SG2214 FLUID MECHANICS

Lecture I.

{ Show Powerpoint presentation as introduction }
~ 30 pages probably ~ 45 min

Vector form & Cartesian tensor form of equations

{ Show OH of N.-S. eq.'s }

x-component of vector form

$$\rho\left[\frac{\partial u}{\partial t}+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)u\right]=-\frac{\partial p}{\partial x}+\mu\underbrace{\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)}_{\nabla^2 u}$$

i=1 for tensor form

$$\rho\left[\frac{\partial u_1}{\partial t}+\underbrace{u_j\frac{\partial u_1}{\partial x_j}}\right]=-\frac{\partial p}{\partial x_1}+\mu\underbrace{\frac{\partial^2 u_1}{\partial x_j\partial x_j}}_{\nabla^2 u_1}$$

$$\sum_{j=1}^{3}u_j\frac{\partial u_1}{\partial x_j}=u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}+u_3\frac{\partial u_1}{\partial x_3}$$

j is summation index, i is free (running) index

Einsteins summation convention: Σ is omitted
(whenever an index appears twice, it is summed over)

Recitation 1 deals with Cartesian tensors,
Kundu & Cohen, chapt. 2 also,
and homework I.2.

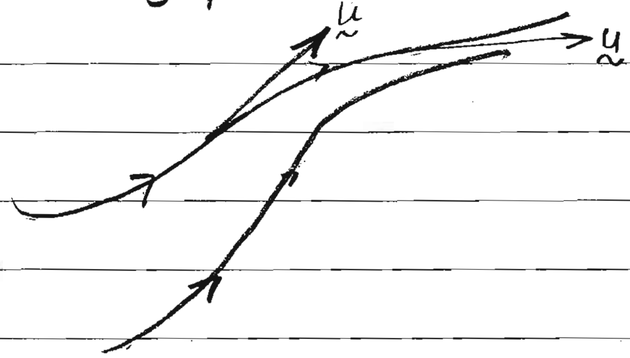
Kinematics:

Description of fluid particle motion, acceleration and deformation.

{Shorter version exists}

Streamlines: tangents to the instantaneous velocity field

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

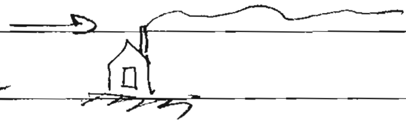


Pathlines: line following the path of a marked fluid particle

$$\frac{d}{dt}\underset{\sim}{x}_p(t) = \underset{\sim}{u}(\underset{\sim}{x}_p(t), t)$$

{For steady state flow these lines are the same}

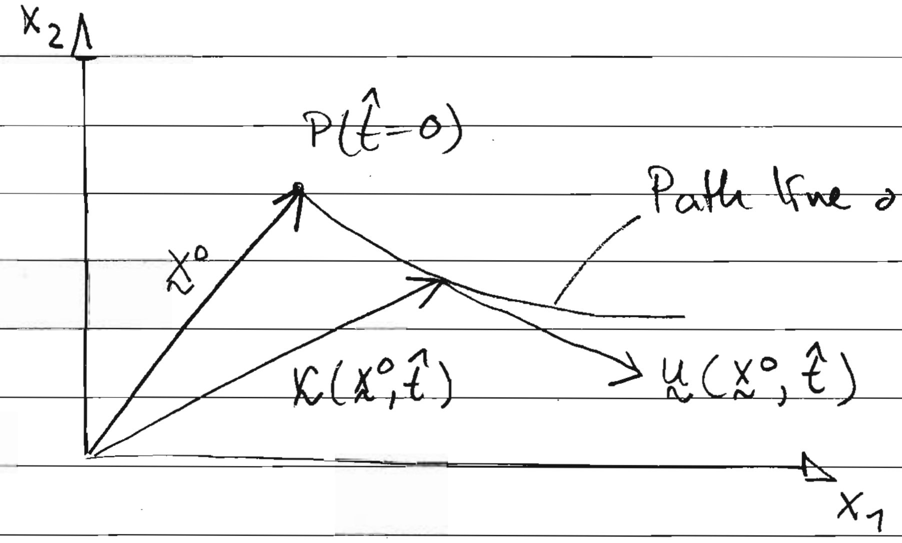
Streaklines: line connecting marked fluid particles that has passed a specific point in the flow

(smoke line )

Timelines: line connecting neighbouring fluid particles at time t that were marked at $t=0$.

