

Exam in Fluid Mechanics SG2214

Final exam in course SG2214, January 14 2008, 9-13 in L52

Examiner: Anders Dahlkild

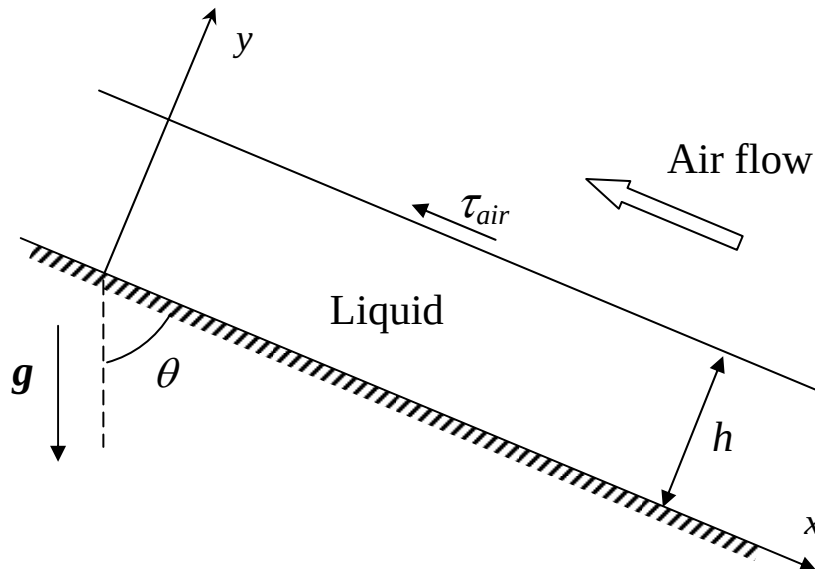
The point value of each question is given in parenthesis and you need more than 20 points to pass the course including the points obtained from the homework problems.

Copies of Cylindrical and Spherical Coordinates, which will be supplied if necessary, can be used for the exam as well as a book of basic math formulas and a calculator.

1. (10p.)

A viscous liquid, of viscosity μ and density ρ , flows due to gravity, g , along an inclined plate. The thickness of the liquid film, h , is constant. An air flow, adjacent to the liquid, flows upwards along the plate. The motion of the air is such that a shear stress $\tau_{xy}(y=h) = -\tau_{air}$ acts on the free surface of the liquid. The pressure in the airflow, p_{air} , is constant.

Calculate the velocity field in the liquid at steady state. For what value of τ_{air} is the shear stress at the plate, $\tau_{xy}(y=0)$, zero? Make a sketch of the velocity profile for this particular case.



2. (10p.)

Consider the complex velocity potential

$$F(z) = \left(\frac{m - i\Gamma}{2\pi} \right) \ln z$$

where $z = x + iy = re^{i\theta}$ and m and Γ are real numbers.

a) (4p.)

What is the equation for the streamlines? Sketch a streamline.

b) (6p.)

Compute the location at time $t > 0$ of a particle that at time $t = 0$ was located at $r = r_0$, $\theta = \theta_0$, where r and θ are polar coordinates.

Hint: $u_r = \frac{dr}{dt}$, $u_\theta = r \frac{d\theta}{dt}$

3. (10p.)

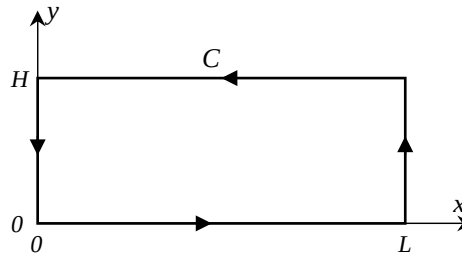
Consider the material time derivative of the x-component of a two-dimensional velocity field

$$\frac{Du}{Dt} = \underbrace{\frac{\partial u}{\partial t}}_I + u \underbrace{\frac{\partial u}{\partial x}}_II + v \underbrace{\frac{\partial u}{\partial y}}_III,$$

and the z-component of the vorticity $\omega_z = \underbrace{(\nabla \times \mathbf{u}) \cdot \mathbf{e}_z}_{IV}$

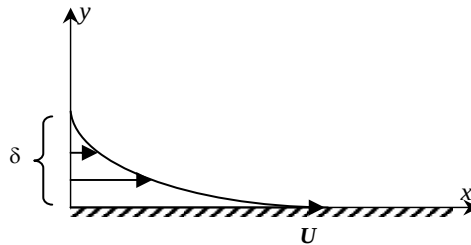
Considering the main features of the different flows below, examined during the course, give the sign of each of the terms, I, II, III and IV, or state if it is zero.

