Homework problem 2(5.5p.), Fluid Mechanics SG2214
Due Oct 10, 2016

Be careful to explain and motivate each non-trivial step of the solution to these problems.

1. (1 p.)
Consider a thin viscous liquid film of constant viscosity μ and depth h_0 initially at rest on an infinite horizontal plate. The liquid is set in motion by the, so called, Marangoni convection effect. This effect is a result of surface tension gradients along a gas-liquid interface. The liquid is “pulled” towards regions of larger surface tension. Surface tension, σ [N/m], (σ is a force per unit length along a cut in the gas-liquid interface) is critically dependent on temperature (σ decreases with temperature) so a temperature gradient will pull the fluid towards regions of lower liquid surface temperature. (Similar decreasing effect on surface tension is given by e.g. increased alcohol content in water or increased surfactant surface concentration.) Suppose then that the temperature decreases (σ increases) in one direction along the liquid film on the plane in question. In the fluid mechanical model of this phenomena the surface tension gradient is equivalent to a shear stress acting on the surface of the liquid film (unit normal perpendicular to the gas-liquid interface) of magnitude $\tau_s = d\sigma/ds$, where s is a coordinate along the surface in the direction of increasing σ.
Suppose the liquid is water and h_0=1 mm, $d\sigma/dT$ = -0.015 N/(m K), dT/ds = -10 K/m.

a) At what angle, α, must the infinite plate be inclined from the horizontal direction in order for gravity, g, to counteract this convection such that at fully developed steady state flow the net volumetric flow rate of liquid is zero?

b) What will be the velocity at the film surface for the condition in a)?

c) What will be the maximum velocity of the liquid in the other direction for the condition in a)?

d) What will be the shear stress at the plate for the condition in a)?

e) What would happen, do you think, if the viscous liquid film was enclosed in a container with a horizontal bottom wall but of finite length L≈1 dm >> h_0 (closed ends)?

2. (1.5 p.)
Consider the steady state, two-dimensional axisymmetric flow between two concentric circular cylinders rotating at angular velocities Ω_1 and Ω_2 respectively. The cylinder walls at $r = r_1$ and $r = r_2$ are porous and a radial flow $u_r(r)$ is also present between the cylinders.

a) Suppose the flow is irrotational and make a superposition of an irrotational line source/sink and an irrotational line vortex. Obtain the flow field $u_r(r)$ and $u_\theta(r)$ if the total volumetric flux of the radial flow (per unit length) is $m = |m|$ ($u_r > 0$) and $m = -|m|$ ($u_r < 0$) respectively. Assume that the angular fluid velocity component at the wall of the *incoming* flow has the same velocity as the feeding cylinder wall. (This means that in the case $u_r > 0$ the speed of the *inner* cylinder wall carries over to the angular fluid velocity component at the *inner* wall. In the case $u_r < 0$ the speed of the *outer* cylinder wall carries over to the angular fluid velocity component at the *outer* wall.)

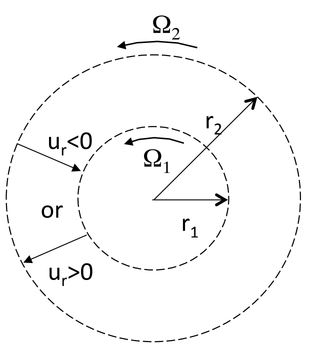
b) Use MATLAB to draw a diagram of $u_\theta(r/r_2)/\Omega r_2$ for the two cases if $r_2 = 2r_1$. Ω_1 and Ω_2 are individual data for each project group.

c) The two solutions in a) do actually satisfy the Navier–Stokes equations of a *viscous* fluid. However, do the solutions in a) satisfy the boundary conditions of a viscous fluid at the boundaries? Why/why not? For a viscous fluid, kinematic viscosity ν, assume $u_r(r)$ is the same as for the irrotational flow in a) and obtain $u_\theta(r)$ by integrating the Navier-Stokes equations and applying the correct boundary conditions of a viscous fluid.

d) Use MATLAB to make a computer plot of $u_\theta(r)/\Omega r_2$ for some different values of $|m|/(2\pi\nu)$=0, 1, 5, 10, 25, 50, 100 if $r_2 = 2r_1$. ($Sign(u_r)$ is individual for each group.) In what limit of the Reynolds number Re= $|m|/(2\pi\nu)$ does the viscous solution approach the irrotational solution in a) away from both boundaries?

e) Plot also the vorticity $\omega_z(r/r_2)/\Omega$ for the different Re. How is the vorticity distributed at different Re?

f) Extra: Try to find a boundary layer approximation to the flow close to one of the cylinders by using the asymptotic suction boundary layer in Chap 10.16 of Kundu&Cohen 4:th ed.



