 4.2$



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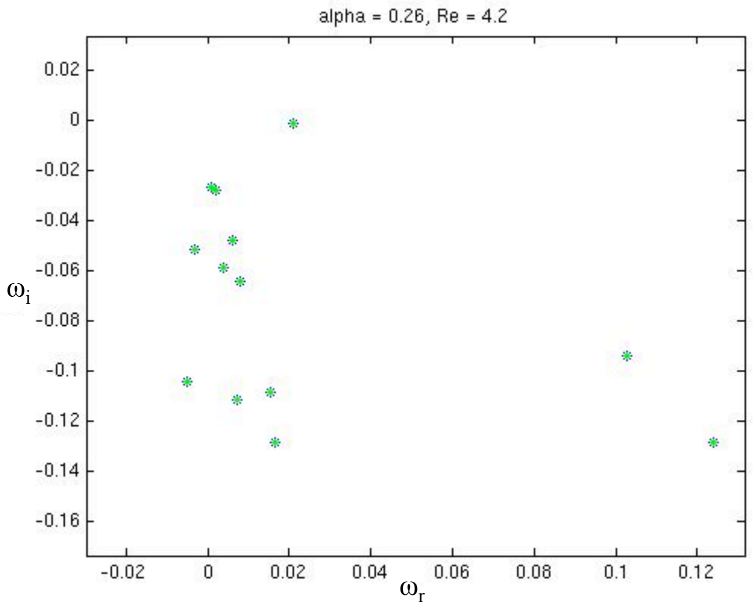
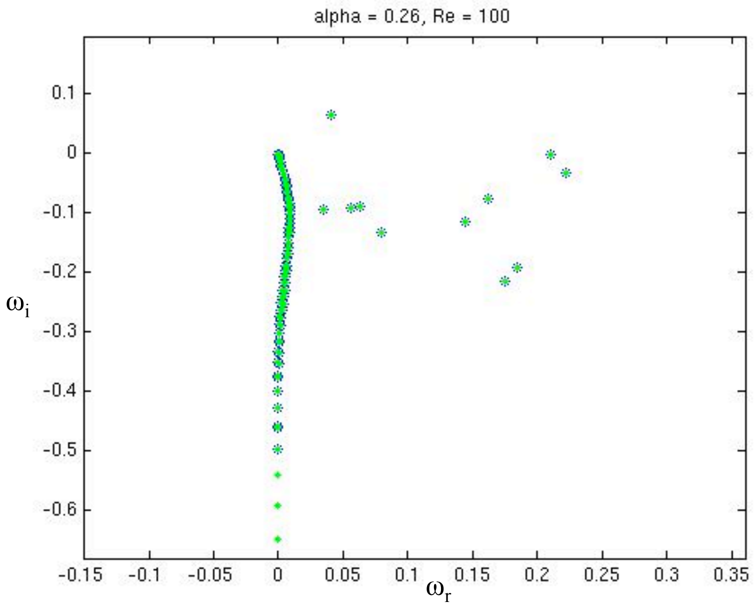
- Sensitivity of eigenvalues for a stable and an unstable case