Lagrangian and Eulerian coordinates.

1. Lagrange coordinates (classical mechanics)



Every particle is marked and followed in the flow

Indep. variables : x_i^0 – initial position $i = 1, 2, 3$

$\hat{t}$ – time

$$r_i = r_i(x_1^0, x_2^0, x_3^0, \hat{t}) = r_i(x_k^0, \hat{t})$$

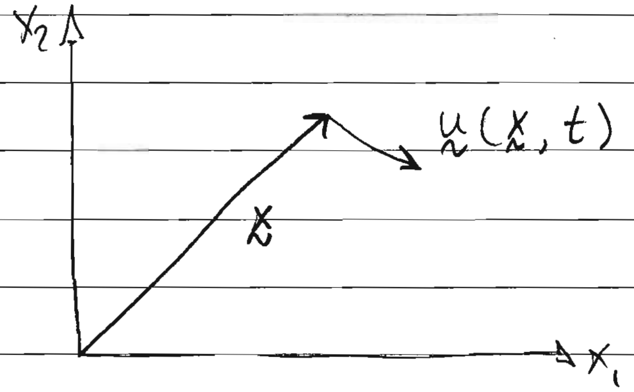
Velocity of particle $u_i = \dfrac{\partial r_i(x_k^0, \hat{t})}{\partial \hat{t}}$

Acceleration of part. $a_i = \dfrac{\partial^2 r_i}{\partial \hat{t}^2} = \dfrac{\partial u_i(x_n^0, \hat{t})}{\partial \hat{t}}$

When x_n^0 changes we consider new particles

Any flow variable $F(x_n^0, \hat{t})$

2. Euler coordinates



Consider fixed point in space. Fluid flows past point $\underset{\sim}{x}$ at time t.

Independent variables : x_i - space coord. $i=1,2,3$

t - time

Fluid velocity at x_k and time t: $u_i(x_k, t)$

Flow variable $F(x_k, t)$

3. Relation between Lagrangian & Eulerian coordinates

$\underset{\sim}{x}^0$ defines a Lagrangian particle passing the point $\underset{\sim}{x} = \underset{\sim}{r}(\underset{\sim}{x}^0, \hat{t})$ at the time $t = \hat{t}$

The Eulerian velocity is $\underset{\sim}{u}(\underset{\sim}{x}, t) = \dfrac{\partial}{\partial \hat{t}} \underset{\sim}{r}(\underset{\sim}{x}^0, \hat{t})$

at $t = \hat{t}$, i.e. the velocity of the Lagrangian particle passing $\underset{\sim}{x}$ at $t = \hat{t}$.

4. Material time derivative

$$F = F_L(\underset{\sim}{X}^0, \hat{t}) = F_E(\underset{\sim}{r}(\underset{\sim}{X}^0; \hat{t}), t) \quad \text{at } t = \hat{t}$$

Rate of change of F for fluid particle

$$\frac{\partial}{\partial \hat{t}} F_L(\underset{\sim}{X}^0, \hat{t})$$

Express this in Euler coordinates

$$\frac{\partial F_L}{\partial \hat{t}} = \frac{\partial F_E}{\partial x_i} \underbrace{\frac{\partial r_i}{\partial \hat{t}}}_{= u_i} + \frac{\partial F_E}{\partial t} \underbrace{\frac{\partial t}{\partial \hat{t}}}_{= 1} = \frac{\partial F_E}{\partial t} + u_i \frac{\partial F_E}{\partial x_i}$$

$$\frac{\partial}{\partial \hat{t}} = \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial t} + u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2} + u_3 \frac{\partial}{\partial x_3}$$

$$= \frac{\partial}{\partial t} + \underset{\sim}{u} \cdot \nabla$$

Material / substantial time derivative

$$\underbrace{\frac{DF}{Dt}}_{\boxed{\text{Rate of change for material fluid particle}}} = \underbrace{\frac{\partial F}{\partial t}}_{\boxed{\text{Rate of change at fixed position in space.}}} + \underbrace{\underset{\sim}{u} \cdot \nabla F}_{\boxed{\text{Advective rate of change as particle moves through spacial gradients of F}}}$$

Let F be fluid velocity $\underset{\sim}{u}(\underset{\sim}{x}, t)$

$$\frac{D}{Dt}\underset{\sim}{u} = \frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{u}$$

Acceleration of material fluid particle.

