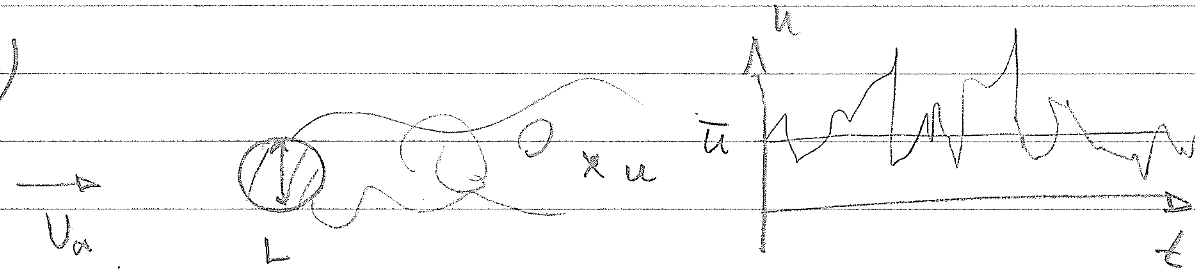
Some properties of turbulent flows.

1)

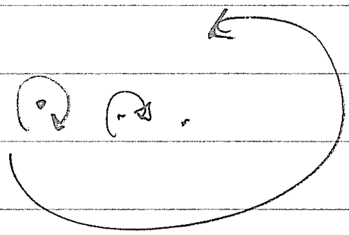


The flow is random in space and time

2) $Re = \frac{U_\infty L}{\nu} \gg 1$

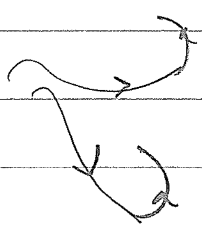
Reynolds number is large and the flow is highly non-linear.

3)



The vorticity, $\underset{\sim}{\omega}$, is a significant part of the flow, where unsteady vortices/eddies of different sizes appear simultaneously

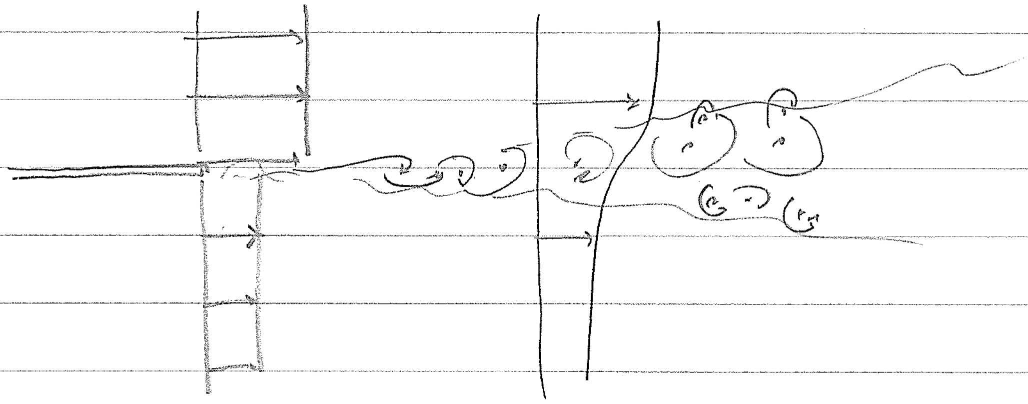
4)



The flow is three-dimensional

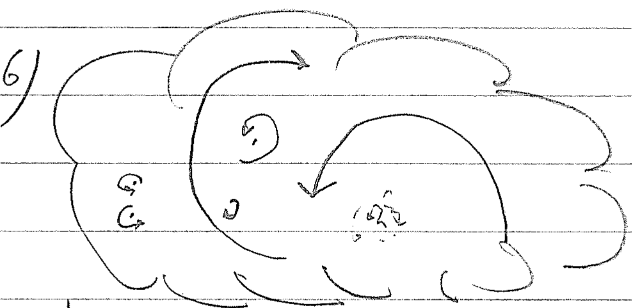
$\underset{\sim}{u} = (u(\underset{\sim}{x},t), v(\underset{\sim}{x},t), w(\underset{\sim}{x},t))$

5)

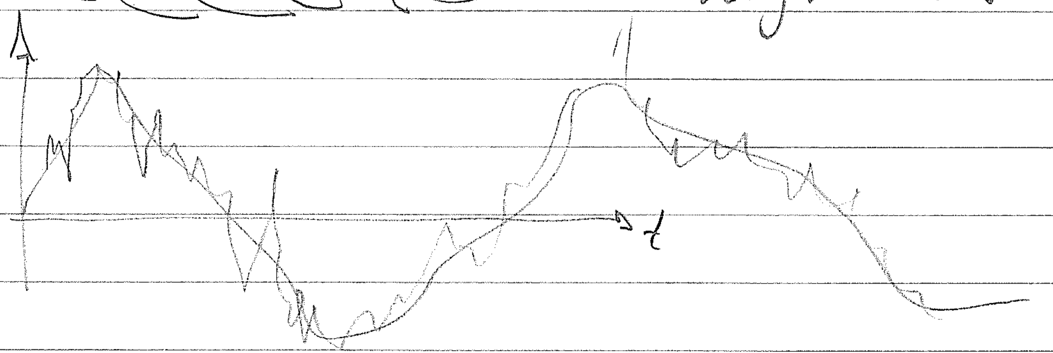


Turbulence has diffusive properties,
Exchange of momentum and energy
between fluid layers.