Stable



Unstable



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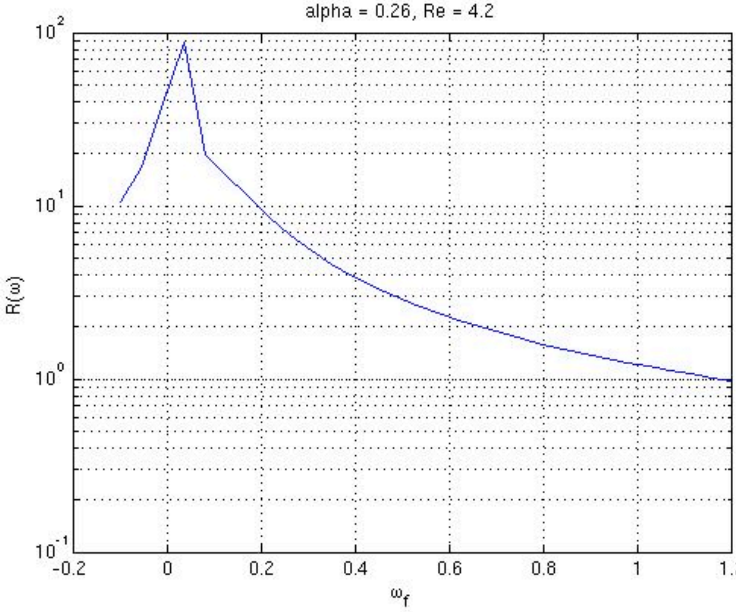
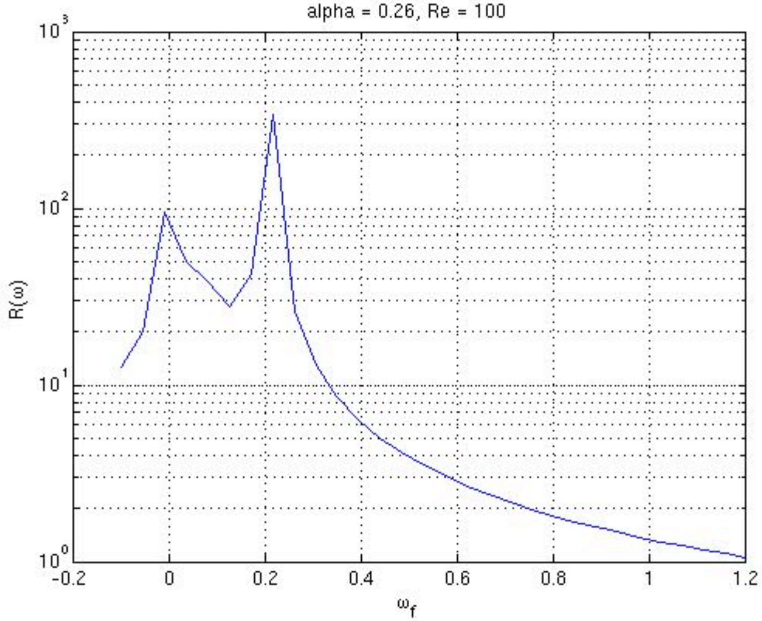
- Optimal response to forcing for stable and unstable mode





- In the stable case the least stable frequency of the homogeneous problem is excited
- This is not the case for the unstable mode

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- Conclusions

- Inflection point in profile, therefore the Jet is inviscidly unstable

- Jet becomes unstable at very low Reynold numbers

- The operator matrix is almost normal and thus the eigenvectors are almost orthogonal

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- Low Reynoldsnumbers quantitavely not relevant as flow is not parallel