Hint: In 2c) make the ansatz $u_\theta(r) = const \times r^n$ and find two possible values of n to obtain $u_\theta(r) = Ar^{n1} + Br^{n2}$ where A and B are constants to be determined. (One particular value of Re needs special treatment, which value? There is no requirement to treat this case.)

Group	Ω_1	Ω_2	sign(u_r)
1	Ω	Ω	+1
2	Ω	Ω	-1
3	0	Ω	+1
4	0	Ω	-1
5	Ω	0	+1
6	Ω	0	-1
7	$-\Omega$	Ω	+1
8	$-\Omega$	Ω	-1
9	Ω	$-\Omega$	+1
10	Ω	$-\Omega$	-1
11	$-\Omega$	0	+1
12	$-\Omega$	0	-1
13	0	$-\Omega$	+1
14	0	$-\Omega$	-1

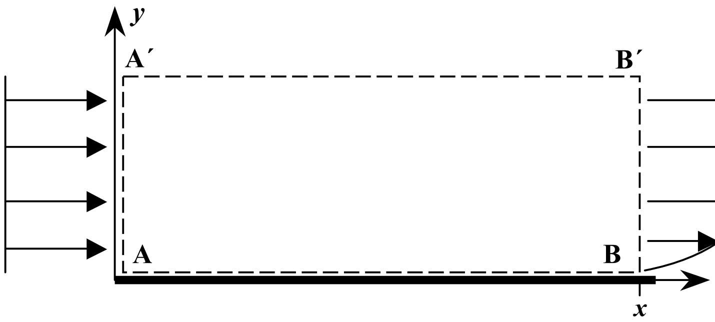
3. (1.5 p.)

Consider the Blausius boundary layer flow over a flat plate. The vertical component of the velocity field is given by

$$v(x,y) = U_\infty \sqrt{\frac{\nu}{U_\infty x}}\frac{1}{2}\big(\eta f'(\eta) - f(\eta)\big), \quad \eta = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}.$$

Far away above the plate we have from the Blausius solution that

$$\frac{f(\eta)}{2} = \frac{\eta}{2} - 0.8604.$$



a) Consider a rectangular control volume of length x adjacent to the plate and extending far away from the edge of the boundary layer. Use the information above to determine the difference in mass flux between that across AA´ and that across BB´.

b) Show that the difference in flux of x-momentum between that across AA´ and BB´ is given by

$$\rho U_\infty^2 2(0.8604 + f''(0))\sqrt{\frac{\nu x}{U_\infty}}. \qquad (f''(0) = 0.332)$$

Hints: Expand the derivative of $(ff')'$ and combine with Blausius equation for f(η).

c) Show that the flux of x-momentum across A´B´ is given by

$$\rho U_\infty^2 2 \cdot 0.8604\sqrt{\frac{\nu x}{U_\infty}}$$

d) Use the results in b) and c) to calculate the drag force per unit width, D', on the upper surface of the plate.

4. (1.5 p.)

Consider plane Couette flow of a viscous incompressible fluid in between two infinite parallel plates at distance h apart. The relative velocity between the plates is U_0. Assume fully developed steady state flow and calculate the temperature field if the plates are kept at constant temperatures by thermal contact with two heat reservoirs at T_0 and $T_0+\Delta T$ respectively. Calculate also the heat flux density vector at each of the plates. If U_0 is given, for what value of ΔT is the heat flux density zero at the upper/lower plate? Sketch the temperature profiles for these values of ΔT. At last, find the total irreversible rate of production of entropy (per unit area of the plates) by evaluating the sum

$$\dot{S} = \frac{q_{\text{in,top}}}{T_{\text{top}}} + \frac{q_{\text{in,bottom}}}{T_{\text{bottom}}}$$

where q_{in} is the heat flux density *into* a reservoir(plate) at temperature T. Try to show from your result that if the viscosity and the coefficient of heat conduction are positive numbers then $\dot{S} \geq 0$, (independent of the signs of U_0 and ΔT).