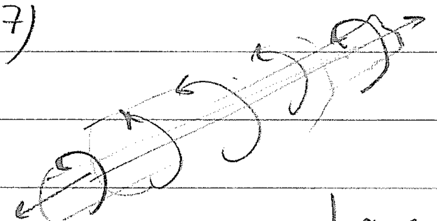
6)



Turbulent motions
appear on largely
different time and
lengthscales



7)



Increasing gradients by
vortex stretching

Large gradients ⇒ Viscous dissipation.

Requires continuous supply of energy to
maintain turbulent flow.

8) Turbulence is a continuum phenomena, i.e.
does not depend on effects at microscopic scales.

Equations of motion for averaged flow.

Ensemble average $U_i = \overline{u_i} = \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{\infty} u_i^{(n)}(\underset{\sim}{x},t)$

Reynolds decomposition $u_i(\underset{\sim}{x},t) = \overline{u_i}(\underset{\sim}{x},t) + u_i'(\underset{\sim}{x},t)$

N.-S. $\Rightarrow$

$$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial u_i'}{\partial t} + \overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j} + \overline{u_j}\frac{\partial u_i'}{\partial x_j} + u_j'\frac{\partial \overline{u_i}}{\partial x_j} + \underbrace{u_j'\frac{\partial u_i'}{\partial x_j}}_{\frac{\partial}{\partial x_j}(u_j'u_i') - \frac{\partial u_j'}{\partial x_j}u_i'} =$$

$$= -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} - \frac{1}{\rho}\frac{\partial}{\partial x_i}p' + \nu\nabla^2\overline{u_i} + \nu\nabla^2 u_i';$$

$$\frac{\partial \overline{u_k}}{\partial x_k} + \frac{\partial u_k'}{\partial x_k} = 0 \quad \Rightarrow \quad \frac{\partial \overline{u_k}}{\partial x_k} = 0 \quad \Rightarrow \quad \frac{\partial u_k'}{\partial x_k} = 0$$

We have $\overline{u_i'} = 0 \Rightarrow \overline{\frac{\partial u_k'}{\partial x_k}} = \frac{\partial}{\partial x_k}\overline{u_k'} = 0$

Also $\overline{v'\overline{v}} = \overline{v'}\,\overline{v} = 0$

Ensemble average of N.-S.

$$\Rightarrow \frac{\partial \overline{u_i}}{\partial t} + \overline{u_j}\frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\overline{u_j'u_i'}\right) - \underbrace{\overline{\frac{\partial u_j'}{\partial x_j}u_i'}}_{=0} =$$

$$= -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \underbrace{\nu\nabla^2\overline{u_i}}_{\frac{1}{\rho}\frac{\partial}{\partial x_j}(\overline{\tau_{ij}})}$$

"Reynolds averaged equations"

Usually drop $'$ and $\overline{u_i} = U_i$

$$\begin{cases} \dfrac{\partial U_i}{\partial t} + U_j \dfrac{\partial U_i}{\partial x_j} = -\dfrac{1}{\varrho}\dfrac{\partial P}{\partial x_i} + \dfrac{1}{\varrho}\dfrac{\partial}{\partial x_j}\left[\overline{\tau}_{ij} - \varrho\overline{u_i u_j}\right] \\ \dfrac{\partial U_k}{\partial x_k} = 0 \end{cases}$$

$$\overline{\tau}_{ij} = 2\mu E_{ij} \quad ; \quad E_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

$-\varrho\overline{u_i u_j}$ Reynolds stress $\gg \overline{\tau}_{ij}$ in turbulent flow
usually

adds on to stress

unknown correlation for random motion
need model for, or <u>new equation</u>

$\downarrow$

new unknown correlations
$\therefore$ closure problem

Reynolds stress tensor $-\varrho R_{ij} = -\varrho\overline{u_i u_j} =$

$$= -\varrho \begin{bmatrix} \overline{u^2} & \overline{uv} & \overline{uw} \\ \overline{uv} & \overline{v^2} & \overline{vw} \\ \overline{uw} & \overline{vw} & \overline{w^2} \end{bmatrix}$$

Average turbulent mechanical energy / unit mass

$$\overline{k} = \frac{1}{2}\overline{(u^2 + v^2 + w^2)} = \frac{1}{2}\left(\overline{u^2} + \overline{v^2} + \overline{w^2}\right) = \frac{1}{2}\overline{u_i u_i} = \frac{R_{ii}}{2}$$

