1. Introduction, tensors, kinematics

Content: Introduction to fluids, Cartesian tensors, vector algebra using tensor notation, operators in tensor form, Eulerian and Lagrangian description of scalar and vector fields, streamlines and pathlines, the relative motion near a point in the fluid.

Literature: Kundu & Cohen (4:th edition) Chapters 1.1-5, 2, 3.1-12
Kundu & Cohen, Dowling (5:th edition) 1.1-5, 2, 3.1-5

Lectures: F1, F2

Recitations: R1, R2

The student should be able to:

- define what is a fluid material
- use tensor formalism to manipulate formulas in vector and matrix algebra
- use tensor formalism to express differential operators
- calculate the time derivative of a vector or scalar field in Eulerian and Lagrangian coordinates and interpret the physical difference between the two descriptions
- define and calculate streamlines and path lines of a given velocity field
- separate the motion of a fluid near a point in a given velocity field in its translation, rotation and deformation using tensor formalism

2 & 5. Conservation laws and Conservation of energy

Content: Force on a fluid surface element and volume element, stress tensor, pressure and viscous stresses, stress in a Newtonian fluid, Reynolds transport theorem, conservation of mass, momentum and energy in a given volume, conservation of mass, momentum and energy in differential form, Navier-Stokes equations, convective and diffusive transport of heat, dissipation of mechanical energy into heat.

Literature: Kundu &Cohen (4:th edition) Chapters 4.1-8, 4.10-11, 4.13-14
Kundu & Cohen, Dowling (5:th edition) 3.6, 4.1-8

Lectures: L3, L8

Recitations: R3, R8, R9

The student should be able to:

- express the force on a fluid volume or fluid element from a given body force
- calculate the force on a fluid surface or fluid surface element with a given unit normal using the stress tensor
- differ between normal stresses and shear stresses, and the isotropic stress tensor and the deviatoric/viscous stress tensor
- calculate the stress tensor for a Newtonian fluid in a given velocity and pressure field, and relate the viscous stresses to the relative motion of the fluid around that point
- formulate the Reynolds transport theorem and apply it to the conservation laws of mass, momentum and energy in a given fluid volume
- derive the differential forms for conservation of mass, momentum and energy in a Newtonian fluid
- give physical interpretations to each of the terms in the conservation laws for mass, momentum and energy in a given fluid volume and in differential form

3. Laminar viscous flow

Content: Boundary conditions in a viscous fluid, thermal boundary conditions, existence of exact solutions to the Navier-Stokes equations, fully developed viscous flow, plane Poiseuille and Couette flow, Poiseuille flow in a pipe and circular Couette flow, other examples of Navier-Stokes equations in curvilinear coordinates, examples of exact solutions to the energy equation, Stoke's problem of an impulsively started plate.

Literature: Kundu & Cohen (4:th edition) Chapters 9.1-11
Kundu & Cohen, Dowling (5:th edition) 8.1-5

Lectures: L4, L5

Recitations: R4, R5, R8

The student should be able to:

- identify the non-linear terms of the Navier-Stokes equations and state under what conditions this mathematical difficulty is absent or can be neglected
- define what is a fully developed viscous flow
- apply boundary conditions for a viscous and heat conducting fluid
- solve the Navier-Stokes equations for fully developed viscous flows in Cartesian or curvilinear coordinates whatever is more appropriate
- solve the Navier-Stokes equations for a fully developed temperature field in Cartesian or curvilinear coordinates whatever is more appropriate
- show how to find a similarity solution in the unsteady example of an impulsively started infinite plate
- identify and calculate each of the terms in the conservations laws for mass, momentum and energy for a given fluid volume of a given example

4. Laminar boundary layers

Content: The boundary layer approximation, the boundary later on a flat plate, Blausius solution, boundary layers with pressure gradient, Falkner-Skan boundary layer solution, separation of the boundary layer, the two-dimensional wake and jets, measurements in a boundary layer with an adverse pressure gradient

Literature: Kundu & Cohen (4:th edition) Chapter 10.1-5, 10.7-9, (10.10-11), 10.12
Kundu & Cohen, Dowling (5:th edition) 9.1-4, 9.7-10

