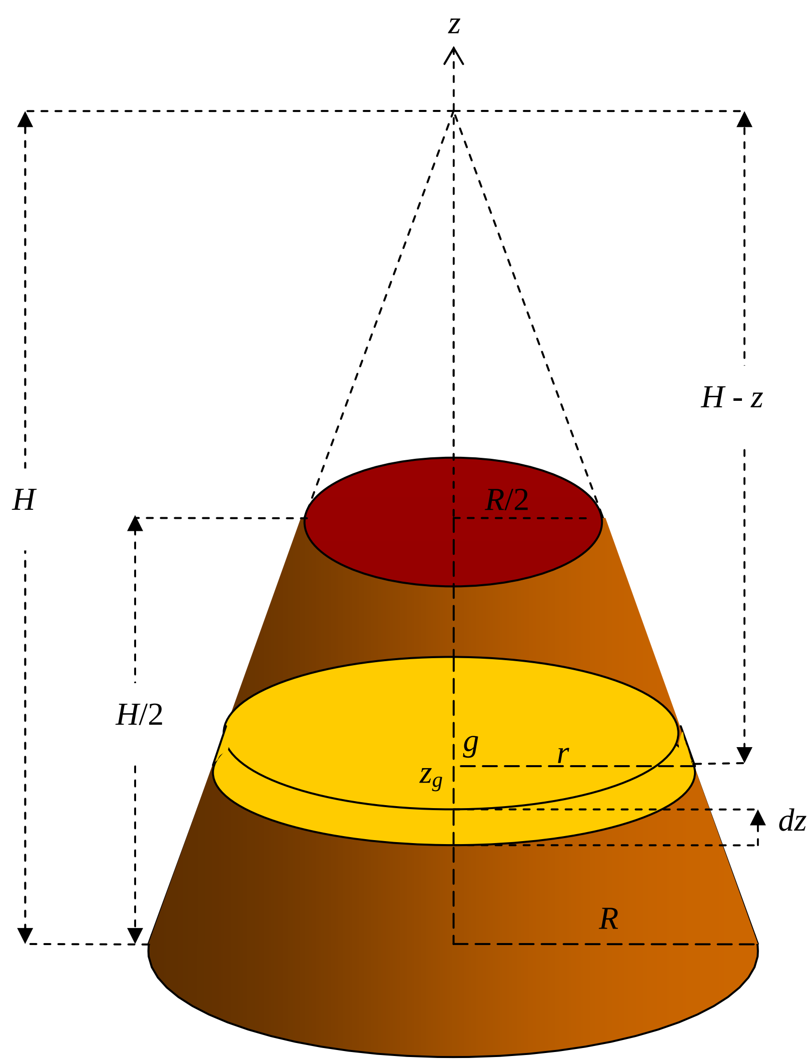
Uppgift 5.12



Välj masselement *dm* som tunn skiva med radien *r* och tjockleken *dz*.

För masselementets masscentrum *g* gäller att det ligger i masselementet, varför $z_g = z$.

Skivans radie $r = r(z)$ är en funktion av z: $\frac{H - z}{H} = \frac{r}{R} \Leftrightarrow r = (H - z)\frac{R}{H}$

$$dm = \rho dV = \rho \pi r^2 dz = \rho \pi (H - z)^2 \frac{R^2}{H^2} dz$$

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$$z_G = \frac{\int z_g dm}{\int dm} = \frac{\int z \rho dV}{\int \rho dV} = \frac{\int_0^{H/2} z\rho\pi (H-z)^2 \frac{R^2}{H^2} dz}{\int_0^{H/2} \rho\pi (H-z)^2 \frac{R^2}{H^2} dz} = \frac{\int_0^{H/2} z(H-z)^2 dz}{\int_0^{H/2} (H-z)^2 dz} = \frac{\int_0^{H/2} z(H^2 - 2Hz + z^2) dz}{\int_0^{H/2} (H^2 - 2Hz + z^2) dz} =$$

$$= \frac{H^2 \int_0^{H/2} zdz - 2H \int_0^{H/2} z^2 dz + \int_0^{H/2} z^3 dz}{H^2 \int_0^{H/2} dz - 2H \int_0^{H/2} zdz + \int_0^{H/2} z^2 dz} = \frac{H^2 \left[\frac{z^2}{2}\right]_0^{H/2} - 2H \left[\frac{z^3}{3}\right]_0^{H/2} + \left[\frac{z^4}{4}\right]_0^{H/2}}{H^2 \left[z\right]_0^{H/2} - 2H \left[\frac{z^2}{2}\right]_0^{H/2} + \left[\frac{z^3}{3}\right]_0^{H/2}} = \frac{\frac{H^4}{8} - \frac{2H^4}{24} + \frac{H^4}{64}}{\frac{H^3}{2} - \frac{2H^3}{8} + \frac{H^3}{24}} =$$

$$= \frac{\frac{24-16+3}{192}}{\frac{12-6+1}{24}} H = \frac{\frac{11}{192}}{\frac{7}{24}} H = \frac{11 \cdot 24}{192 \cdot 7} H = \frac{11}{8 \cdot 7} H = \frac{11}{56} H$$