- Laminar Jet would not be observable in reality

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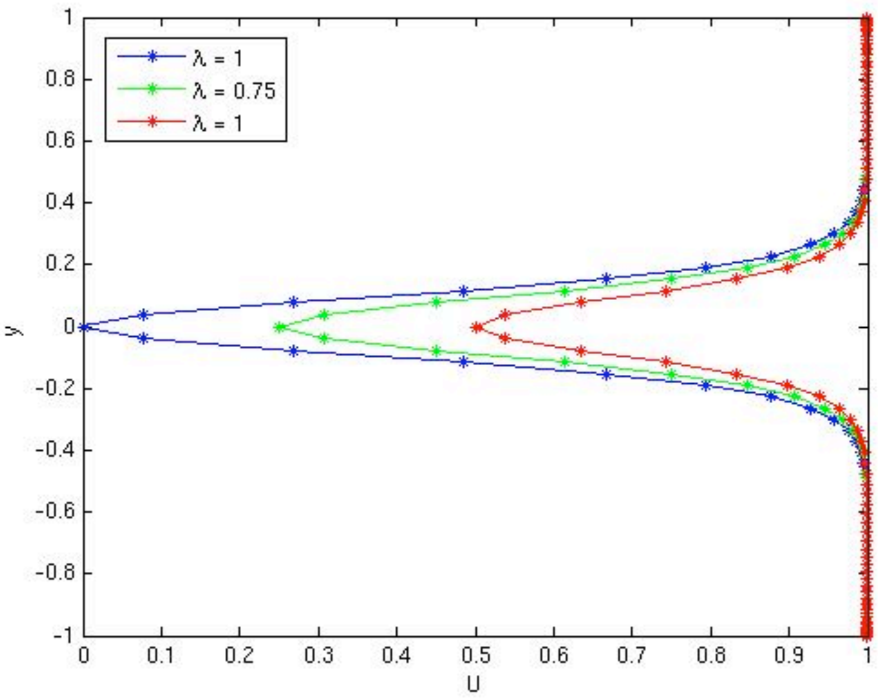
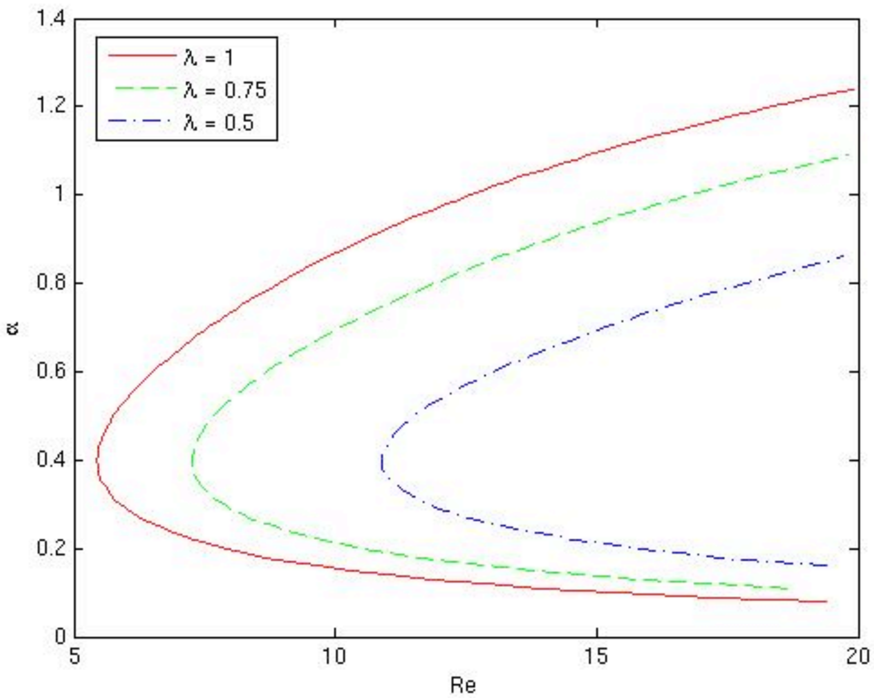
- Base flow of the laminar Wake for different λ