Lectures: L6, L7

Recitations: R6, R7

The student should be able to:

- state the boundary layer approximations and when they are valid
- make an estimate of the boundary layer thickness on a body
- derive the boundary layer equations for two-dimensional flow and state the boundary conditions needed
- derive the Blausius equation for flow over a flat plate and use the Blausius solution to obtain the skin friction and drag on the plate
- define the concepts of displacement thickness and momentum thickness and show how they are related to the conservation of mass and momentum for the flow over a flat plate
- obtain similarity solutions to the boundary layer equations for wake and jet flows
- formulate the thermal boundary layer equations in simple cases
- explain qualitatively the process of separation of the boundary layer
- explain qualitatively the difference between separation of laminar and turbulent boundary layers on a bluff body
- calculate the drag on a body from a diagram of C_D versus *Re*.
- evaluate solutions to the Falkner-Skan similarity solution
- describe a methodology for experimental detection of the velocity in a boundary layer

6. Vorticity dynamics

Content: Vorticity and circulation, rotational and irrotational vortices, Kelvin's Circulation theorem, Helmholtz theorems, the vorticity equation, diffusion and convection of vorticity, generation of vorticity, introduction to irrotational flow, Bernoulli's equation, stream function and velocity potential.

Literature: Kundu & Cohen (4:th edition) Chapter 5.1-5, 4.16-17, 3.4, 3.13
Kundu & Cohen, Dowling (5:th edition) 5.1-5, 4.3, 4.9

Lectures: L9, L10

Recitations: R9, R10

The student should be able to:

- give a mathematical description for the vortex flows of a solid body rotation and an irrotational vortex
- define the concept of circulation in a fluid, state Kelvin's circulation theorem and when it is valid
- state Helmholtz vortex theorems and when they are valid
- derive the vorticity equation from the momentum equation using tensor notation
- make a physical interpretation of the terms in the vorticity equation
- identify the conditions for the mathematical/physical analogy with the energy equation
- explain the consequences of Kelvin's theorem if the fluid is initially irrotational
- derive Bernoulli's equation and state when it is valid
- define stream function and velocity potential and formulate an irrotational flow problem in terms of these functions
- formulate and solve fully developed viscous flow problems in terms of the vorticity equation, identify sources for generation of vorticity

7. 2D irrotational flows

Content: Irrotational flow theory, the complex velocity potential, examples of analytic functions, line source, line vortex, line doublet, the principle of superposition, flow around a cylinder with and without circulation, pressure distribution and force on a circular cylinder and on simple wing profiles through conformal mapping, the Kutta condition

Literature: Kundu & Cohen (4:th edition) Chapter 6.1-15, 15.1-8
Kundu & Cohen, Dowling (5:th edition) 6.1-6, 14.1-5

Lectures: L11, L12

Recitations: R11, R12

The student should be able to:

- state the physical conditions for the validity of irrotational flow in practice
- analyze simple flow fields using the complex potential
- calculate the pressure field for an irrotational flow field
- calculate the force on a cylinder with circulation in irrotational flow
- use the method of conformal mapping
- know the meaning of the Kutta condition and apply it to determine the circulation around an airfoil

8. Introduction to turbulent flow

Content: General properties of turbulent flows, Reynolds averaged conservation equations for mass, momentum and energy, the Reynolds stress tensor, the turbulent kinetic energy, application of boundary layer theory to turbulent flow, plane wall-free shear flow, plane wall bounded shear flow

Literature: Kundu & Cohen (4:th edition) Chapter 13.1, 13.5-7, 13.10-12
Kundu & Cohen, Dowling (5:th edition) 12.1, 12.5, (12.6-10)

Lectures: L13, L14

Recitations: R13, R14

The student should be able to:

- specify the typical properties of turbulent flow
- specify the general conditions for when a flow is expected to be turbulent
- state what is the Reynolds decomposition
- derive the Reynolds averaged conservation equations
- define the Reynolds stress tensor and give a physical interpretation of the Reynolds shear stress
- define the turbulent kinetic energy
- state the boundary layer equations for turbulent flow
- identify the global flow picture in wall-free and wall-bounded turbulent shear flows