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KTH Teknikvetenskap

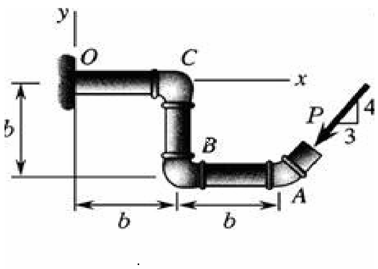
Göran Karlsson

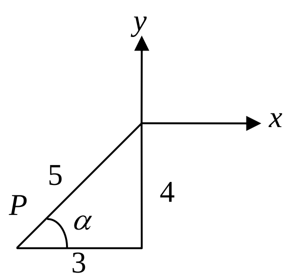
Kontrollskrivning nr 1+2 – KS 1+2 – 2007-05-30

5C1108 Tillämpad fysik, mekanik, 5 poäng
5C1106 Tillämpad fysik, mekanik, 4 poäng

Lösningar

1.





a)

$$\mathbf{P} = P_x \mathbf{e}_x + P_y \mathbf{e}_y = -P\cos\alpha \mathbf{e}_x - P\sin\alpha \mathbf{e}_x = -\frac{3}{5}P\mathbf{e}_x - \frac{4}{5}P\mathbf{e}_y$$

$$\boxed{\mathbf{P} = -\left(\frac{3}{5}P\mathbf{e}_x + \frac{4}{5}P\mathbf{e}_y\right)}$$

b) Kraftmomentet är oförändrat om kraften **P** flyttas längs sin verkningslinje; låt **P** angripa i *A*.

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{P} = (2b\mathbf{e}_x - b\mathbf{e}_y) \times \left(-\frac{3}{5}P\mathbf{e}_x - \frac{4}{5}P\mathbf{e}_y\right) =$$

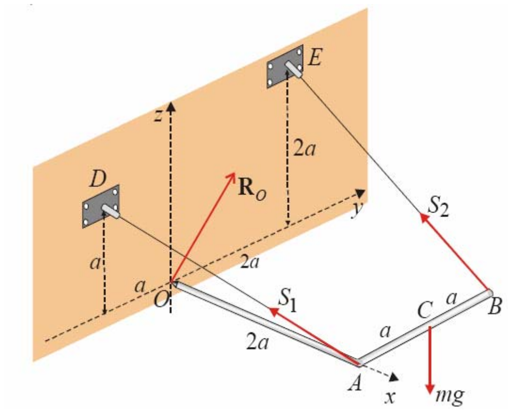
$$= -2b\frac{4}{5}P\mathbf{e}_z - b\frac{3}{5}P\mathbf{e}_z = -\frac{11}{5}P\mathbf{e}_z$$

$$\mathbf{M}_O = -\frac{11}{5}P\mathbf{e}_z$$

$$\boxed{\mathbf{P} = -\left(\frac{3}{5}P\mathbf{e}_x + \frac{4}{5}P\mathbf{e}_y\right), \quad \mathbf{M}_O = -\frac{11}{5}P\mathbf{e}_z}$$

VÄND!

2. a)



$$\mathbf{S}_1 = S_1\mathbf{e}_{AD} = S_1\frac{(-2,-1,1)}{\sqrt{6}},$$

$$\mathbf{S}_2 = S_2\mathbf{e}_{AD} = S_2\frac{(-1,0,1)}{\sqrt{2}};$$

$$\mathbf{M}_O = \mathbf{r}_{OA}\times\mathbf{S}_1 + \mathbf{r}_{OB}\times\mathbf{S}_2 + \mathbf{r}_{OC}\times m\mathbf{g} = \mathbf{0};$$

$$\begin{vmatrix}\mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z\\ 2a & 0 & 0\\ -2 & -1 & 1\end{vmatrix}\frac{S_1}{\sqrt{6}} + \begin{vmatrix}\mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z\\ 2a & 2a & 0\\ -1 & 0 & 1\end{vmatrix}\frac{S_2}{\sqrt{6}} +$$

$$+\begin{vmatrix}\mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z\\ 2a & a & 0\\ 0 & 0 & -1\end{vmatrix} mg = (0,0,0)$$

b) A: $(2a, 0, 0)$, B: $(2a, 2a, 0)$, C: $(2a, a, 0)$, D: $(0, -a, a)$, E: $(0, 2a, a)$

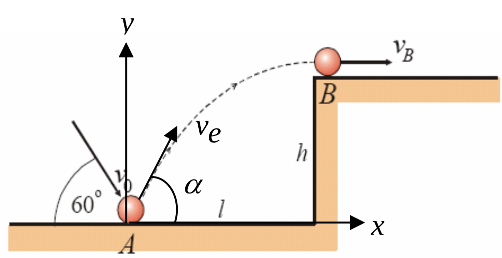
$$\mathbf{e}_{AD} = \frac{(-2,-1,1)}{\sqrt{6}}, \quad \mathbf{e}_{AD} = \frac{(-1,0,1)}{\sqrt{2}}$$

$$\mathbf{e}_x: \; 0 + \frac{2aS_2}{\sqrt{2}} - mga = 0 \Rightarrow S_2 = \frac{mg}{\sqrt{2}}$$

$$\mathbf{e}_z: \; -\frac{2aS_1}{\sqrt{6}} + \frac{2aS_2}{\sqrt{2}} + 0 = 0 \Rightarrow S_1 = \sqrt{3}S_2 = \sqrt{\frac{3}{2}}mg$$

$$\boxed{S_1 = \sqrt{\frac{3}{2}}mg; \quad S_2 = \frac{1}{\sqrt{2}}mg}$$

3.