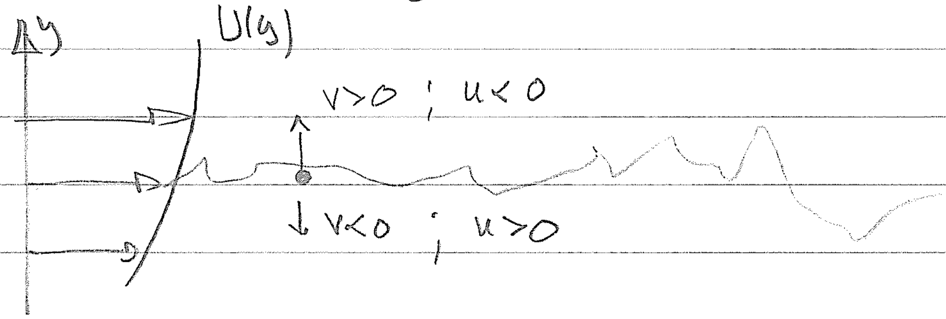
Isotropic part $\varrho\dfrac{2\overline{k}}{3}$ adds to pressure P.

Deviatoric part adds to viscous stress

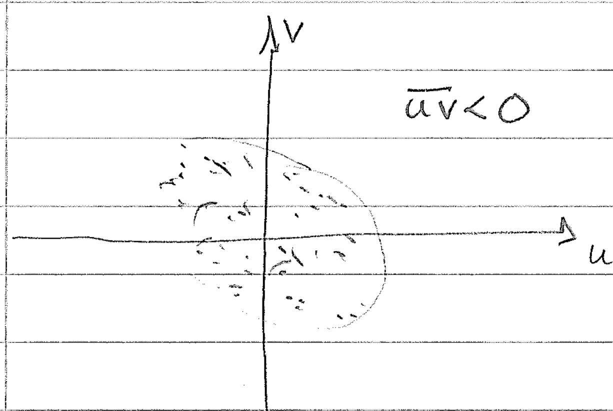
Average total shear stress

$$\overline{\tau_{xy}} - \rho\overline{uv} = \mu\left(\frac{\partial U}{\partial y} + \underbrace{\frac{\partial V}{\partial x}}_{=0}\right) - \rho\overline{uv}$$

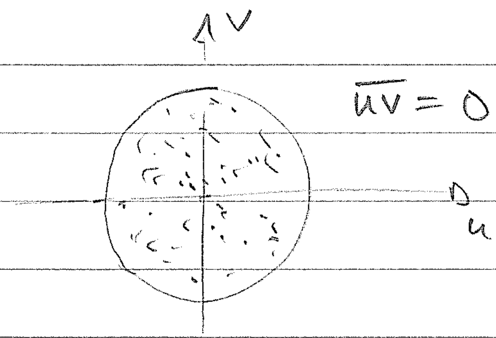
pure shear flow



u & v are correlated



uncorrelated



x-momentum flux in y-direction

$$\rho(U+u)(V+v) = \rho UV + \rho Uv + \rho uV + \rho uv$$

average $\overline{\rho(U+u)(V+v)} = \underbrace{\rho UV}_{\text{flux due to mean flow}} + \underbrace{\rho\overline{uv}}_{\text{flux due to fluctuations}}$

{Discuss "train analogy"}

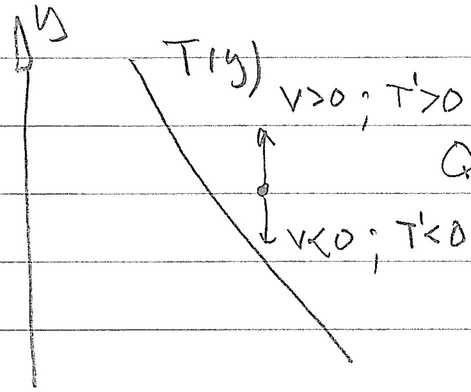
Reynolds averaged heat equation.

$T(\underset{\sim}{x},t) = \overline{T}(\underset{\sim}{x},t) + T'(\underset{\sim}{x},t)$

average velocity $U_i(\underset{\sim}{x},t)$

fluctuating velocity $u_i(\underset{\sim}{x},t)$

$$\varrho c_p \frac{D\overline{T}}{Dt} = -\frac{\partial Q_j}{\partial x_j} = -\frac{\partial}{\partial x_j}\Big(\underbrace{-k\frac{\partial \overline{T}}{\partial x_j}}_{\substack{\text{molecular heat} \\ \text{flux density}}} + \underbrace{\varrho c_p \overline{u_j T'}}_{\substack{\text{averaged turbulent heat} \\ \text{flux density} \\ \text{due to turb. fluctuat.}}}\Big)$$



$$Q_y = \underbrace{-k\frac{\partial \overline{T}}{\partial y}}_{>0} + \varrho c_p \underbrace{\overline{v T'}}_{>0}$$