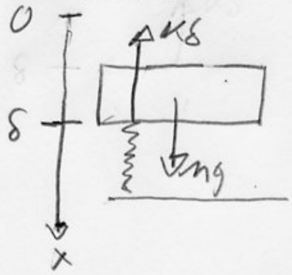
13.18

Amplituden $X = \dfrac{F_0/k}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}}$ (1) enl (13.62) sid 367 i boken

Vila

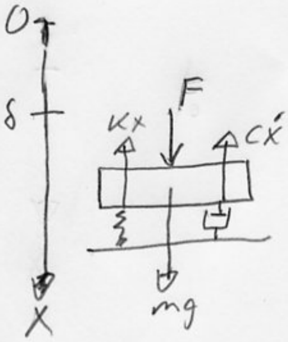


Origo i naturliga längden

$\downarrow: mg - k\delta = 0$

$k = \dfrac{mg}{\delta}$ (2)

Rörelse (Fjädern hopressad, rörels nedåt, återförande kraft)



$\downarrow: -kx - c\dot{x} + mg + F = m\ddot{x}$

$F = pA = \underbrace{A p_0}_{F_0} \sin\omega t = \underbrace{\pi r^2 p_0}_{F_0} \sin\omega t$ (3)

(cylinderns tvärsnittsarea $A = \pi r^2$)

$-kx - c\dot{x} + mg - \pi r^2 p_0 \sin\omega t = m\ddot{x}$

$\ddot{x} + \underbrace{\frac{c}{m}}_{2\zeta\omega_n}\dot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2} x = g + \frac{\pi r^2 p_0}{m}\sin\omega t$

$\omega_n = \sqrt{\frac{k}{m}} \quad 2\zeta\omega_n = \frac{c}{m} \Rightarrow 2\zeta\sqrt{\frac{k}{m}} = \frac{c}{m} \Rightarrow \zeta = \frac{c}{2\sqrt{mk}}$

(3) $F_0 = \pi r^2 p_0 \quad \dfrac{F_0}{k} = \dfrac{\pi r^2 p_0}{k} \underset{(2)}{\overset{\Uparrow}{=}} \dfrac{\pi r^2 p_0 \delta}{mg}$

Ins i (1) ger

$$X = \frac{\pi r^2 p_0 \delta/mg}{\sqrt{\left(1-\left(\frac{\omega}{\sqrt{k/m}}\right)^2\right)^2 + 4\frac{c^2}{4mk}\left(\frac{\omega}{\sqrt{k/m}}\right)^2}} = \frac{\pi r^2 p_0 \delta/mg}{\sqrt{\left(1-\left(\frac{\omega}{\sqrt{g/\delta}}\right)^2\right)^2 + \frac{c^2}{mk}\left(\frac{\omega}{\sqrt{g/\delta}}\right)^2}}$$

$$X = \frac{\pi r^2 p_0 \delta/mg}{\frac{\delta}{g}\sqrt{\left(\frac{g}{\delta}-\omega^2\right)^2 + \left(\frac{c}{m}\omega\right)^2}} = \frac{\pi r^2 p_0/m}{\sqrt{\left(\frac{g}{\delta}-\omega^2\right)^2 + \left(\frac{c}{m}\omega\right)^2}}$$

GK $\boxed{X = \dfrac{\pi r^2 p_0/m}{\sqrt{\left(\frac{g}{\delta}-\omega^2\right)^2 + \left(\frac{c}{m}\omega\right)^2}}}$ $\boxed{\omega_c = \omega_n = \sqrt{g/\delta}}$