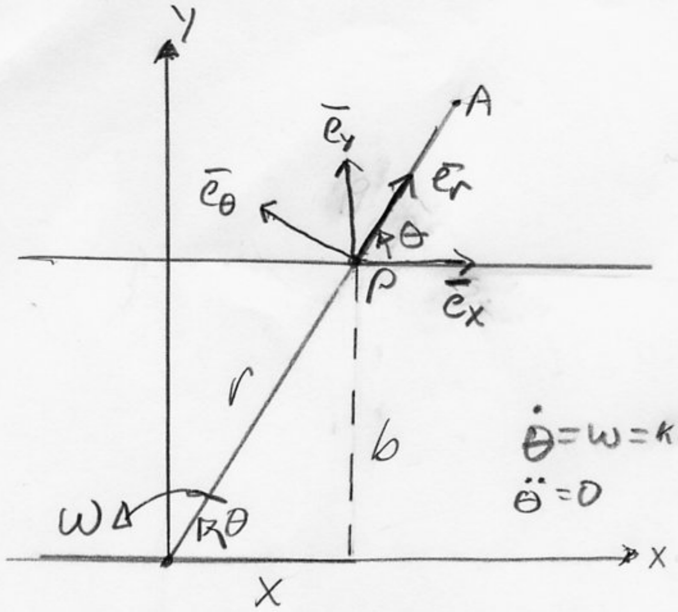
7.15



$\dot\theta = \omega = \text{konst}$

$\ddot\theta = 0$

I. Direkt i Kartesiska Komponenter

$\bar v = \dot x \bar e_x + \dot y \bar e_y$

$\bar a = \ddot x \bar e_x + \ddot y \bar e_y$

$y = b = \text{konst} \Rightarrow \dot y = 0$ och $\ddot y = 0$

$v_x = \dot x \quad a_x = \ddot x \quad v_y = 0 \quad a_y = 0$

$x = r\cos\theta$ Eliminera r!

$r = \frac{b}{\sin\theta} \Rightarrow x = b\frac{\cos\theta}{\sin\theta}$

$\dot x = b\left(-\frac{\dot\theta\sin\theta}{\sin\theta} - \frac{\dot\theta\cos\theta\cos\theta}{\sin^2\theta}\right)$

$\dot x = -b\omega\left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right)$

$\dot x = -b\omega\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}$

$\dot x = -b\omega\frac{1}{\sin^2\theta}$

$\boxed{v_x = -\frac{b\omega}{\sin^2\theta}}$ (1)

$\ddot x = -b\omega\left(-2\omega\frac{\cos\theta}{\sin^3\theta}\right)$

$\boxed{a_x = 2b\omega^2\frac{\cos\theta}{\sin^3\theta}}$ (2)

$\frac{d(\cos\theta)}{dt} = \frac{d(\cos\theta)}{d\theta}\underbrace{\frac{d\theta}{dt}}_{\dot\theta} = -\dot\theta\sin\theta$

$\frac{d}{dt}\left(\frac{1}{\sin\theta}\right) = \frac{d}{d\theta}\left(\frac{1}{\sin\theta}\right)\overbrace{\frac{d\theta}{dt}}^{\dot\theta} =$

$= \frac{d}{d\theta}(\sin\theta)^{-1}\dot\theta = -\dot\theta\cos\theta(\sin\theta)^{-2} =$

$= -\dot\theta\frac{\cos\theta}{\sin^2\theta}$

$\frac{d}{dt}\left(\frac{1}{\sin^2\theta}\right) = \frac{d}{d\theta}(\sin\theta)^{-2}\dot\theta = -2\dot\theta\cos\theta(\sin\theta)^{-3} =$

$= -2\dot\theta\frac{\cos\theta}{\sin^3\theta}$

$\sin\theta > 0$ ty $0 < \theta < \pi$

(1): $v_x < 0$ alltid Partikeln P rör sig åt vänster

$\cos\theta \ge 0$ för $0 < \theta \le \frac{\pi}{2}$

$\cos\theta \le 0$ för $\frac{\pi}{2} \le \theta < \pi$

	x	Accelerationen	Farten $\lvert v_x \rvert$
(2): $a_x \ge 0$ för $0 < \theta \le \frac{\pi}{2}$	>0	Åt höger	Minskar ($>b\omega$)
$a_x \le 0$ för $\frac{\pi}{2} \le \theta < \pi$	=0	Åt vänster	Ökar ($>b\omega$)
$a_x = 0$ för $\theta = \frac{\pi}{2}$	<0	0	Minimum ($=b\omega$)

7.15 (forts)