Also, calculate the circulation Γ around the closed contour in each case: $\Gamma = \oint_C u_i dx_i$



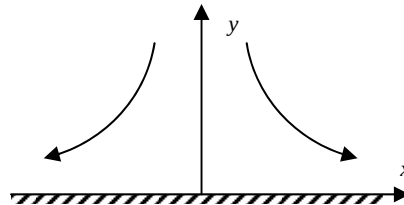
a) (3p.)

Impulsive start from rest of an infinite plate, at velocity U , in an initially stagnant and viscous fluid. Assume that $H \gg \delta$.



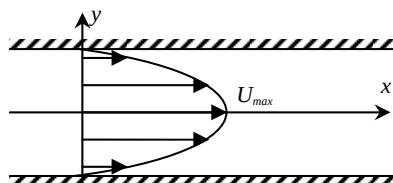
b) (3.5p.)

Steady irrotational flow towards a stagnation point, for $x > 0$ and $x < 0$ respectively. (The circulation is to be considered only for the curve at $x > 0$.)



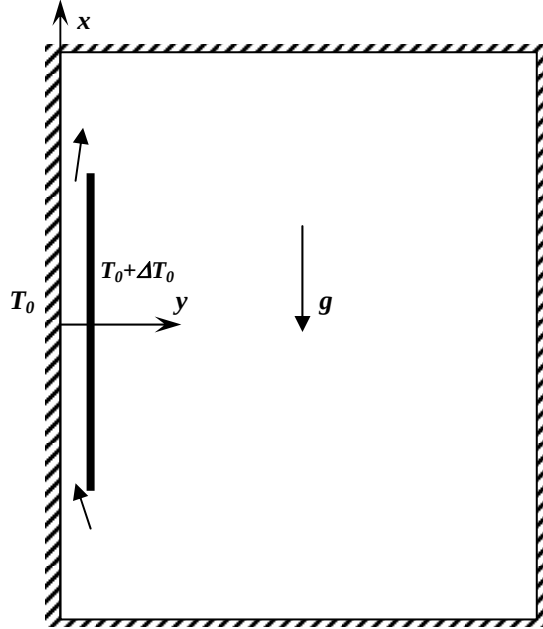
c) (3.5p.)

Fully developed viscous flow in a channel at constant flow rate, for $y > 0$ and $y < 0$ respectively. (The circulation is to be considered only for the curve at $y > 0$.)



4. (10p.)

Consider the 2D flow due to gravity in the narrow channel of width h between a wall and a heated plate submerged in a large container. The wall has temperature T_0 and the plate temperature $T_0 + \Delta T_0$. The overall temperature of the fluid in the container is T_0 , although in the long and narrow channel it is higher.



For 2D, steady state flow and assuming a constant viscosity μ_0 , show that the vorticity equation (in the Boussinesq approximation)

$$\frac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{\omega} \cdot \nabla \mathbf{u} + \frac{1}{\rho_0^2} \nabla \rho \times \nabla p + \frac{\mu_0}{\rho_0} \nabla^2 \boldsymbol{\omega}$$

and the energy equation

$$\frac{DT}{Dt} = \kappa_0 \nabla^2 T$$

may be satisfied if it is given that in the channel

$$u = \frac{U_0}{6} \frac{y}{h} \left(1 - \left(\frac{y}{h} \right)^2 \right),$$

$$v = w = 0,$$

$$\rho(T) = \rho_0 (1 - \alpha_0 (T - T_0)),$$

$$T = T_0 + \Delta T_0 \frac{y}{h},$$

$$p = p_0 - \rho_0 g x.$$

Determine the particular value of the constant U_0 required. (ρ_0 , T_0 , ΔT_0 , α_0 , ρ_0 , κ_0 , and μ_0 are all given constants.) What is the volumetric flow rate (volume flux) in the channel ?

5. (10p.)

Consider the narrow turbulent wake flow downstream of a two-dimensional streamlined body at high Reynolds number. Assume that far downstream of the body the wake is weak, so that the average velocity defect, $U_1(x,y)$, is much smaller than the constant free stream velocity U_∞ , i.e.

$$U(x,y) = U_\infty - U_1(x,y),$$

$$U_1 \ll U_\infty.$$

Start with the boundary layer approximation of the Reynolds averaged Navier-Stokes equations with zero pressure gradient:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} (-\overline{u'v'}),$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0.$$

Derive a linearized equation for the weak wake and obtain the velocity defect, $U_1(x,y)$, if the turbulent stresses are modelled by

$$-\overline{u'v'} = \beta_T U_\infty \theta_w \frac{\partial U}{\partial y},$$

where β_T is an empirical non-dimensional constant of order one and the momentum loss thickness of the wake is given by

$$\theta_w = \int_{-\infty}^{\infty} \frac{U(x,y)}{U_\infty} \left(1 - \frac{U(x,y)}{U_\infty} \right) dy.$$

You may regard the momentum loss thickness of the wake as a given constant. Motivate why the momentum loss thickness of the wake does not vary with x ?

Hint: You may have use of the fact that $\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi}$.