Let F be position vector $\underset{\sim}{x}$

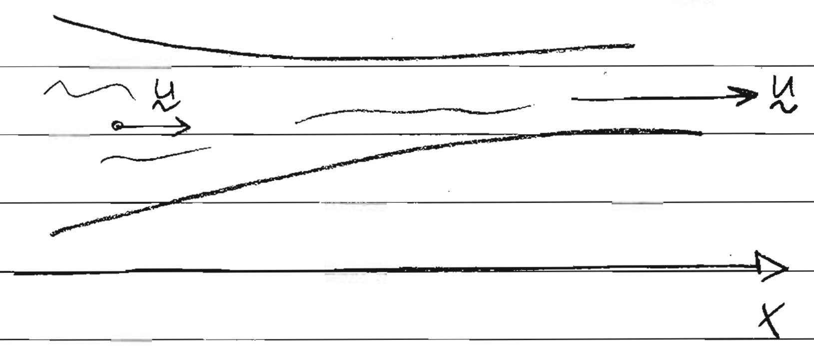
$$\frac{D}{Dt}\underset{\sim}{x} = \underbrace{\frac{\partial \underset{\sim}{x}}{\partial t}}_{=0} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{x} = u \frac{\partial}{\partial x}\underset{\sim}{x} + v \frac{\partial}{\partial y}\underset{\sim}{x} + w \frac{\partial}{\partial z}\underset{\sim}{x}$$

$$= u \underset{\sim}{e}_x + v \underset{\sim}{e}_y + w \underset{\sim}{e}_z = \underset{\sim}{u}$$

$$\frac{D}{Dt} x_i = \underbrace{\frac{\partial x_i}{\partial t}}_{=0} + u_j \frac{\partial x_i}{\partial x_j} = u_j \delta_{ij} = u_i$$

Kronecker delta $\delta_{ij} = \begin{cases} 1 & ; \ i = j \\ 0 & ; \ i \neq j \end{cases}$; $\delta_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

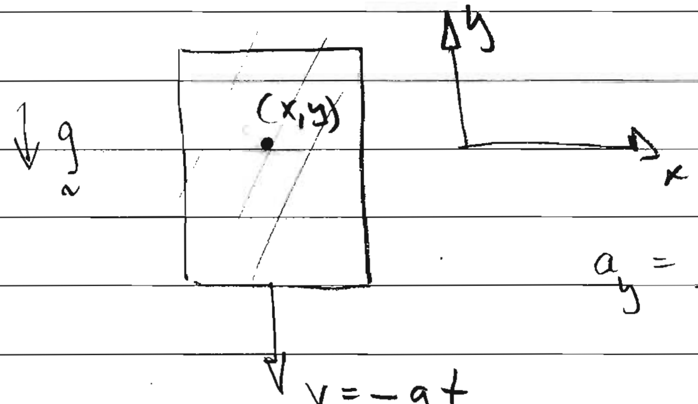
Acceleration of fluid particle in river



$$a_x = \underbrace{\frac{\partial u}{\partial t}}_{=0} + u\frac{\partial u}{\partial x} + \underbrace{\underset{0}{\overset{\shortparallel}{v}}\frac{\partial u}{\partial y} + \underset{0}{\overset{\shortparallel}{w}}\frac{\partial u}{\partial z}}_{\text{assumed}} = \underbrace{u\frac{\partial u}{\partial x}}_{>0}$$

∴ Acceleration of fluid element $\neq 0$ even if velocity at fixed x does not change with time.

Acceleration of falling can filled with fluid.



$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} =$$
$$= -g - gt \cdot 0 = -g$$

∴ No gradients of velocity field in bucket
⇒ advective term is zero.

See also homework I.1.