II Cylinderkomponenter
(planpolära komponenter då $z=0$)

$$\bar{V} = \underbrace{\dot{r}}_{V_r}\,\bar{e}_r + \underbrace{r\dot{\theta}}_{V_\theta}\,\bar{e}_\theta \qquad (\dot{z} = \ddot{z} = 0)$$

$$\bar{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r}\bar{e}_r + \underbrace{(2\dot{r}\dot{\theta} + r\ddot{\theta})}_{a_\theta}\bar{e}_\theta$$

$$\dot{\theta} = \omega \qquad \ddot{\theta} = 0$$

$$r = \frac{b}{\sin\theta} \qquad \frac{d}{dt}\left(\frac{1}{\sin\theta}\right) = \frac{d}{d\theta}\left(\frac{1}{\sin\theta}\right)\frac{d\theta}{dt} = -\dot{\theta}\,\frac{\cos\theta}{\sin^2\theta}$$

enl förra sidan

$$\dot{r} = -\frac{b\dot{\theta}\cos\theta}{\sin^2\theta}$$

$$\dot{r} = -b\omega\,\frac{\cos\theta}{\sin^2\theta} \qquad V_r = -b\omega\,\frac{\cos\theta}{\sin^2\theta} \qquad V_\theta = r\dot{\theta} = b\omega\,\frac{1}{\sin\theta}$$

$$\ddot{r} = -b\omega\left(\frac{-\dot{\theta}\sin\theta}{\sin^2\theta} - \frac{2\dot{\theta}\cos^2\theta}{\sin^3\theta}\right) = b\omega^2\,\frac{\sin^2\theta + 2\cos^2\theta}{\sin^3\theta}$$

se förra sidan

$$a_r = \ddot{r} - r\underbrace{\dot{\theta}^2}_{\omega^2} = b\omega^2\,\frac{\sin^2\theta + 2\cos^2\theta}{\sin^3\theta} - b\omega^2\,\frac{1}{\sin\theta} =$$

$$= b\omega^2\,\frac{\sin^2\theta + 2\cos^2\theta - \sin^2\theta}{\sin^3\theta} = 2b\omega^2\,\frac{\cos^2\theta}{\sin^3\theta}$$

$$a_\theta = 2\dot{r}\underbrace{\dot{\theta}}_{\omega} + r\underbrace{\ddot{\theta}}_{0} = -2b\omega^2\,\frac{\cos\theta}{\sin^2\theta}$$

$$\bar{e}_r = \cos\theta\,\bar{e}_x + \sin\theta\,\bar{e}_y$$

$$\bar{e}_\theta = -\sin\theta\,\bar{e}_x + \cos\theta\,\bar{e}_y$$

$$\bar{V} = V_r\,\bar{e}_r + V_\theta\,\bar{e}_\theta = -b\omega\,\frac{\cos\theta}{\sin^2\theta}(\cos\theta\,\bar{e}_x + \sin\theta\,\bar{e}_y) +$$

$$+ b\omega\,\frac{1}{\sin\theta}(-\sin\theta\,\bar{e}_x + \cos\theta\,\bar{e}_y)$$

$$\bar{V} = \underbrace{-b\omega\left(\frac{\cos^2\theta}{\sin^2\theta} + 1\right)}_{V_x}\bar{e}_x + \underbrace{0}_{V_y}\,\bar{e}_y$$

$$V_x = -b\omega\,\frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta} \qquad \boxed{V_x = -\frac{b\omega}{\sin^2\theta}} \qquad \boxed{V_y = 0}$$

7.15 (forts)

$$\bar{a} = a_r \bar{e}_r + a_\theta \bar{e}_\theta = 2b\omega^2 \frac{\cos^2\theta}{\sin^3\theta}\left(\cos\theta\,\bar{e}_x + \sin\theta\,\bar{e}_y\right) -$$

$$- 2b\omega^2 \frac{\cos\theta}{\sin^2\theta}\left(-\sin\theta\,\bar{e}_x + \cos\theta\,\bar{e}_y\right)$$

$$\bar{a} = \underbrace{2b\omega^2\left(\frac{\cos^3\theta}{\sin^3\theta} + \cos\theta\right)}_{a_x}\bar{e}_x + \underbrace{0}_{a_y}\bar{e}_y$$

$$a_x = 2b\omega^2\,\frac{\cos\theta(\cos^2\theta + \sin^2\theta)}{\sin^3\theta} = 2b\omega^2\,\frac{\cos\theta}{\sin^3\theta}$$

$$\boxed{a_x = 2b\omega^2\,\frac{\cos\theta}{\sin^3\theta}} \quad \boxed{a_y = 0}$$

GK