a) Rörelsemängden bevaras i x-led:

$\rightarrow$: $p_{0x} = p_{ex} \Rightarrow mv_0\cos 60^\circ = mv_e\cos\alpha$

$$v_e\cos\alpha = \frac{1}{2}v_0 \quad (1)$$

Studstalet i y-led:

$$\uparrow: \; e = -\frac{\dot{y}_{1e} - \dot{y}_{2e}}{\dot{y}_{1f} - \dot{y}_{2f}} = -\frac{v_e\sin\alpha - 0}{-v_0\sin 60^\circ} =$$

$$= \frac{v_e\sin\alpha}{v_0\frac{\sqrt{3}}{2}} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow v_e\sin\alpha = \frac{\sqrt{3}}{4}v_0 \quad (2)$$

$(1)^2 + (2)^2$:

$$v_e^2(\cos^2\alpha + \sin^2\alpha) = v_e^2 = \left(\frac{1}{4} + \frac{3}{16}\right)v_0^2 = \frac{7}{16}v_0^2$$

$$\boxed{v_e = \frac{\sqrt{7}}{4}v_0}$$

b) Kastparabel efter studsen:

$$\begin{matrix}\rightarrow: \; \ddot{x} = 0 \\ \uparrow: \; \ddot{y} = -g\end{matrix} \Rightarrow \begin{matrix}\dot{x} = c_1 = \frac{1}{2}v_0 \\ \dot{y} = c_2 - gt = \frac{\sqrt{3}}{4}v_0 - gt\end{matrix} \Rightarrow$$

$$\Rightarrow \begin{matrix} x = \frac{1}{2}v_0 t \\ y = \frac{\sqrt{3}}{4}v_0 t - \frac{1}{2}gt^2\end{matrix}$$

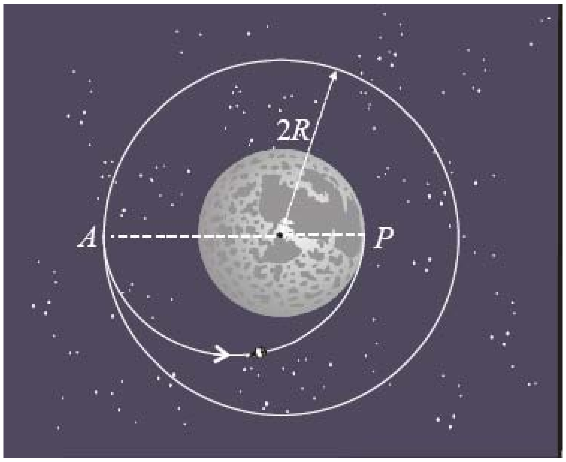
$\dot{y} = 0$ ger tiden t_1 att nå punkten B: $t_1 = \frac{\sqrt{3}}{4}\frac{v_0}{g}$.

Insättning ger sedan

$$\boxed{l = x(t_1) = \frac{\sqrt{3}}{8}\frac{v_0^2}{g}; h = y(t_1) = \frac{3}{32}\frac{v_0^2}{g}}$$

VÄND!

4.



a) Bestäm rymdfarkostens hastighet i den cirkulära banan med hjälp av NII:

$$\mathbf{e}_n:\ G\frac{mM}{(2R)^2} = m\frac{(v_{cirk})^2}{2R}$$

Eftersom $G\frac{mM}{R^2} = mg_0$ är $GM = g_0R^2$

Insättning ger då $v_{cirk} = \sqrt{\frac{GM}{2R}} = \sqrt{\frac{g_0R}{2}}$

$$\boxed{v_{cirk} = \sqrt{\frac{g_0R}{2}}} \quad (1)$$

b) Bestäm sondens absoluta hastighet:

Rörelsemängdsmomentet bevaras:

$$r_{OA}v_A = r_{OP}v_P \Rightarrow 2Rv_A = Rv_P \Rightarrow v_P = 2v_A \quad (2)$$

Mekaniska energin bevaras:

$$T_A + V_A = T_A + V_A \Rightarrow$$

$$\frac{1}{2}mv_A^2 - mg_0\frac{R^2}{2R} = \frac{1}{2}mv_P^2 - mg_0\frac{R^2}{R} \Rightarrow$$

$$\Rightarrow v_P^2 - v_A^2 = 2g_0R\left(1 - \frac{1}{2}\right) = g_0R$$

Insättning av (2) ger

$$3v_A^2 = g_0R \Rightarrow v_A = \sqrt{\frac{g_0R}{3}} \quad v_A < v_{cirk}$$

$$v_{rel} = v_{cirk} - v_A \quad \boxed{v_{rel} = \sqrt{g_0R}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)}$$

$\boxed{\mathbf{v}_{rel} \text{ är parallell med men motriktad } \mathbf{v}_A.}$

GK