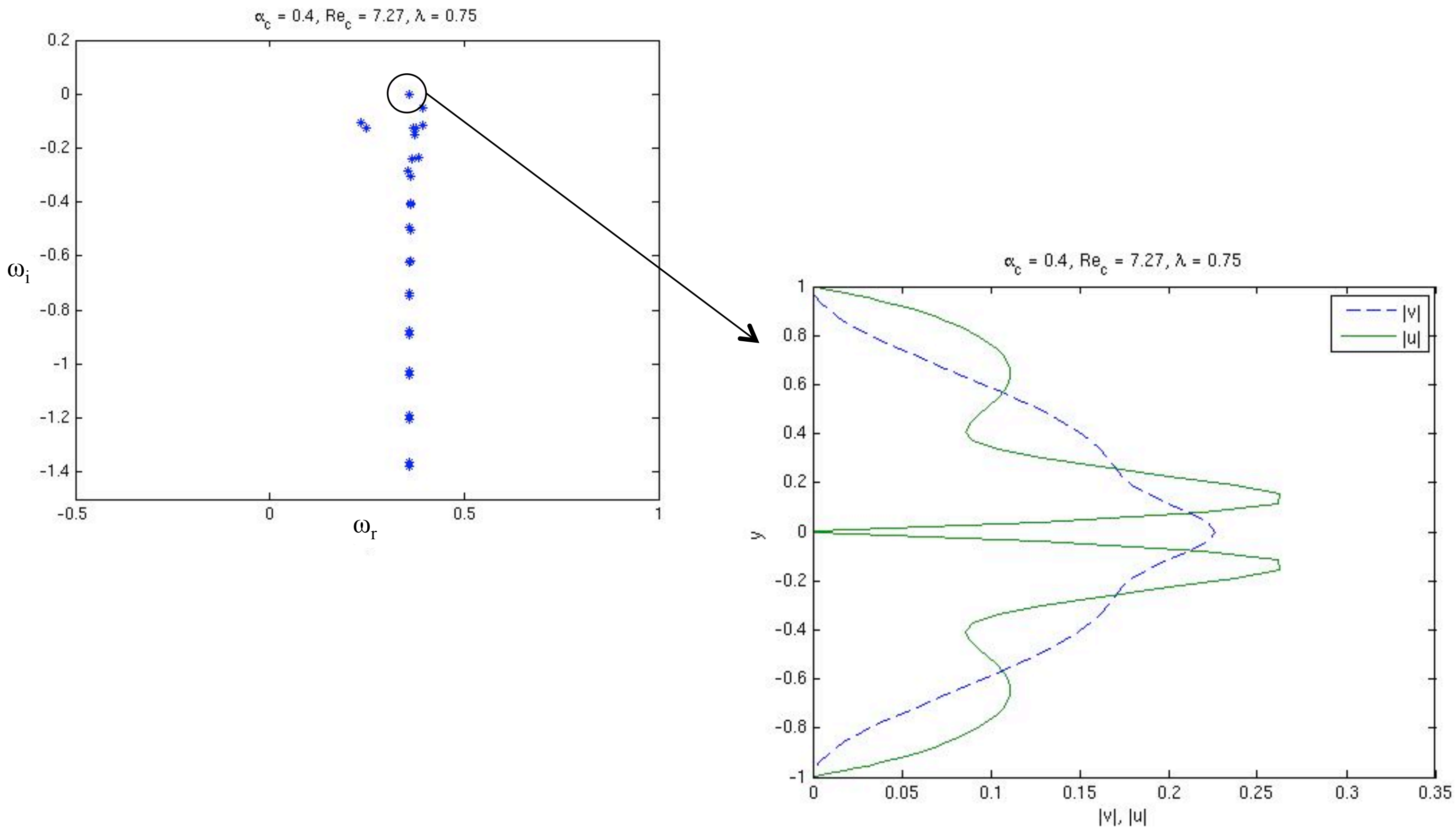
- Chebycheff collocation points also unsuitable in this case

- Neutral stability of laminar wake



- $\alpha_c = 0.4$, $Re_c = 7.27$

- Eigenspectrum and least stable eigenmode for the neutral case $\alpha_c = 0.4$, $Re_c = 7.27$



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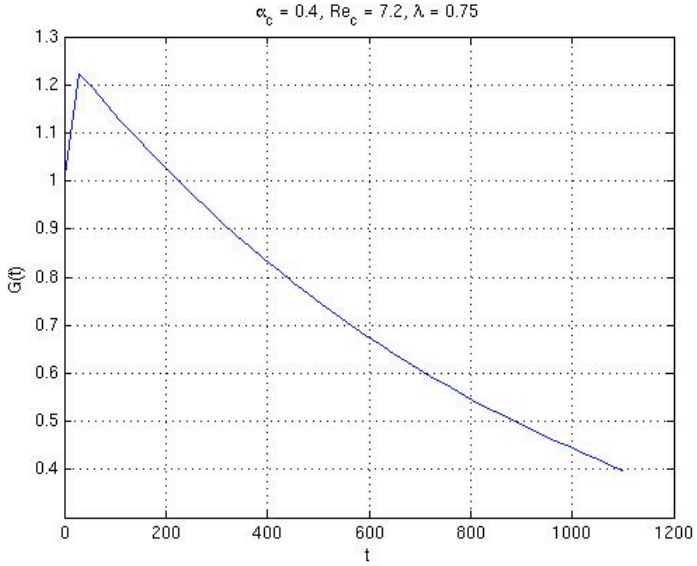
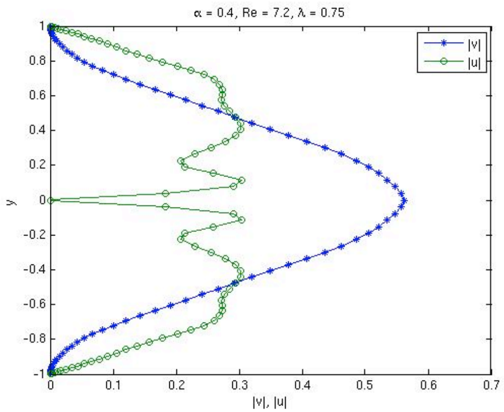
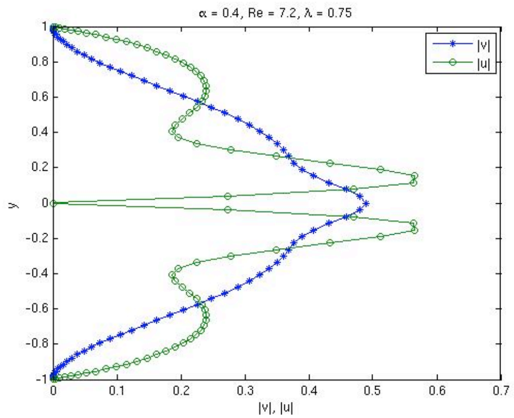
- Transient Growth, optimal disturbance and optimal response for the stable case α = 0.4







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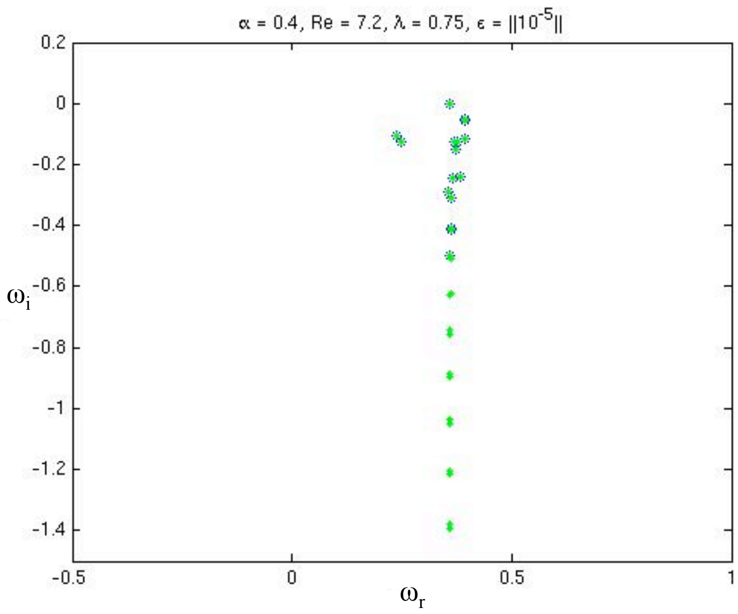
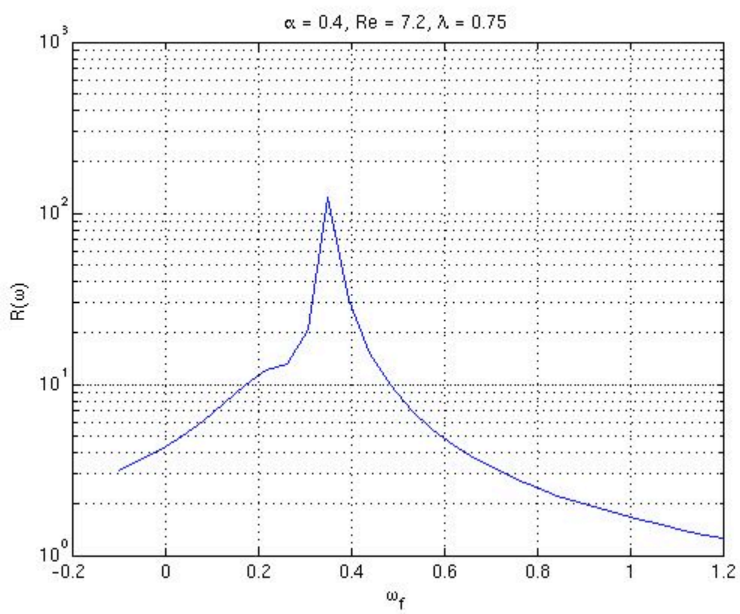
- Eigenvalue sensitivity and Optimal response to time harmonic forcing for a stable case $\alpha = 0.4$





- Least stable mode of homogeneous problem is excited most

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- Conclusions

- The wake behaves similar to the jet

- Jet becomes unstable at very low Reynold numbers → inviscidly unstable

- The operator matrix is almost normal and thus the eigenvectors are almost orthogonal

- Low Reynoldsnumbers quantitavely not relevant as flow is not parallel

- Wake becomes more stable for increasing λ